Assignments on Tolerances, Lagrangian relaxation and Branch and Bound

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Introduction

- This seminar contains a set of exercises on the second lecture.
- So the topics of the exercises are tolerances, Branch and Bound and Lagrangian relaxation.

Assignments

1. Follow a branch and bound approach to solve a small ATSP instance to optimality.
2. Compute upper and lower tolerances in AP and MSTP examples.
3. Take a small example and try to determine an optimal solution by applying Lagrangian relaxation yourselves (find optimal multiplier values).
4. Show that Lagrangian relaxation optimality conditions hold, one may use complementary slackness.
5. Show that if the upper tolerance value of a Minimum Spanning Tree edge is larger than the additions of its endpoints, the edge remains an each optimal solution to the MSTP.
6. Show that lower tolerance based bound are tighter than upper tolerance-based.
7. Determine multiplier values based on upper tolerances for two examples.

ATSP example

In the figure below, you'll find an AP solution.
Solve this example using Branch and Bound.
Specify the upper and lower bounds, the search strategy and the branching rule.

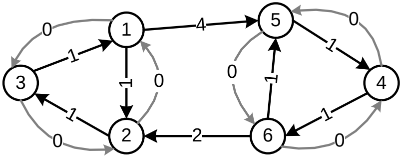


Figure: ATSP instance

Compute upper and lower tolerance values

In the following AP instance, compute upper and lower tolerance values.

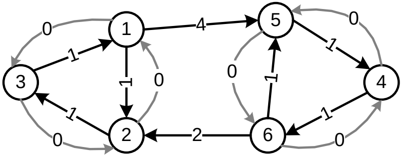


Figure: ATSP instance

Compute upper and lower tolerance values

In the following MSTP instance, compute upper tolerance values of the edges (3,4), (2,4) and (6,9), and compute the lower tolerance values of (5,7), (4,6), (2,3) and (2,4).

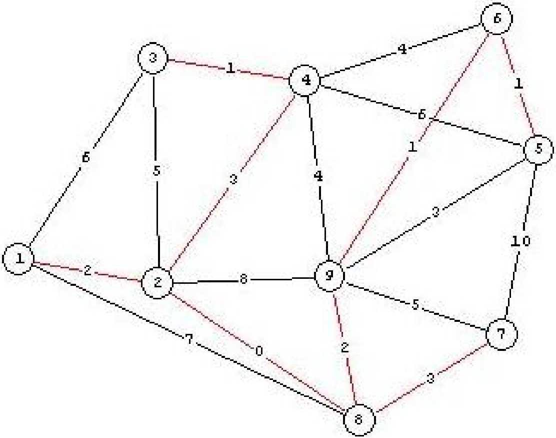


Figure: MSTP instance

Applying Lagrangian relaxation

$$\begin{aligned} \min \quad & \textstyle\sum_{ij} c_{ij}x_{ij} && (1)\\ s.t. \quad & \textstyle\sum_i x_{ij} \geq 1 && (2)\\ & \textstyle\sum_j x_{ij} \leq 1 && (3)\\ & all\ x_{ij} \in \{0,1\} && (4) \end{aligned}$$

This is an Assignment Problem, but solve it with Lagrangian relaxation.
Relax the constraints (3) on the column total and solve the resulting Row Minimum Problem.
Define the resulting Lagrangian relaxation problem and indicate how it should be solved.

Applying Lagrangian relaxation (2)

Extra: solve the previous problem with following c_{ij}:

$$\begin{matrix} 0 & 4 & 3 & 1 \\ 2 & 1 & 4 & 10 \\ 3 & 4 & 6 & 4 \\ 5 & 2 & 5 & 7 \end{matrix}$$

Lagrangian relaxation optimality

Show that for the following LP problem, optimality of the LRP corresponds to complementary slackness.

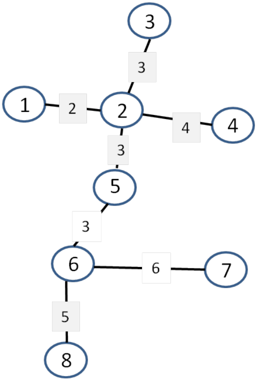
$$\begin{aligned} \min \quad & c^T x && (5) \\ s.t. \quad & A_1 x \leq b_1 && (6) \\ & A_2 x \leq b_2 && (7) \\ & && (8) \end{aligned}$$

... where we take the following LRP.

$$\begin{aligned} \min \quad & c^T x + \lambda(b_2 - A_2 x) && (9) \\ s.t. \quad & A_1 x \leq b_1 && (10) \\ & && (11) \end{aligned}$$

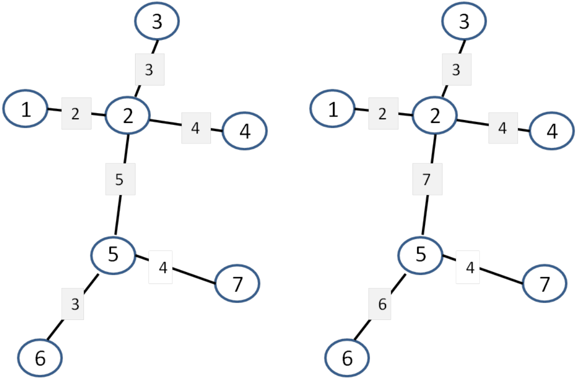
Increase of λ

Let $\lambda 2$ be the Lagrangian multiplier of the degree-constraint of vertex 2.



Set λ_2 to resp. 0, 1,2,3,4,5 and estimate how many edges will be adjacent to v.

Increase of λ (2)



Determine λ_2 and λ_5, where $b = 2$ for each vertex in both networks.

Bottleneck tolerances

In the previous AP instance, determine the bottleneck tolerances.

Bottleneck tolerances

Show that the bottleneck lower tolerance into or out of a cycle is higher than the bottleneck upper tolerance of breaking that cycle.