Star Clustering by Primal-Dual Variable neighborhood search

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(joint work with J Brimberg, P Hansen and D Urosevic)

- Variable neighborhood search algorithms
  - ▷ VN Decomposition search (VNDS);
  - ▷ Primal-Dual VNS;
  - ▷ Simple plant location problem
  - ▷ p-median problem
- Application:
  - ▷ Data Mining, Color image quantization;
- Conclusions

Variable neighborhood decomposition search (VNDS)

- VNDS extends the basic VNS into a two-level VNS scheme based upon decomposition of t problem.
- t_d is an additional parameter and represents the running time given for solving decompos (smaller sized) problems by VNS.
- For ease of presentation, but without loss of generality, we assume that the solution represents the set of some elements.
- In Step 4 we denote with y a set of k solution attributes present in x' but not in $(y = x' \setminus x)$.

Function `VNDS` $(x, k_{max}, t_{max}, t_d)$
repeat

$k \leftarrow 2$
repeat

$x' \leftarrow$ `Shake` (x, k); $y \leftarrow x' \setminus x$
$y' \leftarrow$ `VNS`(y, k, t_d); $x'' = (x' \setminus y) \cup y'$
$x''' \leftarrow$ `FirstImprovement`(x'')
`NeighborhoodChange`(x, x''', k)
until $k = k_{max}$;
until $t > t_{max}$;

Primal-dual VNS

- For most modern heuristics, the difference in value between the optimal solution and t obtained one is completely unknown.
- Guaranteed performance of the primal heuristic may be determined if a lower bound on t objective function value is known.
- When the dimension of the problem is large, even the relaxed problem may be impossible solve exactly by standard commercial solvers.
- Therefore, it seems a good idea to solve dual relaxed problems heuristically as well. In t way we obtain guaranteed bounds on the primal heuristics performance.
- The next problem arises if we want to reach an exact solution within a Branch and bou framework, since having the approximate value of the relaxed dual does not allow us branch easily, e.g., by exploiting complementary slackness conditions.
- Thus, the exact value of the dual is necessary.
- In Primal-dual VNS (PD-VNS) one possible general way to attain both the guaranteed boun and the exact solution is proposed.

Function `PD-VNS` $(x, k'_{max}, k_{max}, t_{max})$
`BVNS` $(x, k'_{max}, k_{max}, t_{max})$ /* Solve primal by VNS */
`DualFeasible`(x, y) /* Find (infeasible) dual such that $f_P = f_D$ */
`DualVNS`(y) /* Use VNS do decrease infeasibility */
`DualExact`(y) /* Find exact (relaxed) dual */
`BandB`(x, y) /* Apply branch-and-bound method */

Primal-Dual VNS - cont.

- In the first stage an heuristic procedure based on VNS is used to obtain a near optim solution.
- In Hansen *et al.* (2007) it is shown that VNS with decomposition is a very powerful techniq for large-scale simple plant location problems (SPLP) with up to 15 000 facilities and 15 0 users.
- In the second phase, this approach is designed to find an exact solution to the relaxed du problem.
- Solving SPLP is accomplished in three stages:
  - (i) find an initial dual solution (generally infeasible), using the primal heuristic soluti and complementary slackness conditions;
  - (ii) improve the solution by applying VNS to the unconstrained nonlinear form of t dual;
  - (iii) solve the dual exactly using a customized "sliding simplex" algorithm which appl "windows" to the dual variables, substantially reducing the size of the problem.
- In all the problems tested, including instances much larger than previously reported in t literature, the procedure was able to find the exact dual solution in reasonable computi time.
- In the third and final phase armed with tight upper and lower bounds, obtained respectiv from the heuristic primal solution in phase one and the exact dual solution in phase tw we apply a standard branch-and-bound algorithm to find an optimal solution of the origin problem.

- The lower bounds are updated with the dual sliding simplex method and the upper bounc whenever new integer solutions are obtained at the nodes of the branching tree.
- In this way it is possible to solve exactly problem instances with up to $7\,000 \times 7\,000$ f uniform fixed costs and $15\,000 \times 15\,000$ otherwise.