Condorset’s paradox and probability of its occurence

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49% vote for A, tolerate B and hate C

30% vote for C, tolerate B and hate A

21% vote for B, tolerate C and hate A

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Simple majority: A wins – 49% of voters are satisfied.

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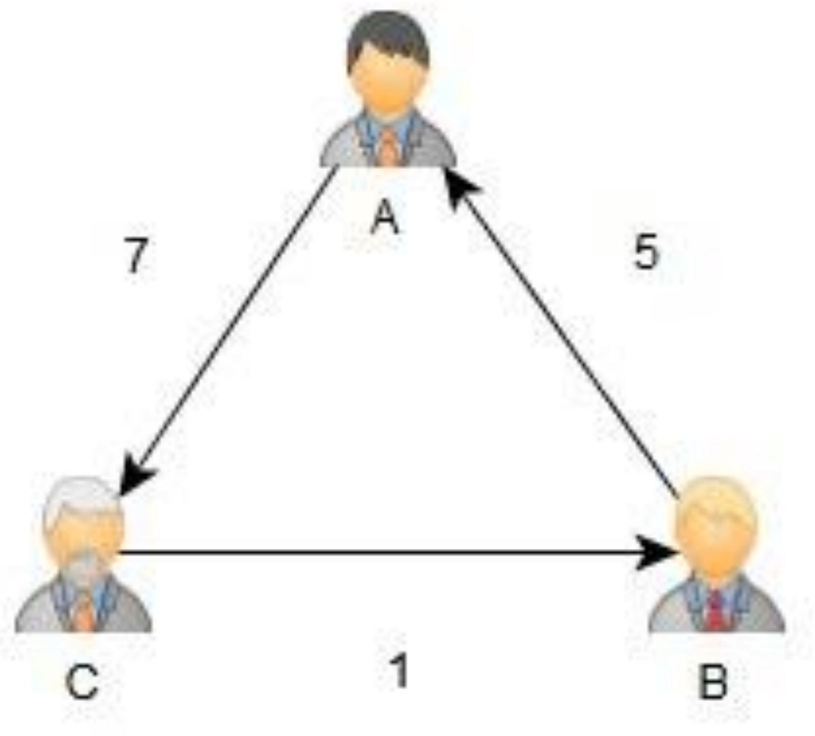
Two-round system: C wins – 51% satisfied.

A>B>C	5
A>C>B	4
B>A>C	2
B>C>A	8
C>A>B	8
C>B>A	2



B beats C by 1 vote

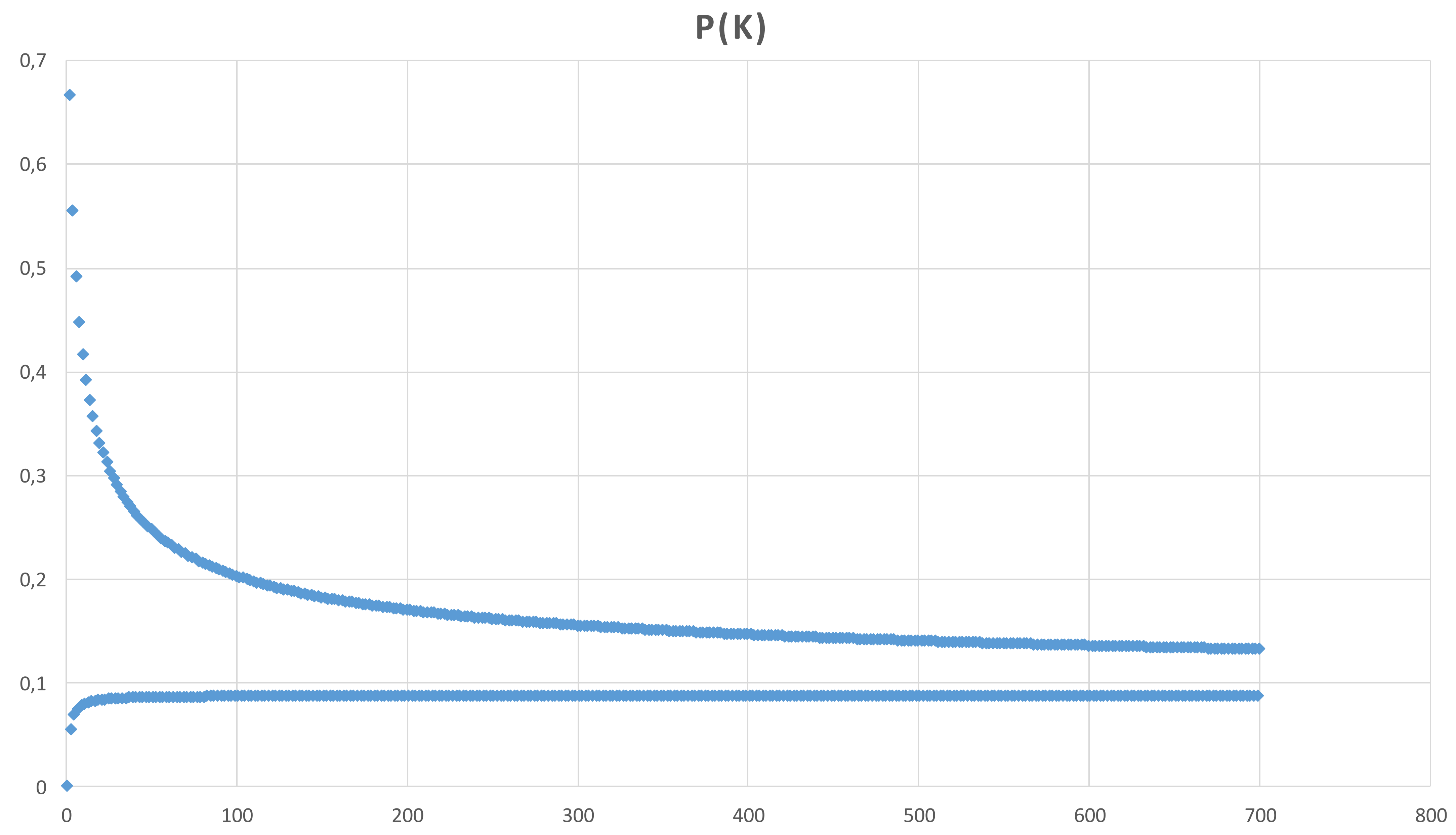
C beats A by 7 votes

A beats B by 5 votes

A>B>C	a
A>C>B	b
B>A>C	c
B>C>A	d
C>A>B	e
C>B>A	f

A beats B by $a+b+e-c-d-f$ votes
A beats C by $a+b+c-d-e-f$ votes

$$p(K)=1-3\cdot\sum_{i=0}^{M}\sum_{j=0}^{M-i}\sum_{k=0}^{M-i}\frac{N!}{i!\cdot j!\cdot k!\cdot(N-i-j-k)!}\cdot\left(\frac{1}{3}\right)^{i}\cdot\left(\frac{1}{6}\right)^{j}\cdot\left(\frac{1}{6}\right)^{k}\cdot\left(\frac{1}{3}\right)^{N-i-j-k};\quad M=\left[\frac{N-1}{2}\right]$$


P(K)
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
0
100
200
300
400
500
600
700
800