Patrolling Games

Katerina Papadaki

LSE

Steve Alpern

University of Warwick

Alec Morton

University of Strathclyde

Tom Lidbetter

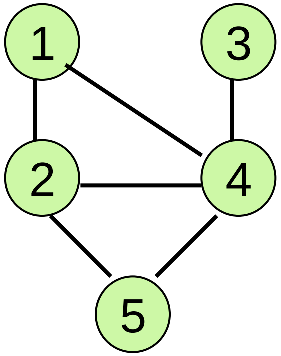
LSE

Outline

- Introduce Patrolling Games on a graph.
- Applications
- Results for all graphs.
- Strategy reduction techniques.
- Types of strategies.
- Solutions for special graphs.
- LP formulations
- The discrete line
- The continuous line
- Current and future work

Patrolling Game on a Graph

Graph: Q=(N,E)
Nodes: N ={1,2,…,n}
Edges: E



T = time horizon of the game
t = 1,…,T

Players

Attacker: picks a node i and time τ to perform the attack and needs m uninterrupted periods at the node for the attack to be successful

Patroller: picks a walk w on the graph that lasts T time periods and is successful if the walk intercepts the Attacker during the attack.

Pure Strategies

Attacker: (i, τ)
Patroller: w

Mixed Strategies:

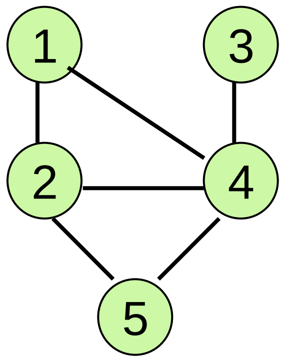
Playing (i, τ) with probability p(i, τ)
Playing w with probability p(w)

We assume: $T \geq m$

Patrolling Game on a Graph

Space-time Network:

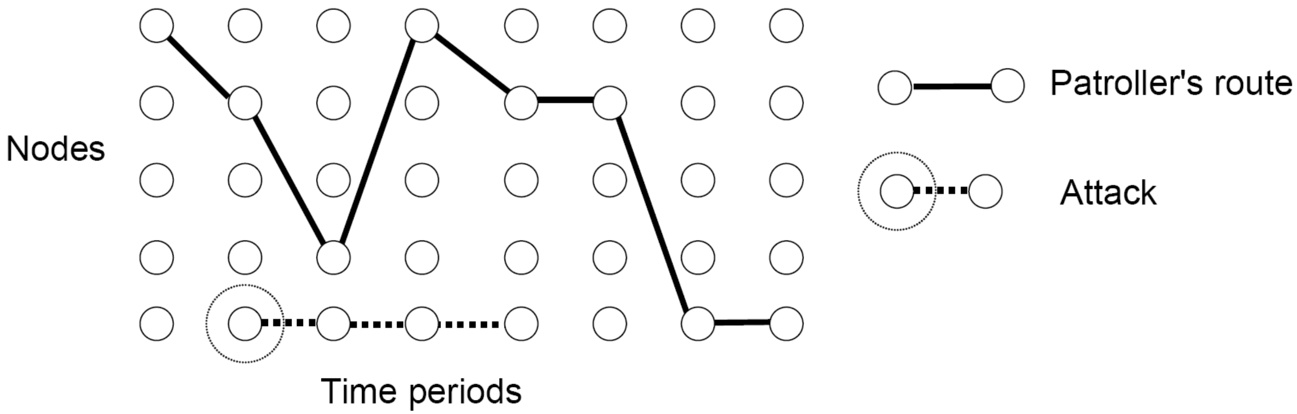
n=5, T=8, m=4



patroller picks: w = 1-2-4-1-2-2-5-5

attacker picks: (i, τ) =(5,2)

a. Successful Attack

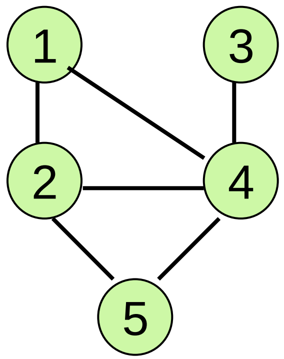


Since the patroller's walk does not intercept the attacker the attack is successful.

Patrolling Game on a Graph

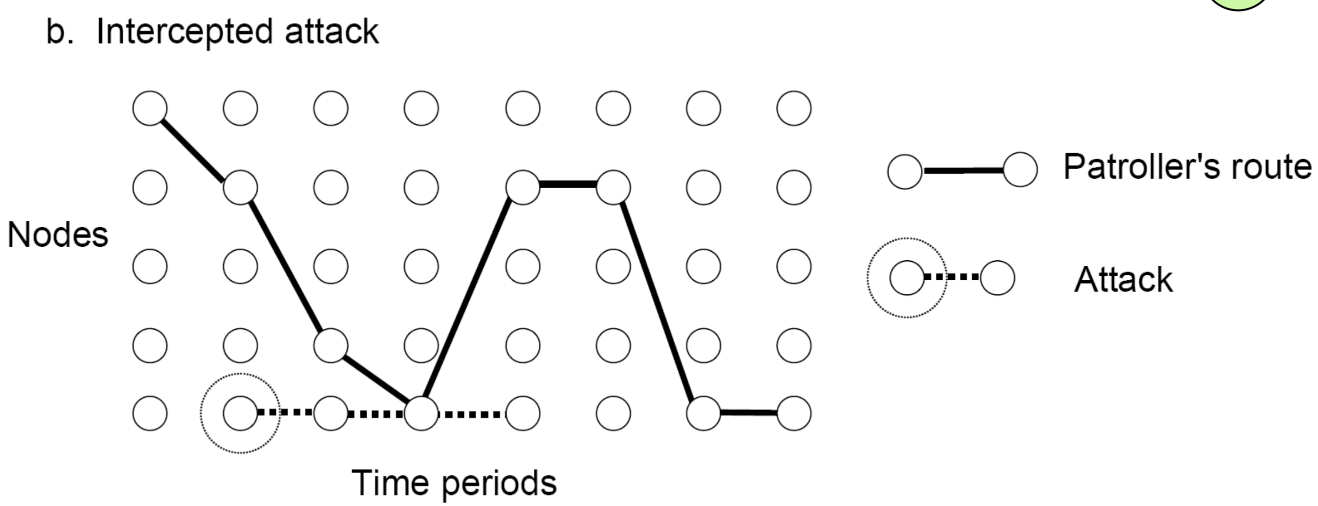
Space-time Network:

n=5, T=8, m=4



patroller picks: w = 1-2-4-5-2-2-5-5

attacker picks: (i, τ) =(5,2)



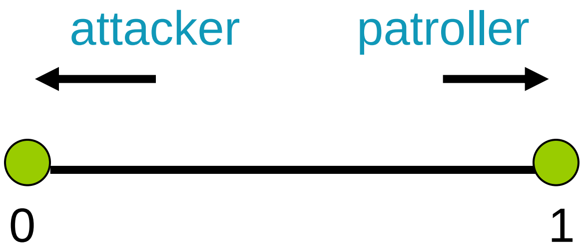
Since the patroller’s walk intercepts the attacker the attack is not successful.

Patrolling Game on a Graph

The game is a zero-sum game with the following payoff:

$$\text{Payoff to the patroller} = \begin{cases} 1 & \text{if } (i, \tau) \text{ is intercepted by } w \\ 0 & \text{otherwise} \end{cases}$$

Value of the game = probability that the attack is intercepted



We denote the value of the game V or V(Q, T, m).

Assumptions

We make some simplifying assumptions:

- The attacker will attack during the time interval:

  By patrolling as if an attack will take place, the patroller deters the attack on this network and gives an incentive to the attacker to attack another network.

- The nodes have equal values:

  Nodes with different values can be easily modelled in the mathematical programming formulations of the game. All games that can be solved computationally, can also be solved using different valued nodes.

- The nodes on the network are equidistant:

  This can also be modelled in the mathematical programming formulations.

Applications

- Security guards patrolling a museum or art gallery.
- Antiterrorist officers patrolling an airport or shopping mall.
- Patrolling a virtual network for malware.
- Police forces patrolling a city containing a number of potential targets for theft, such as jewellery stores.
- Soldiers patrolling a military territory.
- Air marshals patrolling an airline network.
- Inspectors patrolling a container yard or cargo warehouse.

Types of Games

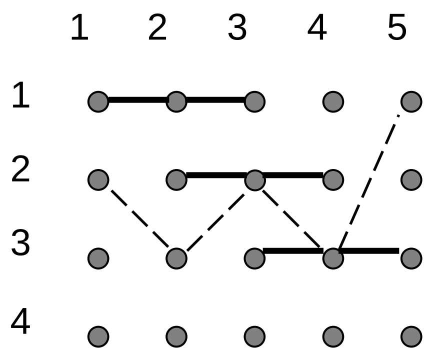
- Patrolling a Gallery:

  T = fixed shift

  (e.g. one working day)

  We call this the one-off game and denote it G^o with value V^o.

one-off game:



attacker can only start attack at nodes 1,2,3.

- Patrolling an Airport :

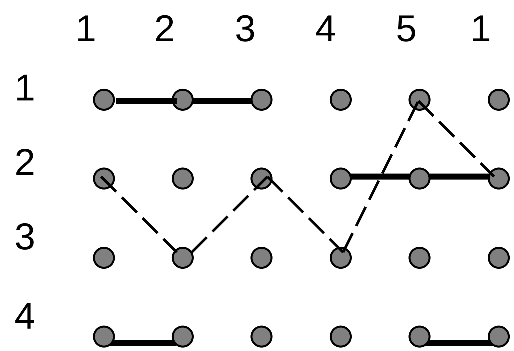
  continuous patrolling

  We call this the periodic game

  and we let T be the period.

  We denote it with G^p, V^p.

periodic game:



patroller must return to starting node.

Results for all Graphs

Monotonicity Results

1. The Value of the game is non-decreasing in m:

$$V(Q,T,m) \leq V(Q,T,m') \text{ for } m \leq m'$$

- the longer the attacker takes to complete the attack, the higher the probability of the attack being intercepted.

2. The Value of the game is non-decreasing in the number of edges |E|:

$$V(Q,T,m) \leq V(Q',T,m)$$

$$E \subseteq E'$$
$$N = N'$$

- with more edges there are more patrolling paths and thus better for the patroller

Results for all Graphs

Monotonicity Results

3. The Value of the periodic game is less than or equal to the value of the one-off game:

$$V^p(Q, T, m) \leq V^o(Q, T, m)$$

- the one-off game has more patroller strategies and less attacker strategies, so it is better for the patroller

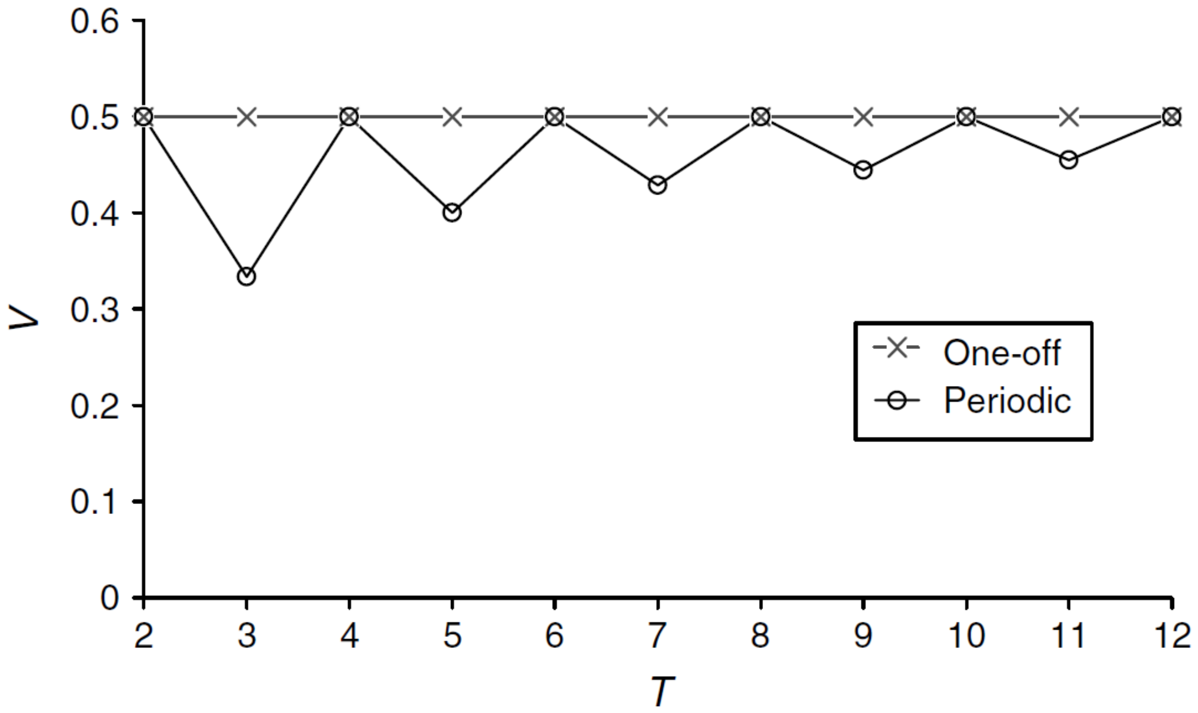
Results for all Graphs

Monotonicity Results

4. The Value of the one-off game is non-increasing in T:

$$V^o(T+1) \leqslant V^o(T)$$

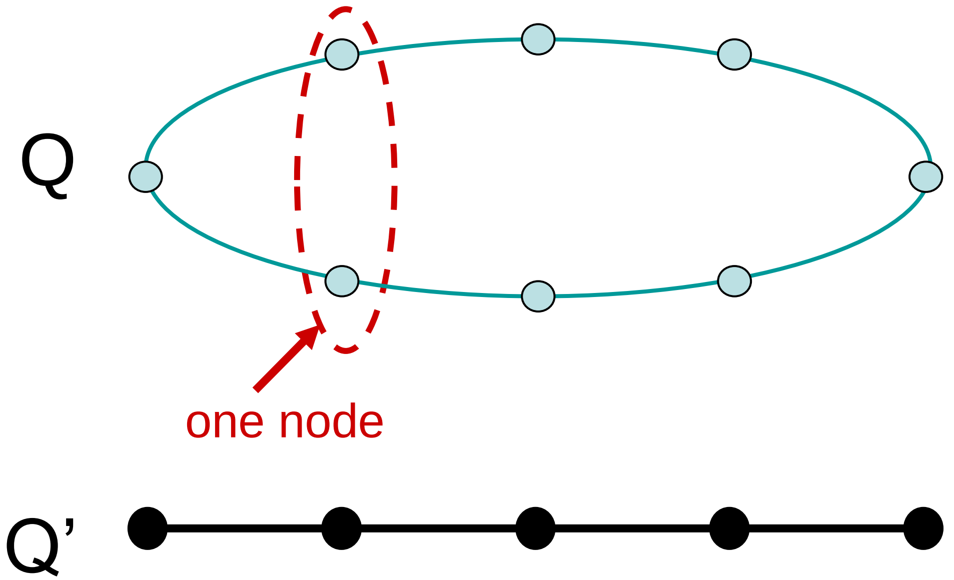
The Value of the periodic game comes closer to the one-off game as T goes to infinity:



C_4, m=2

Results for all Graphs

Node Identification



5. If Q’ is obtained from Q by node identification, then

$$V(Q') \geq V(Q)$$

since any patrol on Q that intercepts an attack, has a corresponding patrol on Q’ that intercepts the same attack, thus Q’ is at least as good as Q for the patroller

Results for all Graphs

Bounds on Value

6. We have: $$\frac{1}{n} \leq V \leq \frac{m}{n}$$

The patroller can guarantee the lower bound by:

- picking a node equiprobably and
- waiting there

The attacker can guarantee the upper bound by:

- fixing an attack time interval and
- attacking at a node equiprobably during that interval;
- out of these n pure attacker strategies, the patroller can intercept at most m of them, in a time interval of length m

The lower bound can be achieved for the disconnected graph D_n with n nodes:

$$V(D_n, T, m) = \frac{1}{n}$$

Results for all Graphs

Game with m=1

7. For the special case where K_n is the complete graph with n nodes, Ruckle (1983) has shown that:

$$V^o(K_n, T, 1) = \frac{1}{n}$$

Hence, $$\frac{1}{n} = V^o(K_n, T, 1) \geq V(Q, T, 1) \geq \frac{1}{n}$$

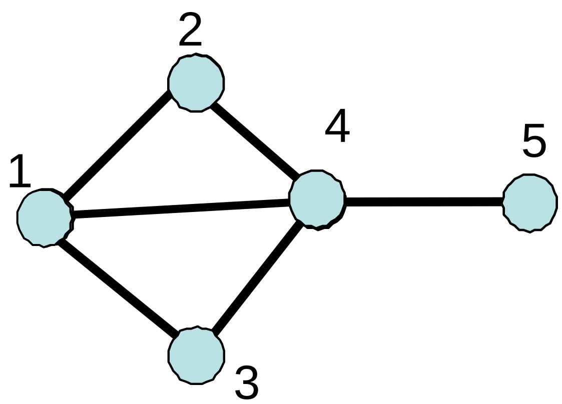
Result: For m=1: $V(Q, T, 1) = \frac{1}{n}$ for all Q and T

Henceforth we assume $m \geq 2$

Strategy Reduction Techniques

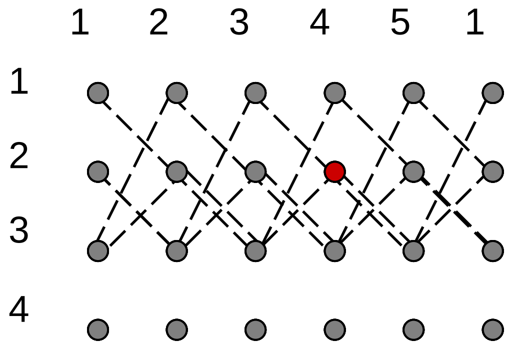
Symmetrization

Graph symmetrization:



Adjacency preserving bijections on Q:
- Nodes 2 and 3 are equivalent
- There exists an optimal attack strategy that attacks nodes 2 and 3 equiprobably

Time symmetrization:



For the periodic game,

- the time shifted patrols are equivalent
- the attack intervals on the same node are equivalent under some rotation of the time cycle.
- we only need to consider the attack node not the attack interval.

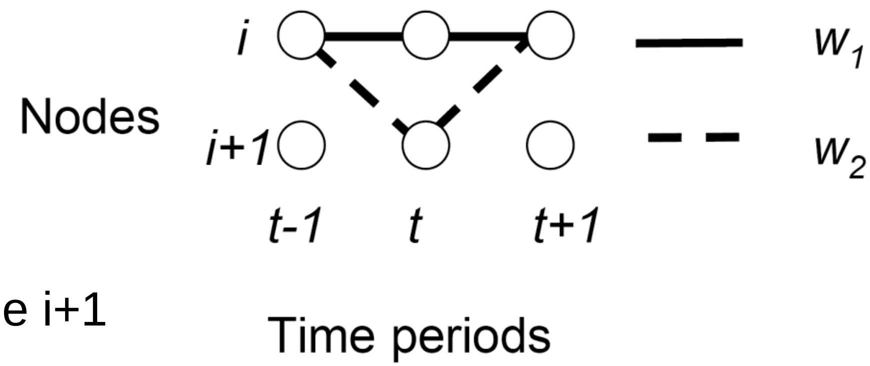
Symmetrical Strategies: mixed strategies which give equal probability to equivalent strategies

Strategy Reduction Techniques

Dominance

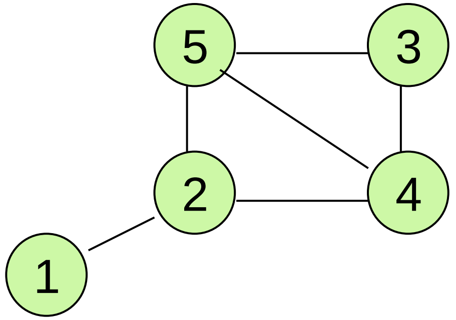
For $m \geq 2$:

Walks w_1, w_2 same except on (t-1, t, t+1).
• walk w_2 dominates w_1:
If w_1 intercepts an attack (i, τ) then w_2 also intercepts (i, τ) and at least one more at node i+1



For $m \geq 3$:

Let 1 be a leaf node connected to node 2:
We call node 2 a penultimate node.
• the attacker should not attack at penultimate nodes.



From above, walk w does not duel at a node for 3 consecutive periods.

If (2, τ) wins against a patrol then (1, τ) will also win but (1, τ) also wins against patrols that pass only through 2.

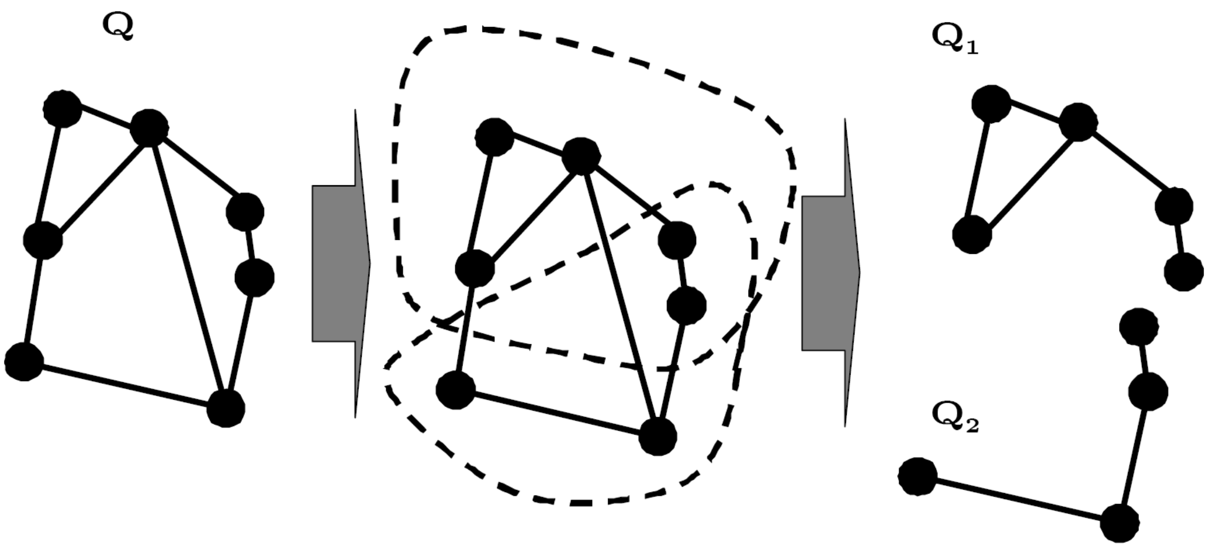
Strategy Reduction Techniques

Decomposition

The set of graphs $Q_k = (N_k, E_k)$, $k = 1 \ldots K$ is a decomposition of graph Q if:

$\cup N_k = N$

If both $i, j \in N_k$ and $(i, j) \in E$, then $(i, j) \in E_k$.



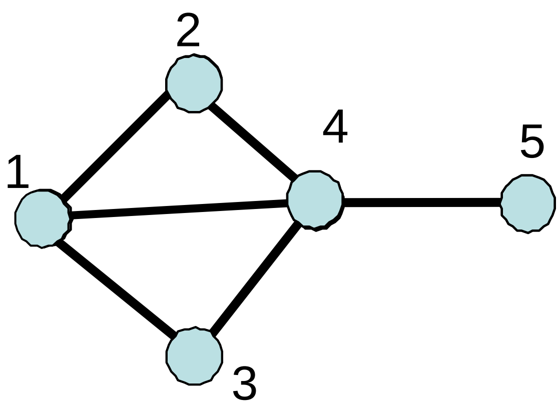
Decomposition Result: We have $V(Q) \geq \dfrac{1}{\sum_k 1/V(Q_k)}$,

which holds with equality if the Q_k are disjoint in Q .

Proof Techniques: example

Kite Graph

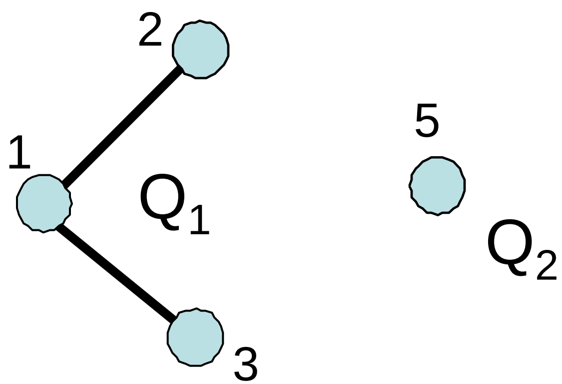
Periodic game on Q, with T=3 and m=3:



From dominance, we know that attacker would never attack at penultimate node 4, since it is always better to attack at the adjacent leaf node 5.

No feasible patroller strategy that visits both node 5 and any one of 1,2 or 3.

Without node 4 the graph decomposes into two graphs Q_1 and Q_2 shown below.



From decomposition we have:

$$V^p(Q) = \frac{1}{1/V^p(Q_1) + 1/V^p(Q_2)} = \frac{1}{2+1} = \frac{1}{3}$$

$$V^p(Q_1) = \frac{1}{2} \qquad V^p(Q_2) = 1$$

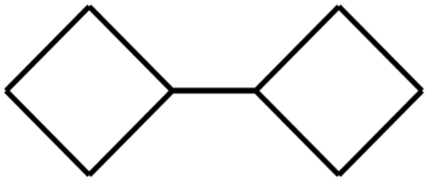
Generic Strategies

Uniform Attacker Strategy

The attacker attacks equiprobably over all time intervals and over all nodes. This guarantees the attacker the upper bound of m/n.

Attacker's Diametrical Strategy

d(i,j) = minimum number of edges between nodes i and j
d = diameter of Q = maximum d(i,j) for all pairs i, j.



The attacker picks random attack time τ and attacks equiprobably nodes i and j that have distance d.

We have: $V \leq \max\left[m/2d, 1/2\right]$

The diametrical strategy guarantees the above upper bound:

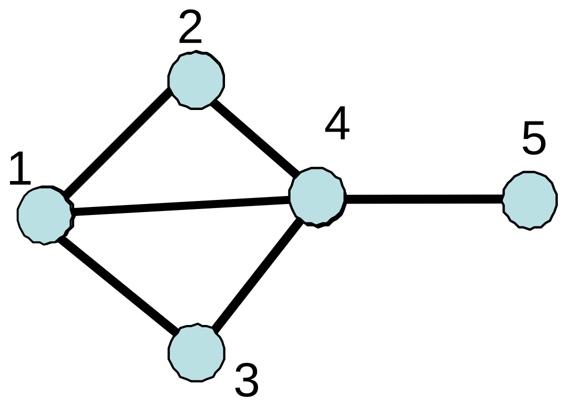
- If m, T are large as compared to d, the best the patroller can do against the diametrical strategy is to go back and forth across the graph diameter (m/2d)

- If d is large as compared to m, T, the best the patroller can do against the diametrical strategy is to stay at the diametrical nodes and win half the time (1/2).

Generic Strategies

Independent strategies

Independent set: set of nodes where no simultaneous attacks at any two nodes of the set can be covered by the same patrol during any fixed time interval (of length m).



Periodic Game for Kite Graph with T=3, m=3.

Independent Sets: {2,3} {1,5} {2,3,5}
(since the patrol needs to return to the initial node)

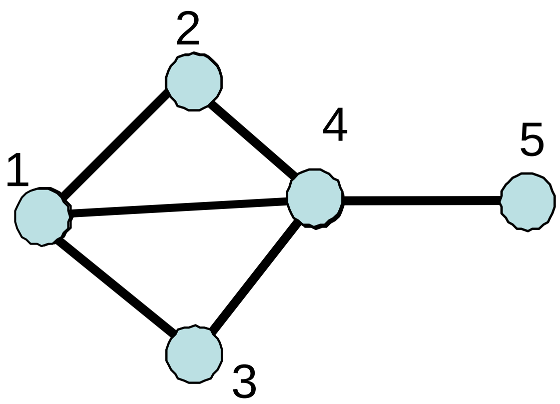
Independence number I : the size of the maximal independent set.

Independent attack strategy: attack equiprobably nodes in the maximal independence set.

Generic Strategies

Covering strategies

Intercepting Patrol: a patrol w that intercepts every attack on a node that it contains.



Periodic Game for Kite Graph with T=3, m=3.

Intercepting patrols: 1-1-2-1
1-3-4-1
4-5-5-4

covering set

Covering set of Q: a set of intercepting patrols such that every node of Q is contained in at least one of the patrols.

Covering number J: the size of the minimal covering set.

Covering patrol strategy: choose equiprobably from the minimal set of covering patrols.

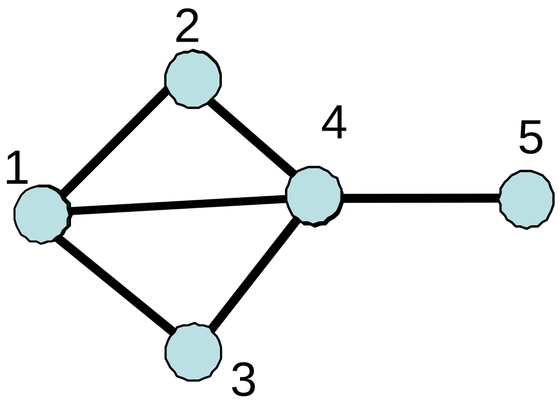
Generic Strategies

Independent and Covering strategies

$$\frac{1}{\mathcal{J}} \leq V \leq \frac{1}{\mathcal{I}}$$

Upper bound: independent attack strategy
Lower bound: covering patrol strategy

When I = J we can determine the value of the game:



Periodic Game for Kite Graph with T=3, m=3.

Maximal Independent Set = {2,3,5}
Minimal Covering Set = {1-1-2-1, 1-3-4-1, 4-5-5-4}

We have I = J = 3:

V(Q) = 1/3

Independence/Covering strategies

Example: The line

m=3, L7 (n=7)

Maximum Independence set
= {1,4,7}
I = 3 ⇒ V ≤ 1/3

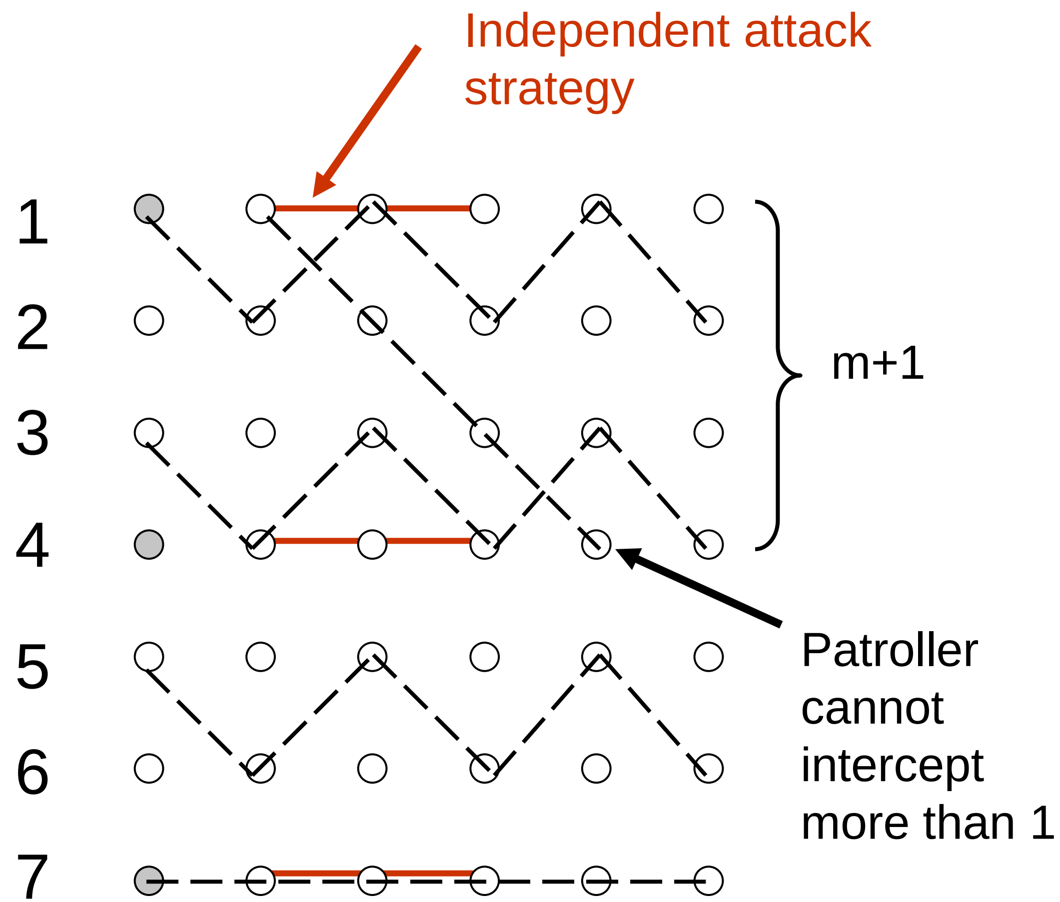
Minimum covering set of walks:

J = 4 ⇒ V ≥ 1/4

$\boxed{1/4 \leq V \leq 1/3}$

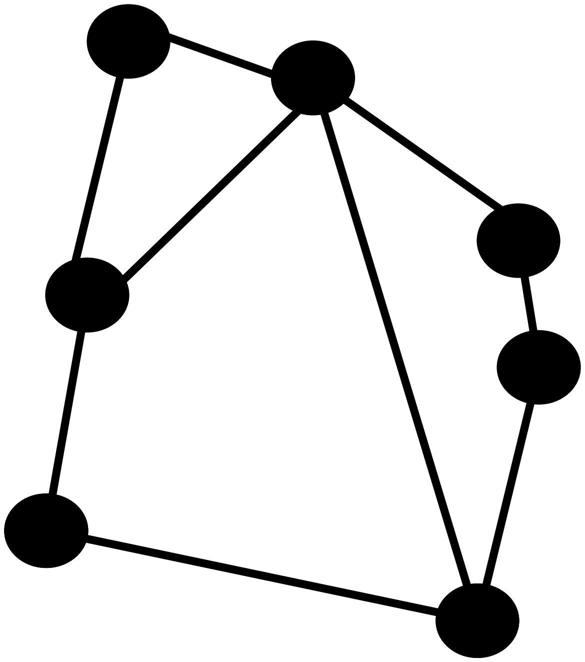
patroller
can do better

optimal



Solutions for Special Graphs

Hamiltonian Graph



Any graph with a Hamiltonian cycle:

- Value (of V^o) is $\frac{m}{n}$
- Patroller - Random Hamiltonian patrol: pick a node at random and follow the Hamiltonian cycle in a fixed direction

  For any attack interval, the nodes visited by the patroller form an m-arc of the Hamiltonian cycle, which contains attack node i with probability m/n.

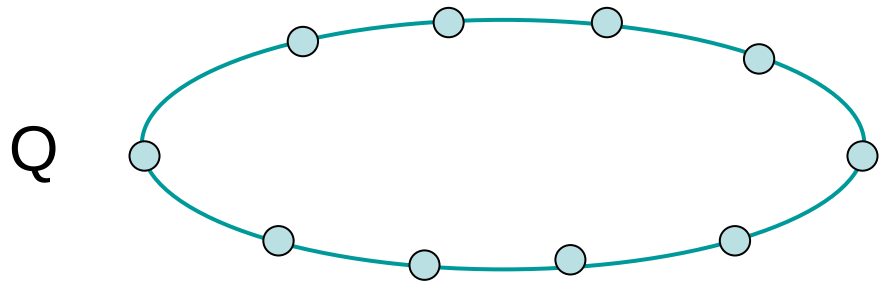
- Attacker - uniform attacking strategy, attack equiprobably over time and nodes

1. $V^o = \frac{m}{n}$;
2. $V^p \le \frac{m}{n}$ *with equality if* T *is a multiple of* n, *and* $V^p \to m/n$ *as* $T \to \infty$.

Solutions for Special Graphs

Hamiltonian Graphs: example

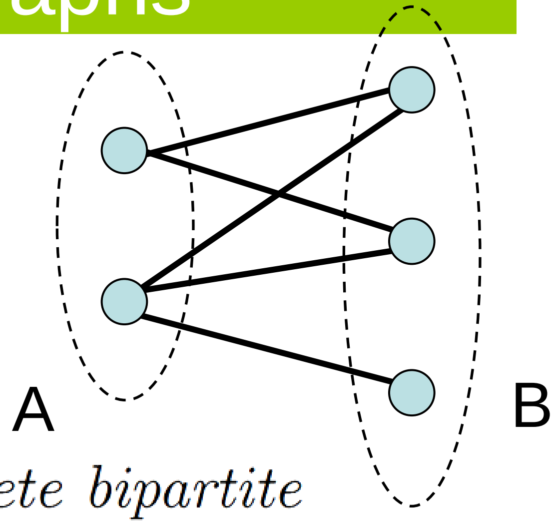
Periodic game on Q, T=10, m=4:



C_{10} has a Hamiltonian cycle and T=10 is a multiple of n=10:

$$V(C_{10}) = \frac{m}{n} = \frac{4}{10}$$

Solutions for Special Graphs



Bipartite Graphs $K_{a,b}$

- $a = |A|, b = |B|, a \le b$

We assume: $m \le 2b$

1. $V^o \le m/(2b)$, *with equality if* Q *is complete bipartite*

2. $V^p \le m/(2b)$, *with equality if* Q *is complete bipartite and* T *is a multiple of* $2b$.
   if Q *is complete bipartite then* $V^p \to m/(2b)$ *as* $T \to \infty$.

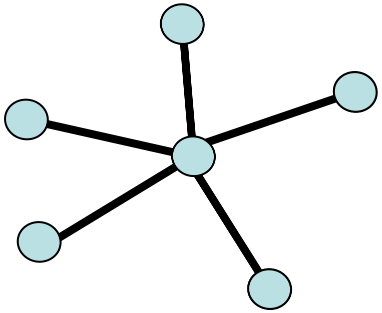
Attacker can guarantee $m/2b$, if he fixes the attack interval and attacks equiprobably on each node of the larger set B.

$$V^o(K_{a,b}) \geqslant V^o(K_{b,b}) = m/2b$$

When Q is complete bipartite and a=b, there exists a Hamiltonian cycle and the value is achieved; $K_{a,b}$ can be obtained from $K_{b,b}$ by node identification.

Solutions for Special Graphs

Bipartite Graphs: The Star Graph



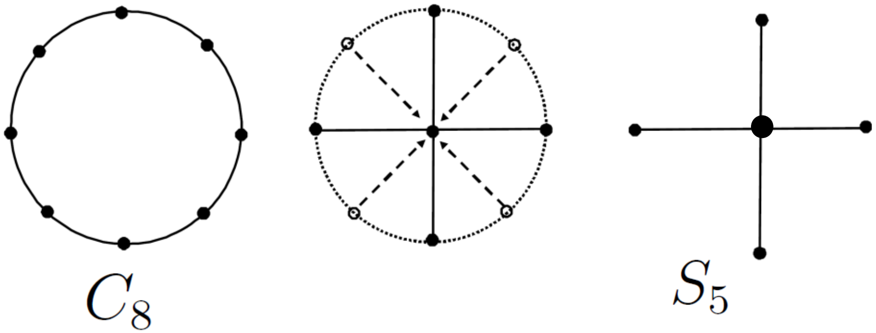
S_n : star graph with n nodes

$C_{2(n-1)}$: cycle graph with 2(n-1) nodes

a = 1, b = n-1
T is a multiple of 2(n-1)

By node identification:

$$V(S_n) \geq V(C_{2(n-1)}) = \frac{m}{2(n-1)}$$



Since S_n is bipartite:

$$V(S_n) \leq \frac{m}{2b} = \frac{m}{2(n-1)}$$

Thus, $V(S_n) = \dfrac{m}{2(n-1)}$

- attack leaf nodes equiprobably
- patrols leaf nodes every second period

Mathematical Programming

LP Formulation

Let A be the set of attacker strategies for G(Q,T,m)

Patroller's game:

$$\max_{x,v} v$$

$$s.t. \quad \sum_{w \in \mathcal{W}} P(w,a)x(w) \geq v \quad \text{for all} \quad a \in \mathcal{A}$$

$$\sum_{w \in \mathcal{W}} x(w) = 1$$

$$x(w) \geq 0, \quad \text{for all} \quad w \in \mathcal{W}$$

Num. of attacker strategies:	n	(periodic game)
(constraints)	n(T-m+1)	(one-off game)
Num. of patroller strategies:	number of circuits of length T	(periodic game)
(variables)	number of paths of length T	(one-off game)

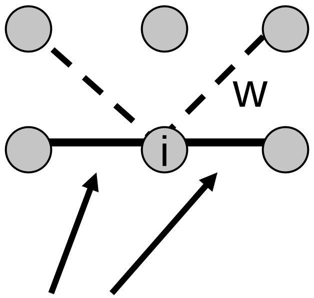
Flow formulation

Case: Periodic game, Q bipartite, m=2, T even

Proposition: A walk that dwells at a node for more than one period is dominated walks that do not dwell at a node.

Thus, we can count the number of attacks intercepted:

- each visit at a node will intercept exactly two attacks
- the attacks intercepted from visits to different nodes are disjoint

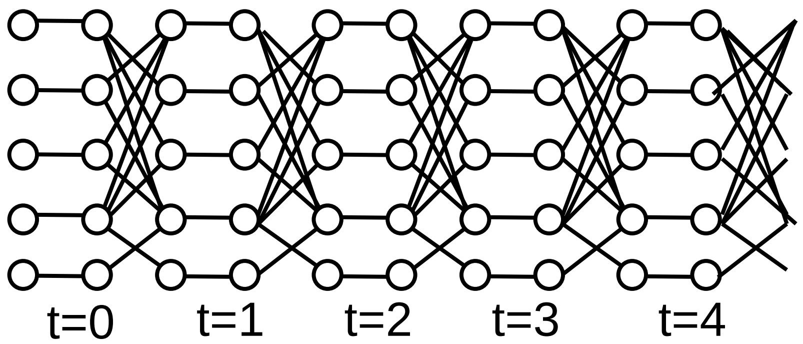


attacks intercepted from the visit of walk w to node i and not intercepted by any other visit of w

Flow formulation

Case: Periodic game, Q bipartite, m=2, T even

kite graph, T=5



Split space-time network Q_S:
- introduce split arcs
- no arc joining the same node in consecutive time periods

$$N(i,e) = \begin{cases} 1 & \text{if } e \text{ is a split arc for node } i, \\ 0 & \text{otherwise.} \end{cases}$$

- i nodes in Q
- e arc of Q_S:

$$B(e,w) = \begin{cases} 1 & \text{if arc } e \text{ of } Q_S \text{ is on the walk } S \text{ of } Q_S, \\ 0 & \text{otherwise.} \end{cases}$$

$NB(i,w)$ = number of visits of walk w to node i during the time horizon

$2NB(i,w)$ = number of attacks at node i intercepted by walk w

Probability attack at node i is intercepted by w = $\dfrac{2}{T} NB(i,w)$

Flow formulation

Case: Periodic game, Q bipartite, m=2, T even

$$\max_{x,v} v$$

$$s.t. \quad \frac{2}{T} N B x \geq v$$

$$\sum_{w} x(w) = 1$$

$$x(w) \geq 0$$

- Substitute: Bx with z
- Then z(e) is the probability flow on arc e.
- Using flow conservation constraints: $Fz = 0$ we can guarantee that the flow z forms walks

$$\max_{z,v} v$$

$$s.t. \quad Fz = 0$$

$$\frac{2}{T} N z \geq v$$

$$\sum_{e \in \mathcal{S}} z(e) = 1$$

flow value equals 1

$$z(e) \geq 0$$

size of x: no. of walks
x gives probability of each walk
x is a flow on each walk

Flow formulation

Case: Periodic game, Q bipartite, m=2, T even

$$\max_{z,v} v$$

$$s.t. \quad Fz = 0$$

$$\frac{2}{T} N z \geq v$$

$$\sum_{e \in \mathcal{S}} z(e) = 1$$

$$z(e) \geq 0$$

num. of variables: (2E+n)T + 1
num of constraints: 2nT+n+1

Linear in the problem parameters.

We can solve games with large n and T.

Further, it is easy to introduce different attack values at each node.

Flow formulation

Case: Periodic game, Q bipartite, m=2, T even

Multi-valued Nodes

$$\min_{z,v} v$$

$$s.t. \quad Fz = 0$$

$$d - \frac{2}{T} DNz \leq v$$

$$\sum_{e \in \mathcal{S}} z(e) = 1$$

$$z(e) \geq 0$$

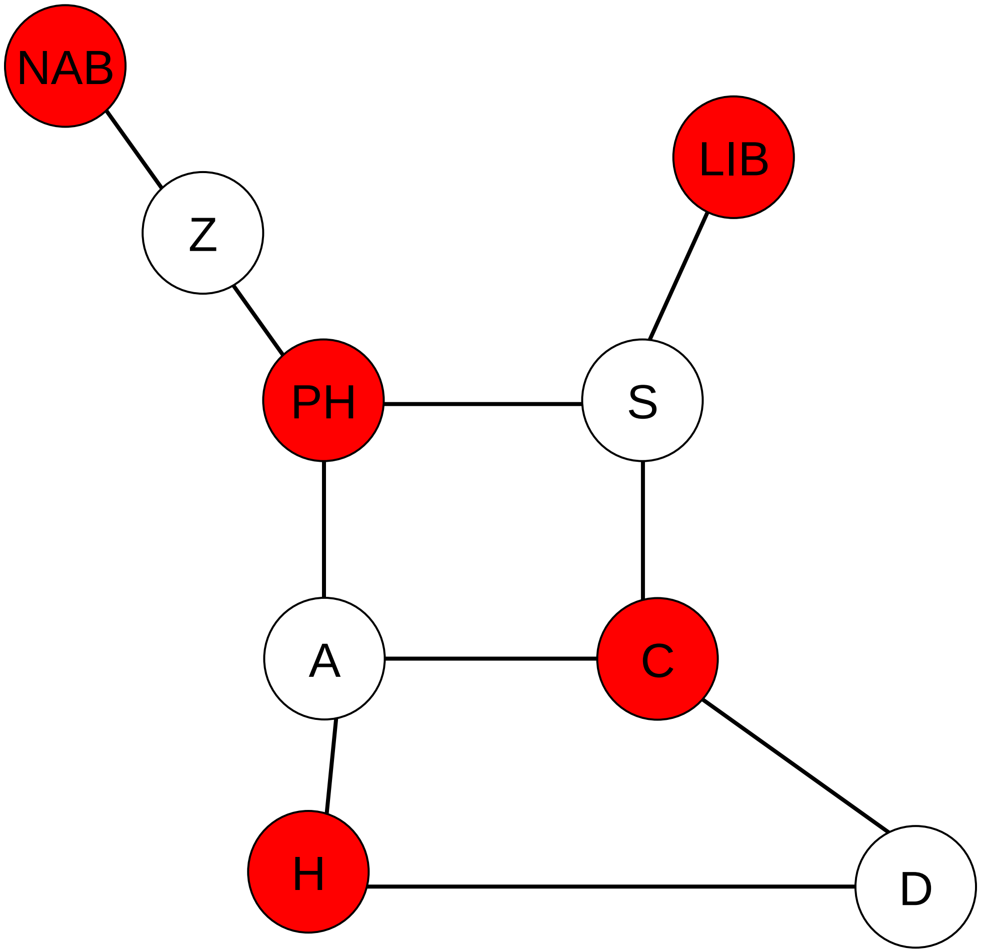
d = vector of node values
D = diagonal matrix with d on the diagonal

Reverse the payoff:
0 when attack is intercepted
d(i) when attack at node i is successful

Flow formulation

Single-valued Nodes: (value = 0 attack intercepted)

LSE network, m=2, T=20.



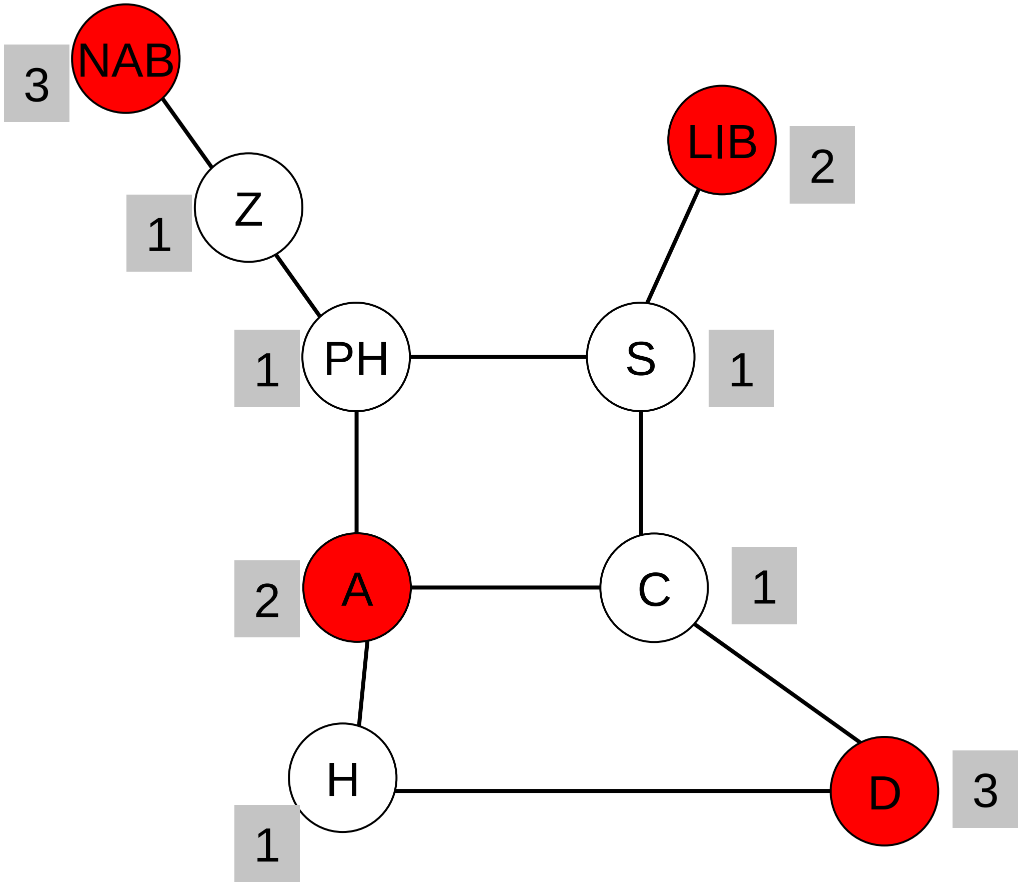
Optimal Attacker strategy:
attack red nodes equiprobably
with probability 1/5

Game Value = 4/5
(1 is best for attacker)

Flow formulation

Multi-valued Nodes (value = 0 attack intercepted)

LSE network, m=2, T=20.



Optimal Attacker strategy:

- attack NAB, D with prob. 2/10
- attack A, LIB with prob. 3/10

Game Value = 1.8
(0 is best for patroller)

The discrete line - results

$Case\ A\colon\ If\ n \leq m+1\colon$

$$V^o = \frac{m}{2(n-1)}$$

n small compared to m

$Case\ B\colon\ If\ n = m+2\ \ and\ n, m\ \ even,$

$$V^o = \frac{1}{2}$$

n similar compared to m

$Case\ C\colon\ If\ n = m+2\ \ and\ at\ least\ one\ of\ n, m\ \ odd,\ \ or\ n \geq m+3\colon$

$$V^o = \frac{m}{n+m-1}$$

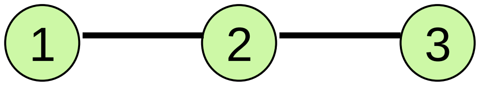
n large compared to m

We concentrate on the one-off game. The value for the periodic game is the same when either T goes to infinity, or when T is the appropriate multiple otherwise this is just an upper bound.

The discrete line – Case A

n small compared to m

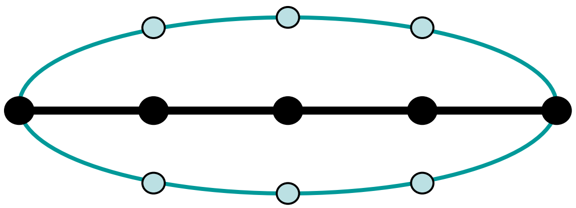
$$If\ n \leq m+1\ \ V^o = \frac{m}{2(n-1)}$$



- d = diameter = n-1
  The diametrical attacker strategy guarantees the upper bound for the attacker

- We use node identification, to show that
  the upper bound is achieved:

$$V(L_n) \geq V(C_{2(n-1)}) = \frac{m}{2(n-1)}$$

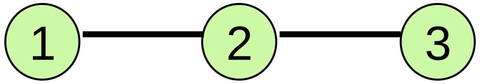


The Hamiltonian patrol on the cycle graph is equivalent to walking up and down the line graph (oscillation strategy).

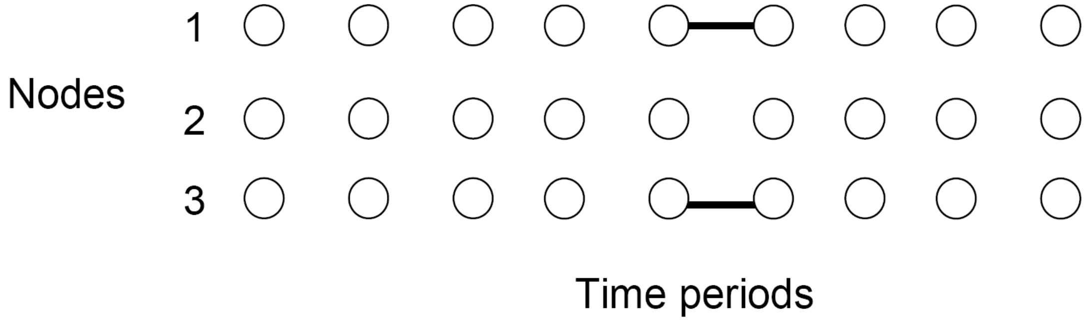
The discrete line – Case A

n small compared to m:

Consider L_3 the line graph with n=3. Let m=2.

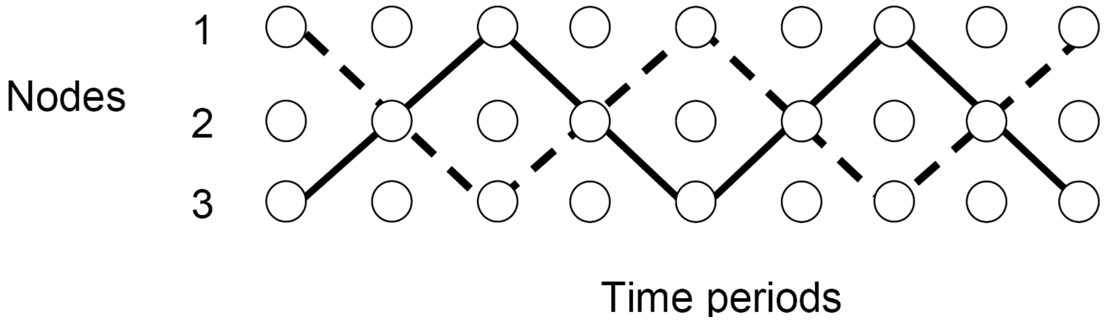


Attacker can guarantee ½ by attacking at the endpoints equiprobably: no walk can intercept both.



$$V \leq \frac{1}{2}$$

Patroller can guarantee ½ by playing equiprobably the following oscillations: every attack is intercepted by at least one oscillation.

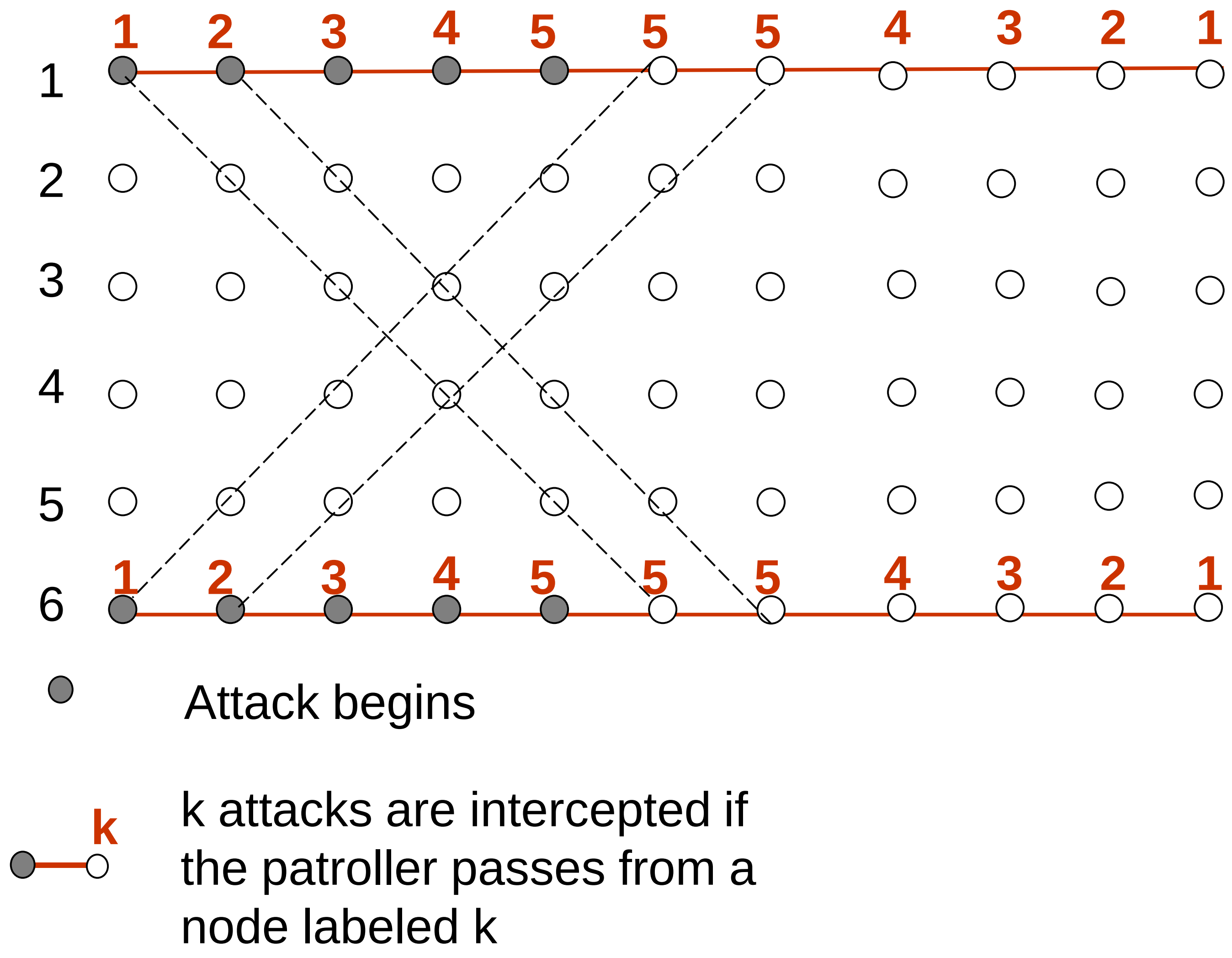


$$V \geq \frac{1}{2}$$

The discrete line – Case A

n=6, m = 7

Time-dependent attacker strategies



The discrete line – Case B

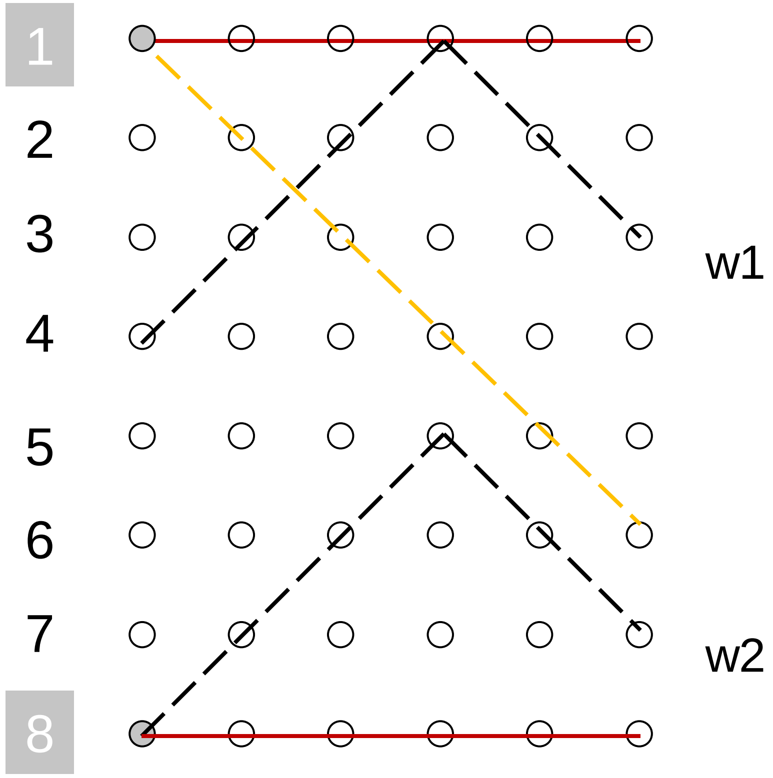
n similar compared to m: n=m+2 and both even

V= 1/2

Patrols:
w1 oscillate between 1 and n/2
w2 oscillate between n/2+1 and n
w1, w2 are intercepting patrols
{w1,w2} is a covering set
J ≤ 2 and thus V ≥1/2

Attacks:
nodes {1,n} are an independent set
I ≥ 2 and thus V ≤ 1/2

Example: n=8, m=6

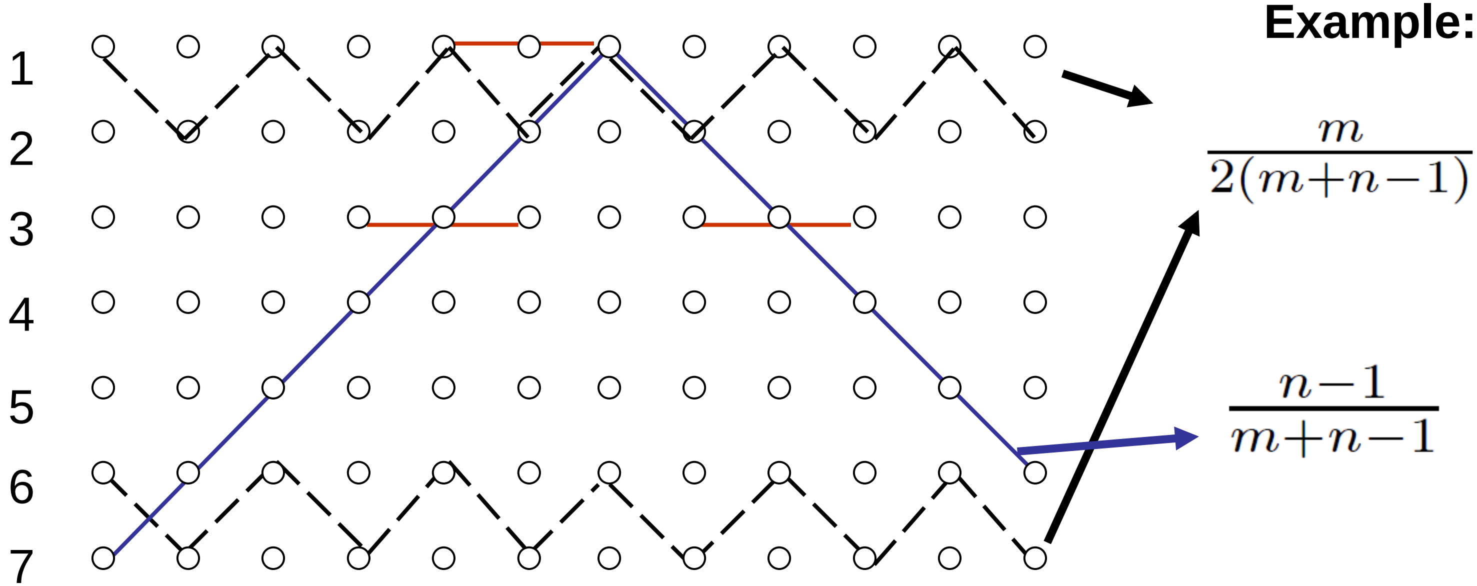


The discrete line – Case C

n large compared to m

Patroller Strategy – Lower bound

Example: m=3, L7



Pr(interception at end node) = $\frac{n-1}{m+n-1}\,\frac{m}{2(n-1)} + \frac{m}{2(m+n-1)} = \frac{m}{n+m-1}$

Pr(interception at nodes 3-5) = $\frac{n-1}{m+n-1}\,\frac{2m}{2(n-1)} = \frac{m}{n+m-1}$

Pr(interception at nodes 2 and 6) ≥ Pr(interception at end node)

$$\boxed{V \geq \frac{m}{n+m-1}}$$

V ≥ 1/3

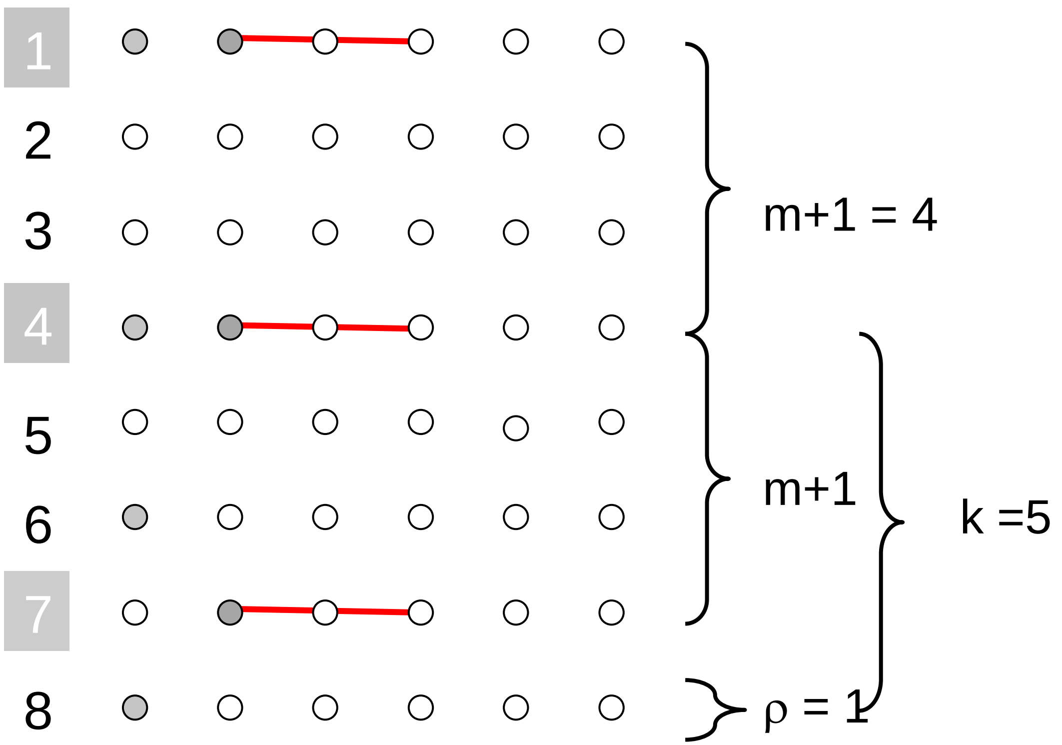
The discrete line – Case C

Attacker Strategies – Upper bound

Let q be the quotient and ρ be the remainder when $n-1$ is divided by m:

$$n - 1 = qm + \rho$$

$$k = m + 1 + \rho.$$



Cases for attacker strategies:

1. $\rho = 0$.
2. $\rho > 0$ and k odd.
3. $\rho > 0$ and k even, m odd
4. $\rho > 0$ and k even , m even and $k > m+2$.
5. $\rho > 0$ and k even , m even and $k = m+2$.

The discrete line – Case C1

n large compared to m: $If\ \rho = 0,\ we\ have\ V^o \leq \frac{m}{n+m-1}.$

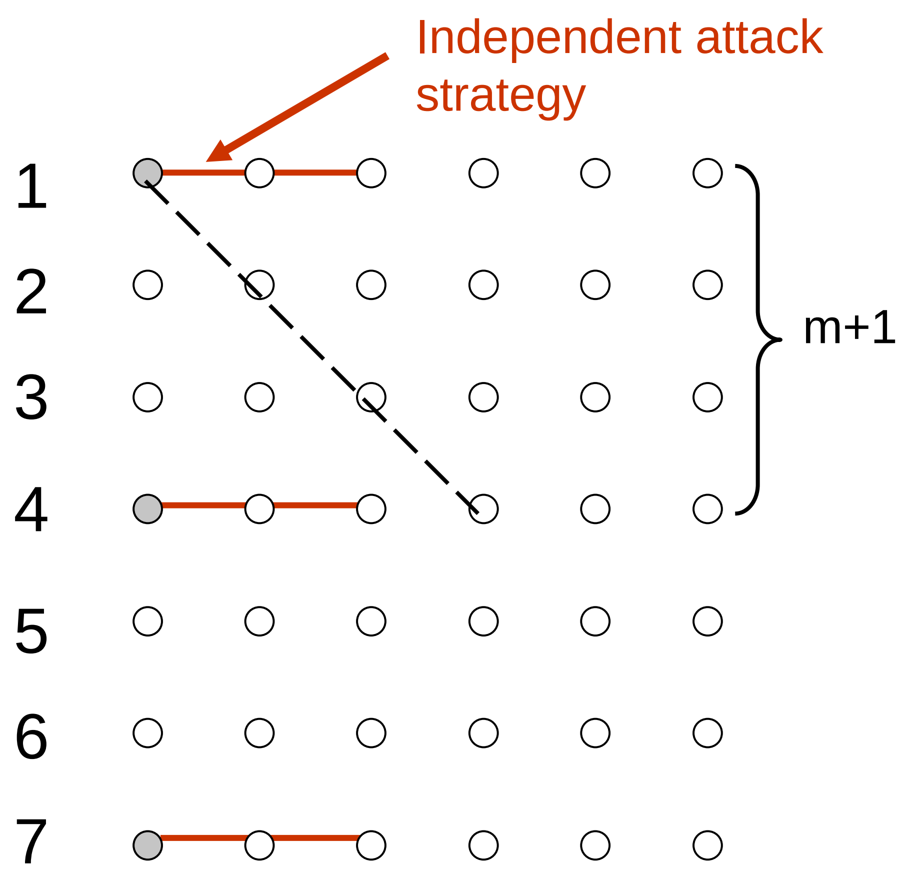
Attacker plays Independent strategy:

Attack at equiprobably at nodes {1, m+1, 2m+1,…,qm+1=n}. Patroller can intercept at most 1 out of q+1 attacks, where q = (n-1)/m :

$$\frac{m}{n+m-1} = \frac{1}{\frac{n-1}{m}+1} = \frac{1}{q+1}$$

Example with ρ = 0:
n = 7, m=3

Maximal Independence set
= {1,4,7}
I = 3 ⇨ V ≤ 1/3



The discrete line – Case C2

n large compared to m

If $\rho > 0$ and k odd, we have $V^o \leq \frac{m}{n+m-1}$

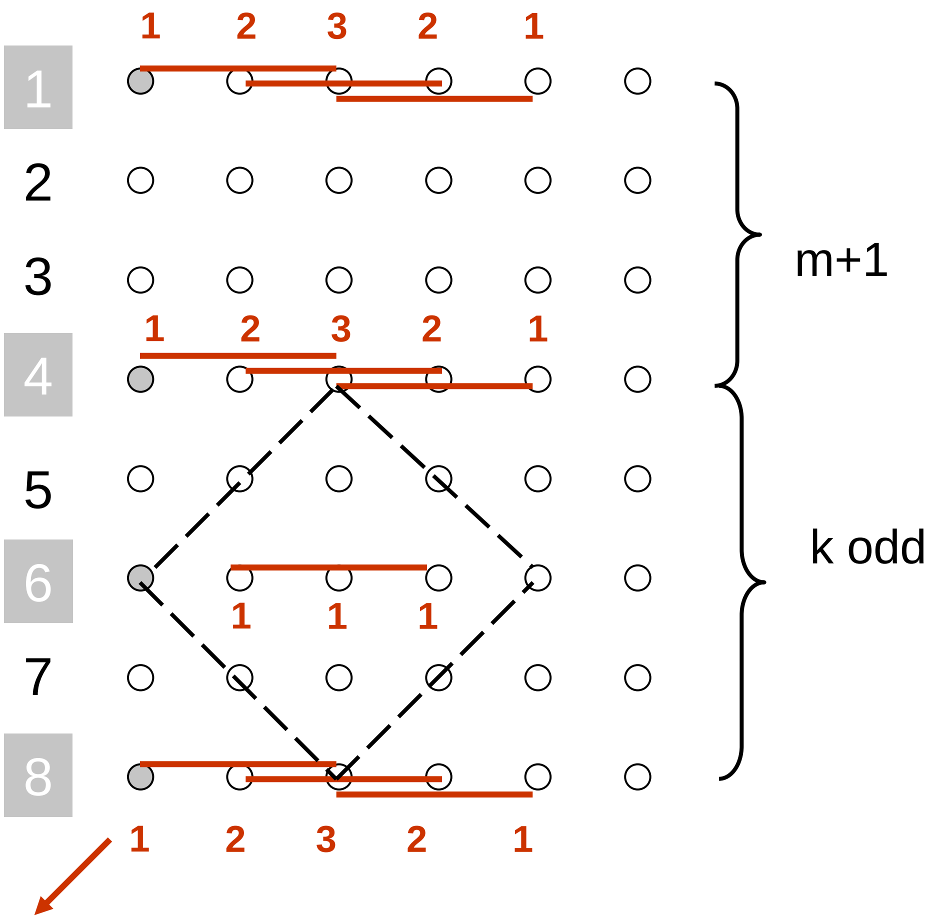
Example with ρ > 0 and k odd:
m=3, L8

Patroller cannot intercept more than 3 out of 3(3)+1 = 10 attacks.

Attacker can guarantee:
Value ≤ 3/10

Patroller can guarantee:

Value ≥ $\frac{m}{n+m-1}$ = 3/10



Number of attacks = n + m - 1

The discrete line – Case C2

n large compared to m

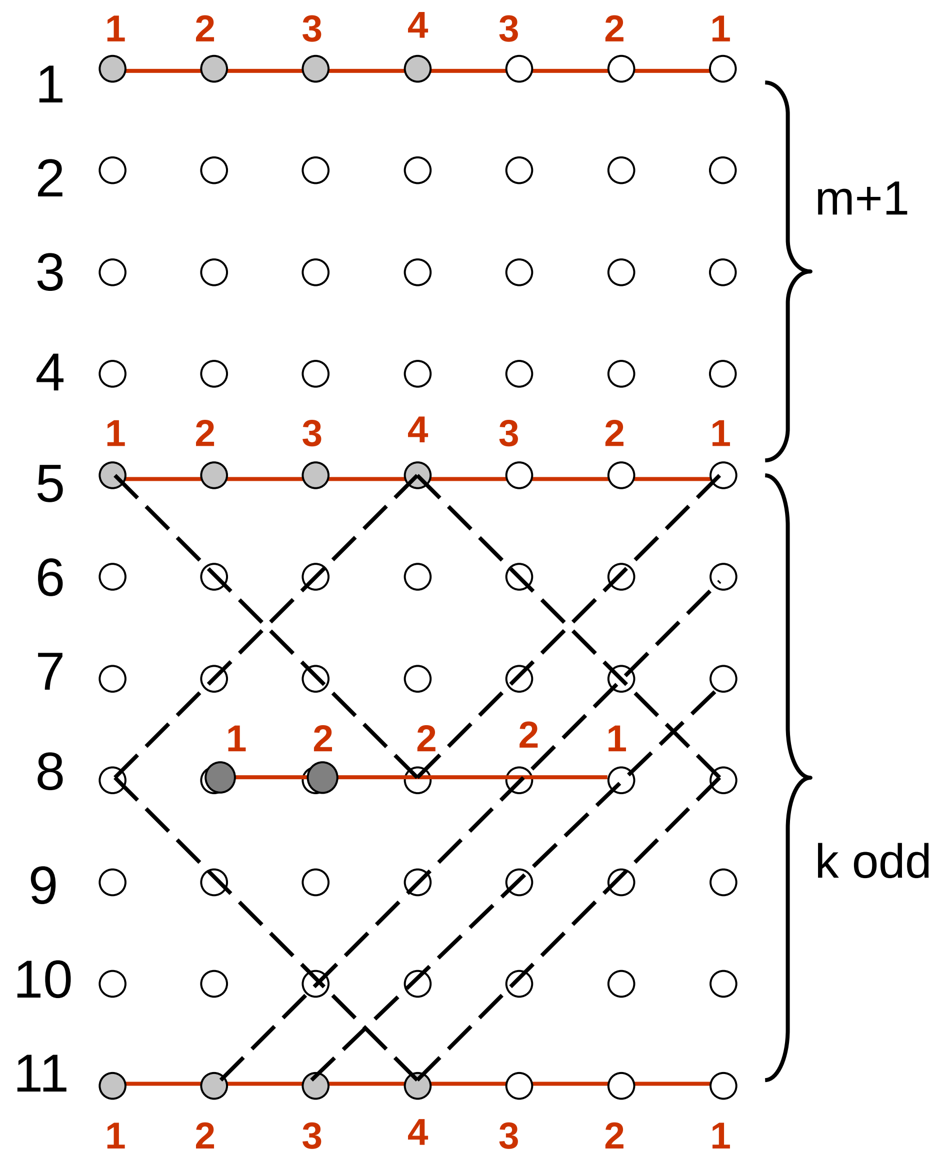
$$\textit{If } \rho > 0 \textit{ and } k \textit{ odd, we have } V^o \leq \frac{m}{n+m-1}$$

Example with ρ > 0 and k odd:
m=4, L11 (n=11)

Can we place n+m-1 attacks such that only m are intercepted by a single patrol?

Divide n-1 by m:
quotient q, remainder d.

- attack at nodes {1,m+1, …,(q-1)m+1,n} m times with attacks shifted by 1 time step
- attack at node in the middle of the odd interval



The discrete line – Case C3

n large compared to m

If $\rho > 0$, k is even and m is odd,

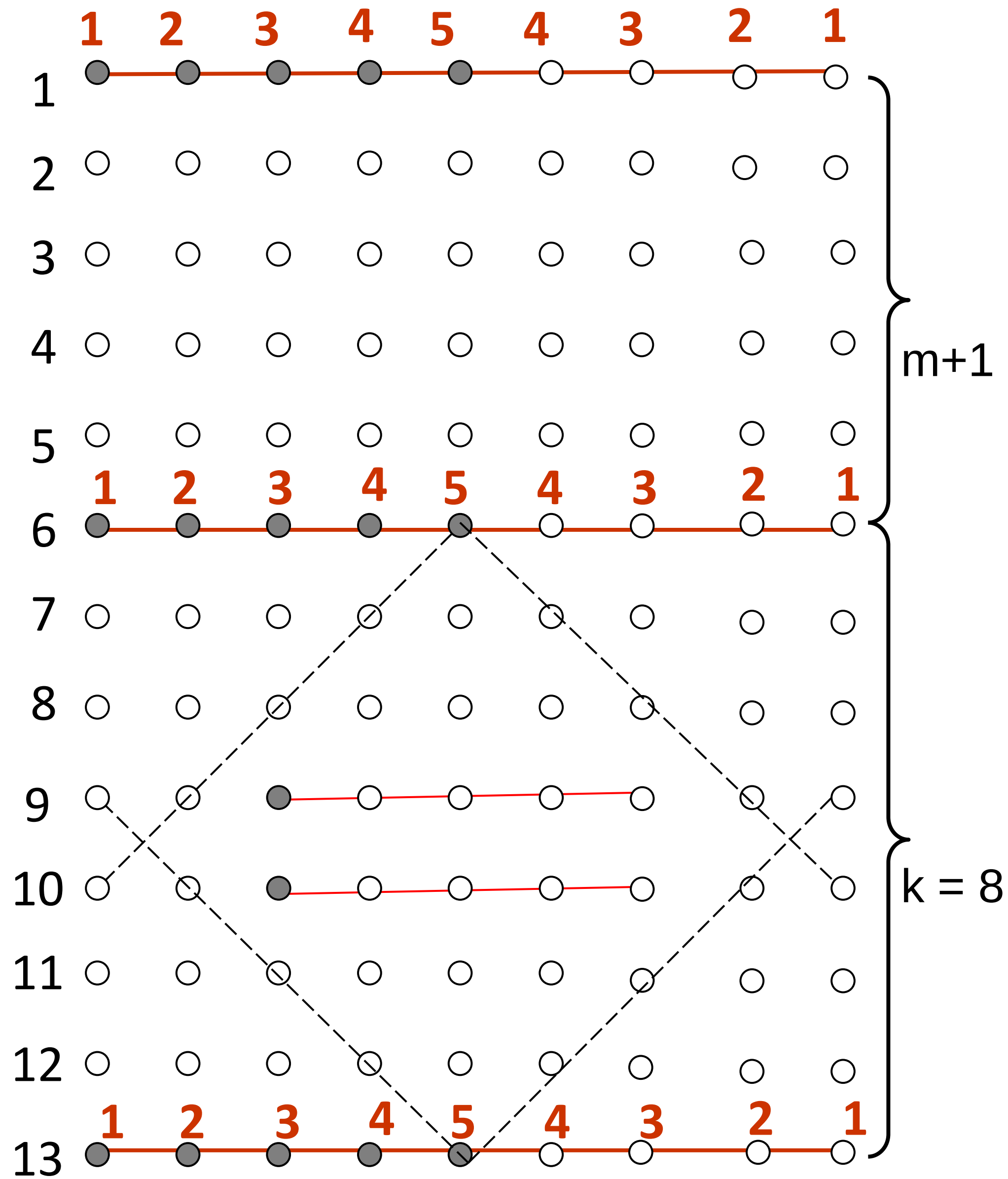
$$V^o \leq \frac{m}{n+m-1}$$

Example with ρ > 0 and k even, m odd: n = 13, m = 5
Thus, q = 2 and ρ = 2, k = 8.

Can we place n+m-1 attacks such that only m are intercepted by a single patrol?

External attacks: at nodes {1,6,13} at time periods {1,2,3,4,5}

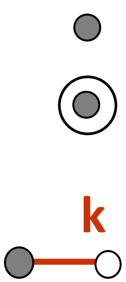
Internal attacks: nodes {9,10} at time period 3.



The discrete line – Case C4

n large compared to m

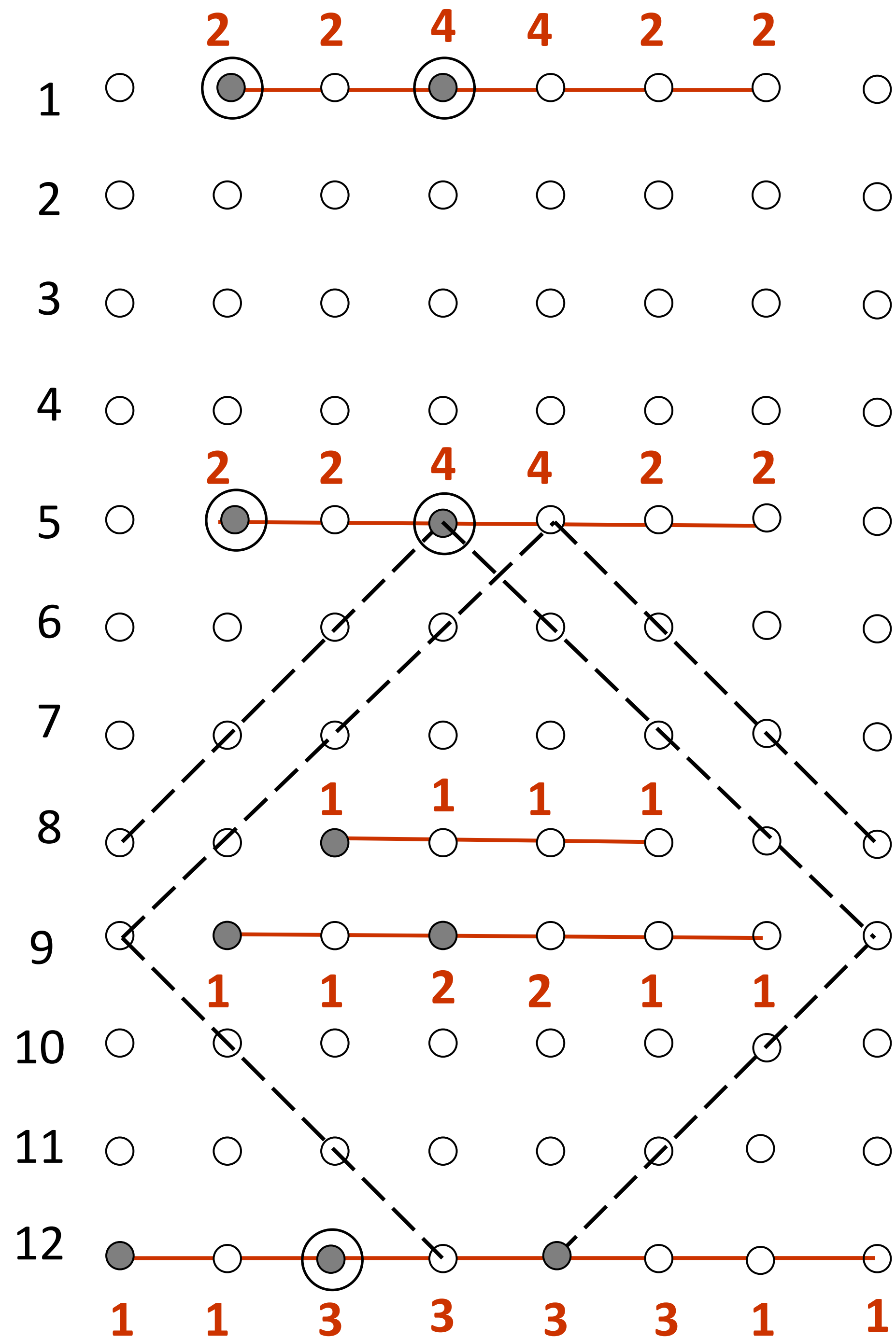
Example with $\rho > 0$ and k even, m even, k > m+2:
n = 12, m = 4, k = 8.



One attack

Two attacks

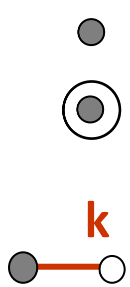
k attacks are intercepted If the patroller passes from a node labeled k

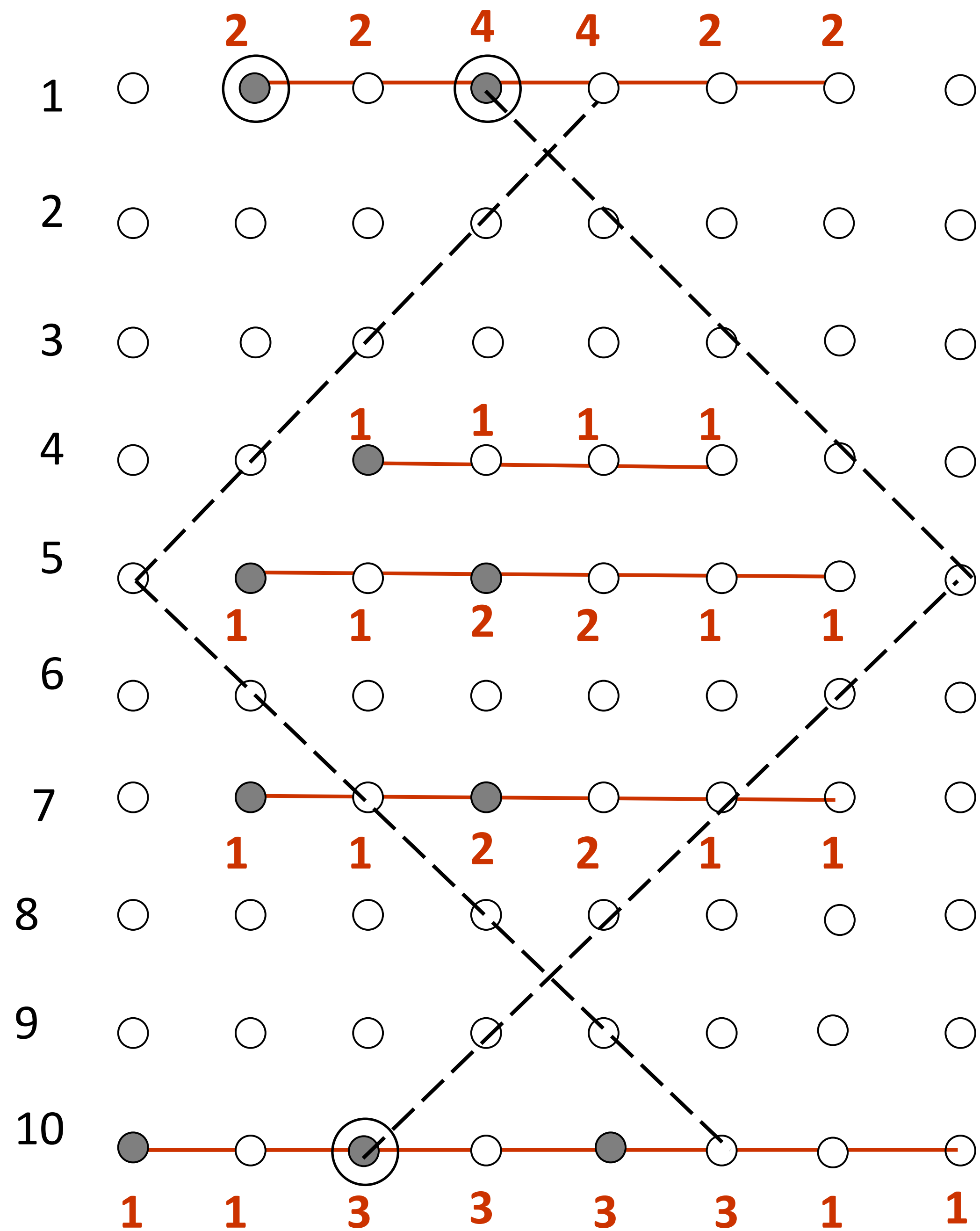


The discrete line – Case C5

n large compared to m

Example with $\rho > 0$ and k even, m even, $k = m+2$:
$n = 10$, $m = 4$, $k = 6$.





The continuous line

The game is played on the unit interval [0,1] over a time horizon T.

Patroller: patrols at unit speed, picks a walk $w: t \rightarrow [0,1]$

Attacker: picks a point x in [0,1] and a time τ, and stays there for time r.
Thus the attack interval is $[\tau, \tau + r]$.

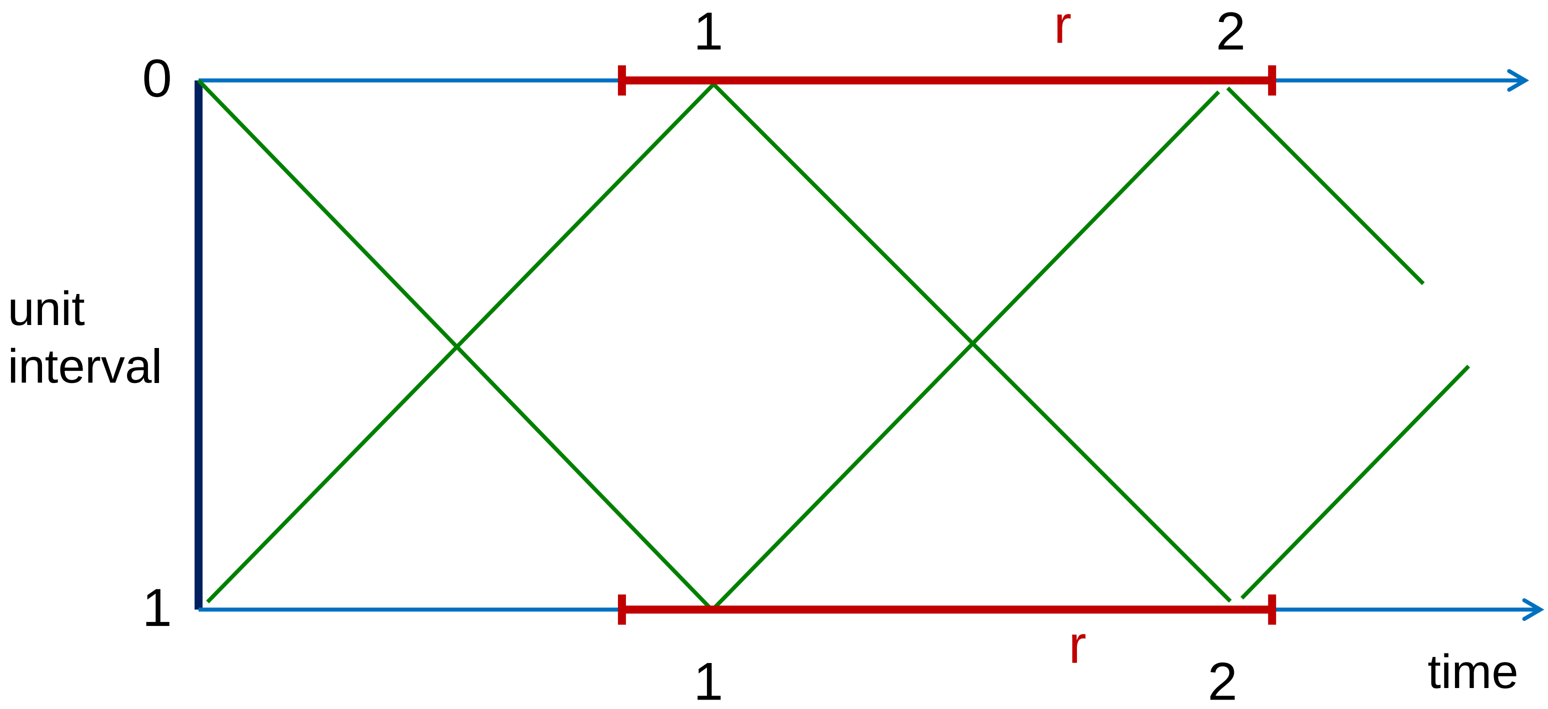
The attack is intercepted if $w(t) = x$ for some t in $[\tau, \tau + r]$.

Value of the game is 1 is the attack is intercepted, otherwise it is 0.

We assume $0 \le r \le 2$, otherwise the patroller can always intercept the attacker by going up and down the unit interval.

The continuous line

$$If\ r \geq 1,\ then\ V = \frac{r}{2}.$$



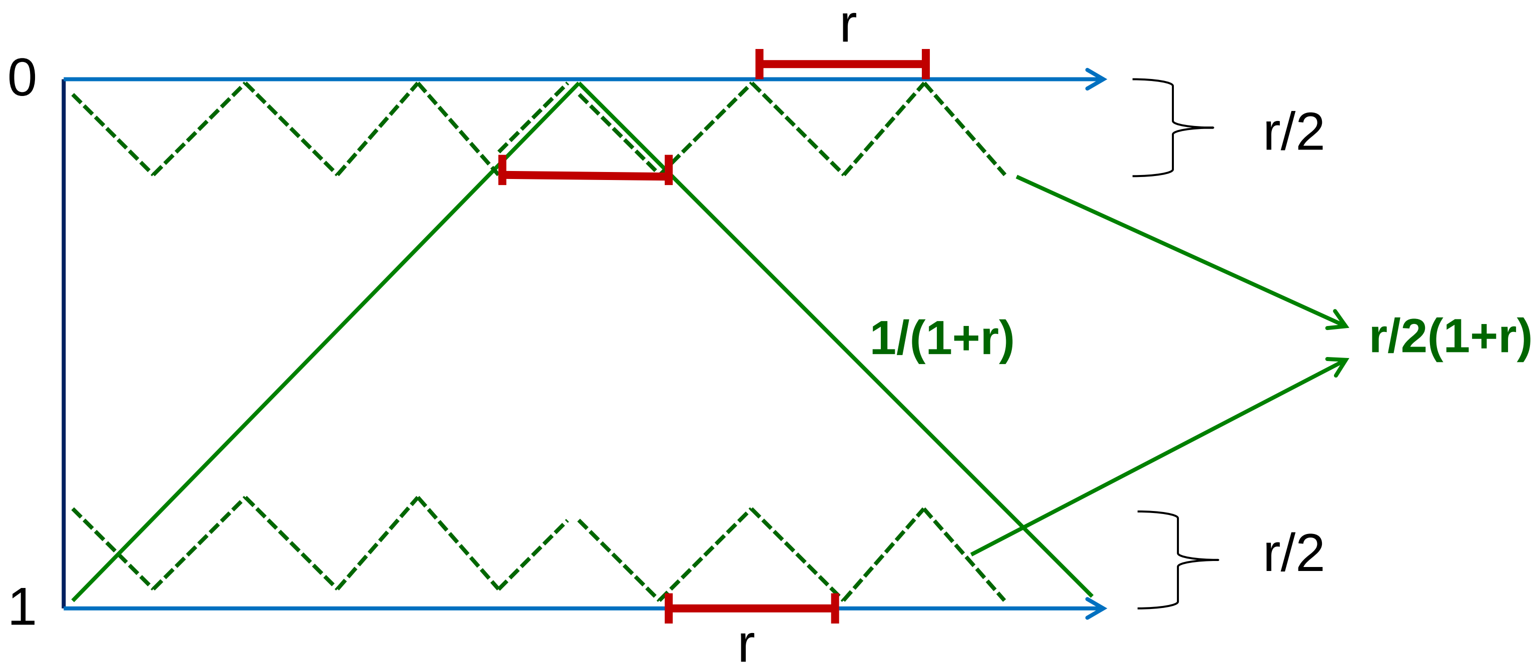
the patroller: picks a point at random and a random direction and oscillates from one endpoint to the other

the attacker: Diametrical strategy: pick a point y in the [0,1] time interval and attack equiprobably between the two endpoints during attack time interval [y,y+r].

The continuous line

$$If\ r \leq 1,\ then\ V = \frac{r}{1+r}$$

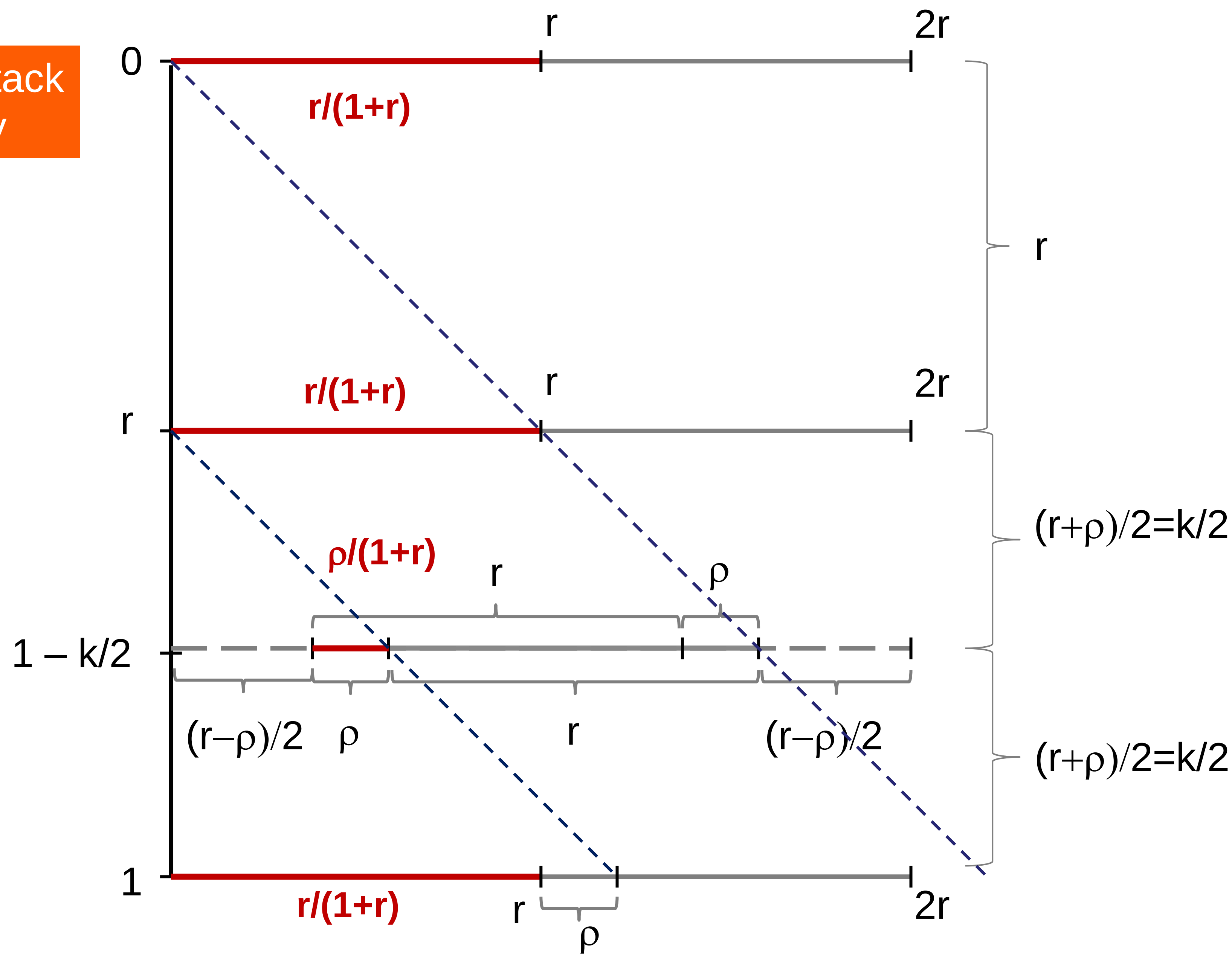
the patroller strategy



The continuous line

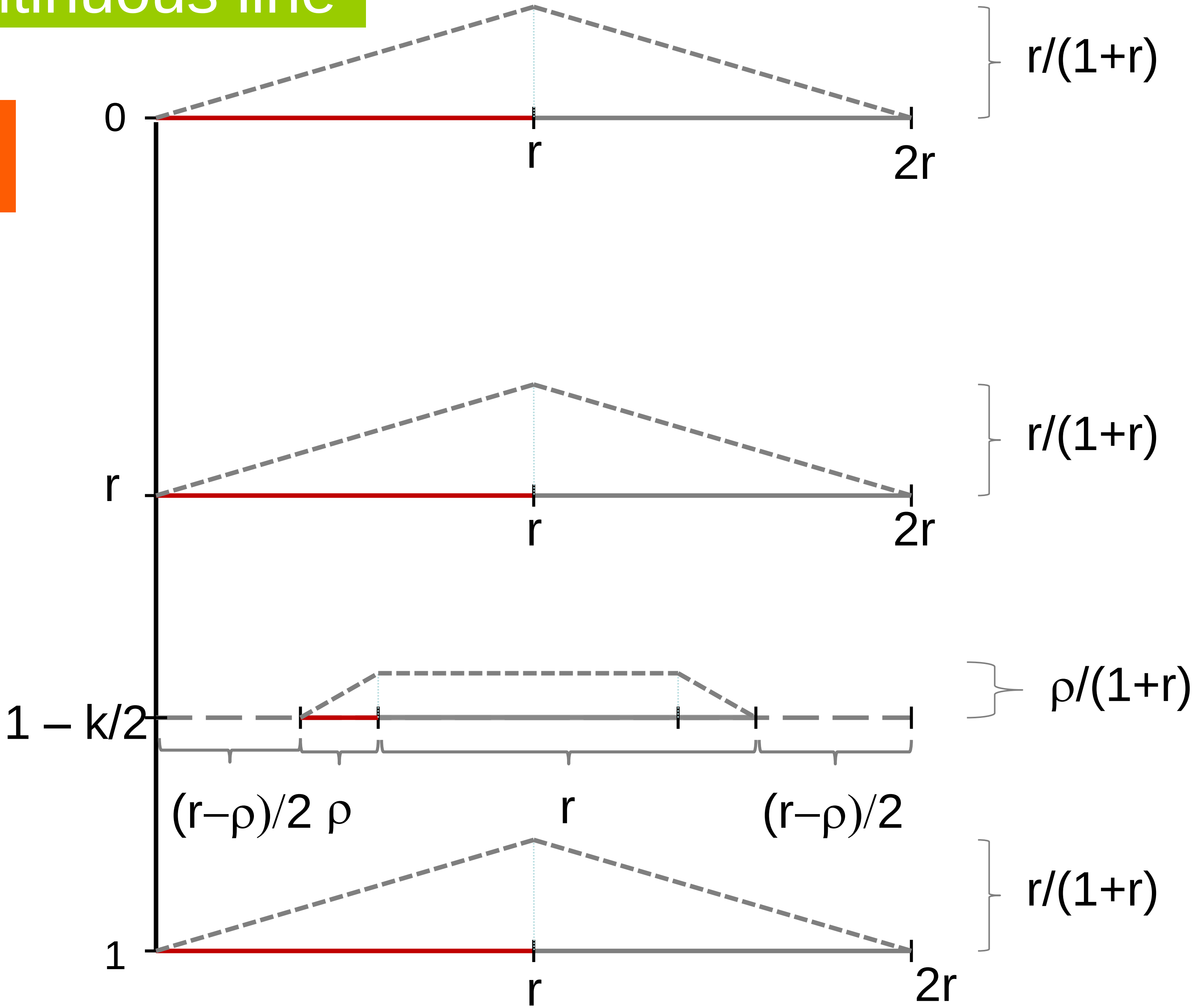
$$If\ r \leq 1,\ then\ V = \frac{r}{1+r}$$

the r-attack strategy



The continuous line

the r-attack strategy



Current and Future Work

Current work: Other graphs: Trees.

Computational work:

Show that the problem is its general form is NP-complete: Hamiltonian graphs can have optimal strategies that do not use the Hamiltonian cycle.

For m=2, the game can be formulated as a network flow problem for cases where dwelling at a node is a dominated strategy:
G bipartite and T even.

Constraint generation methods where the most violated constraints are generated:

- mixed integer programming is used to find the most violated constraint
- a heuristic to find a violated constraint

Extended Patrolling Games:

Multiple patrollers/attackers.

Version with in-game observation: uniformed patroller

The End

Thank you.