Facility Location.
From the Simple to Bi-Level Models

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Contents

Uncapacitated Facility Location Problem

- Input: J is the set of users;

  I is the set of potential facilities;

  f_i is the fixed cost of opening facility i;

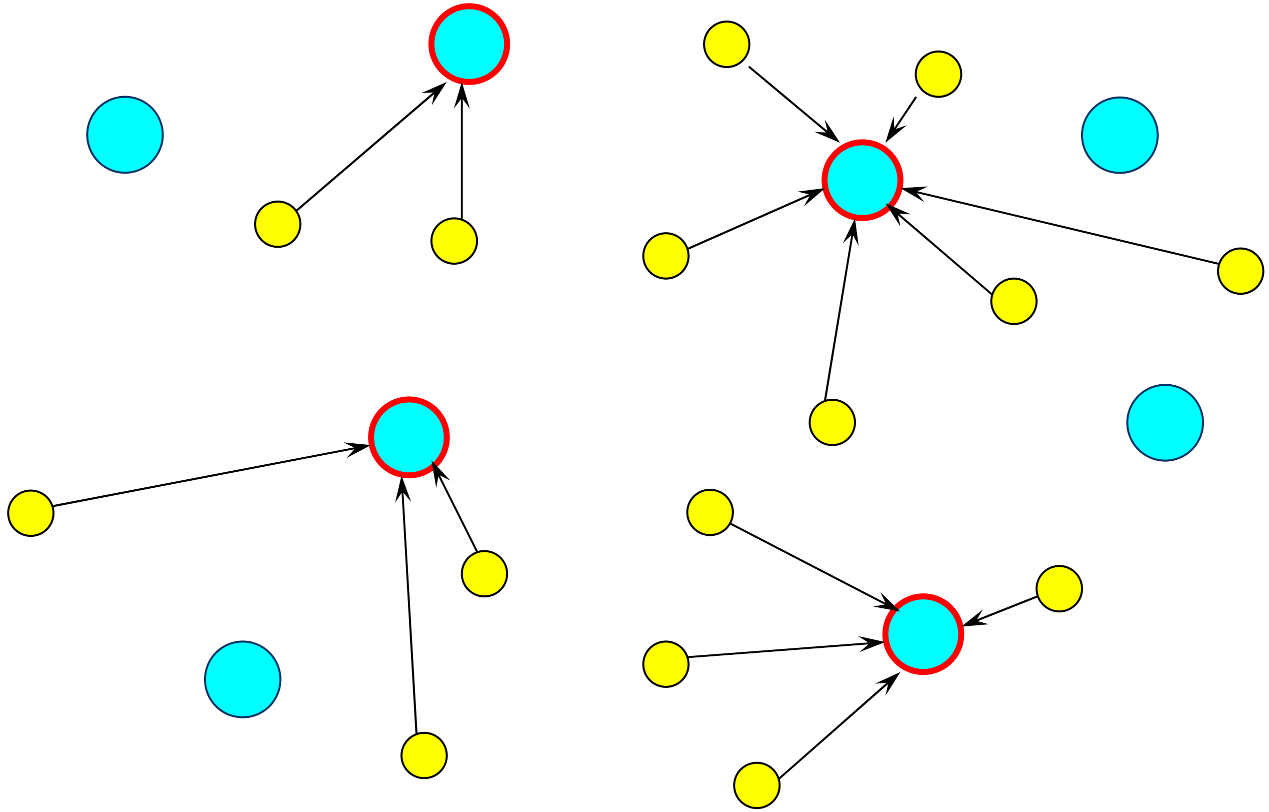
  c_{ij} is the cost for servicing user j from facility i;

- Output: a set $S \subseteq I$ of opened facilities;

- Goal: minimize the total cost of opening facilities and servicing users

$$F(S) = \sum_{i \in S} f_i + \sum_{j \in J} \min_{i \in S} c_{ij}.$$

Example

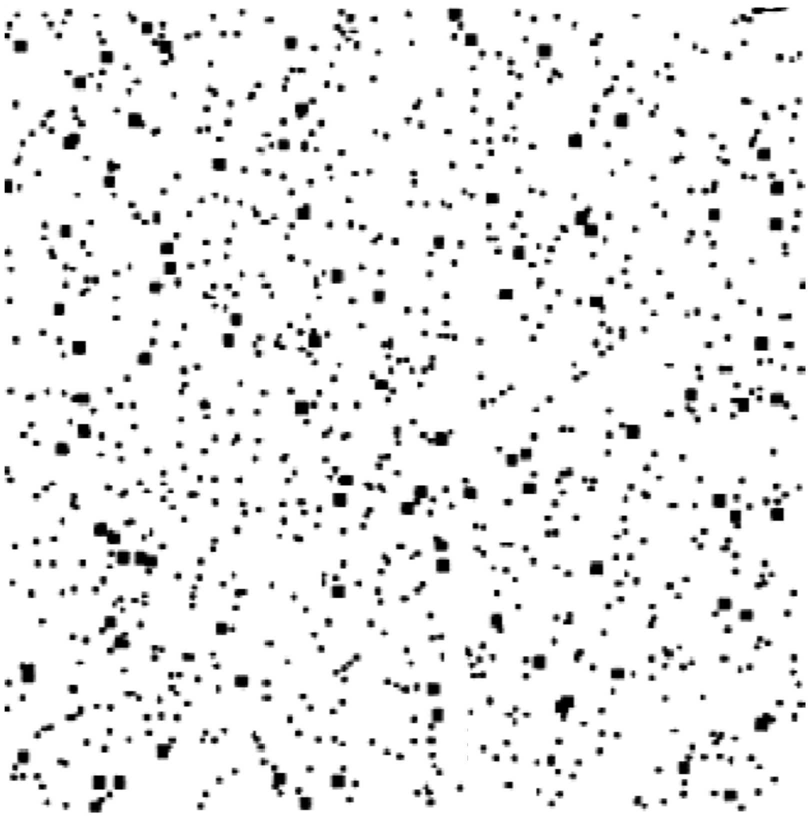


I = {1,…, 8} is potential facility locations;

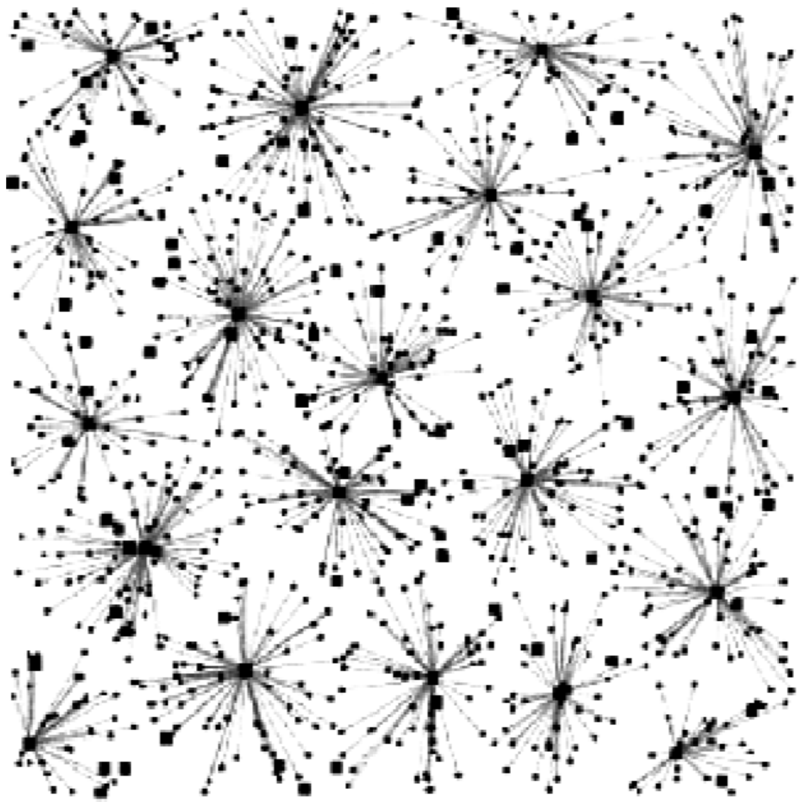
J = {1,…, 15} is set of users

Users are serviced from nearest facility

Example $|I| = 100; |J| = 1000$



Instance



Solution

Integer Programming Formulation

Variables: $x_i = \begin{cases} 1 & \text{if facility } i \text{ is opened} \\ 0 & \text{otherwise} \end{cases}$

$$y_{ij} = \begin{cases} 1 & \text{if user } j \text{ is served by facility } i \\ 0 & \text{otherwise} \end{cases}$$

Mathematical model:

$$\{\min \sum_{i \in I} f_i x_i + \sum_{i \in I} \sum_{j \in J} c_{ij} y_{ij} \}$$

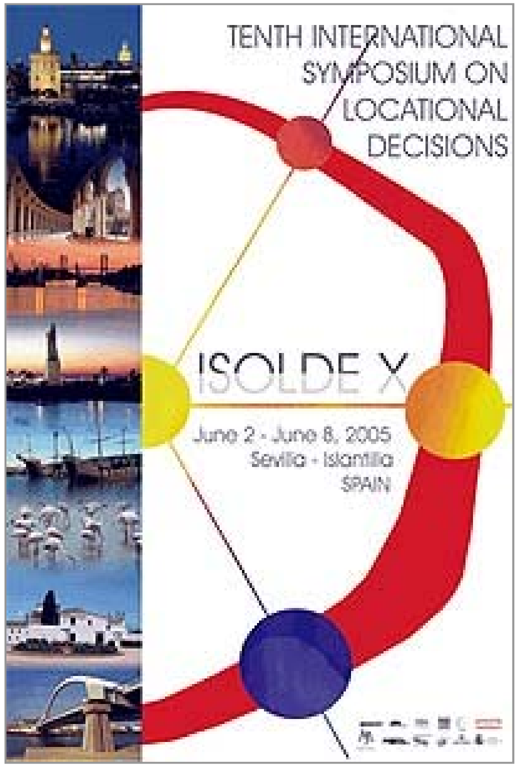
$$s.t. \quad \sum_{i \in I} y_{ij} = 1, \qquad j \in J;$$

$$x_i \geq y_{ij}, \qquad i \in I, j \in J;$$

$$x_i, y_{ij} \in \{0,1), \qquad i \in I, j \in J.$$

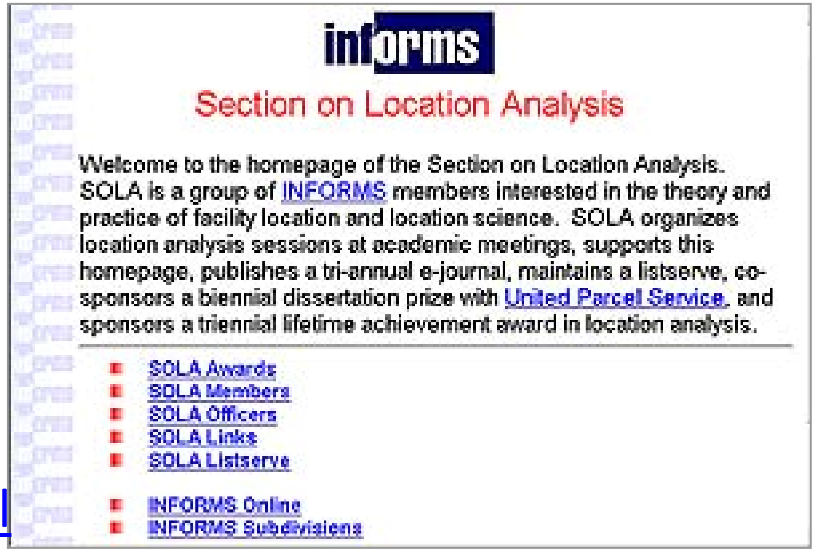
Societies and conferences on Location

Euro Working Group on Locational Analysis
http://www.vub.ac.be/EWGLA/



International Symposium on Location Decisions
http://www.aloj.us.es/isolde/

INFORMS Section on Location Analysis
http://www.ent.ohiou.edu/~thale/sola/sola.html



Books and Journals

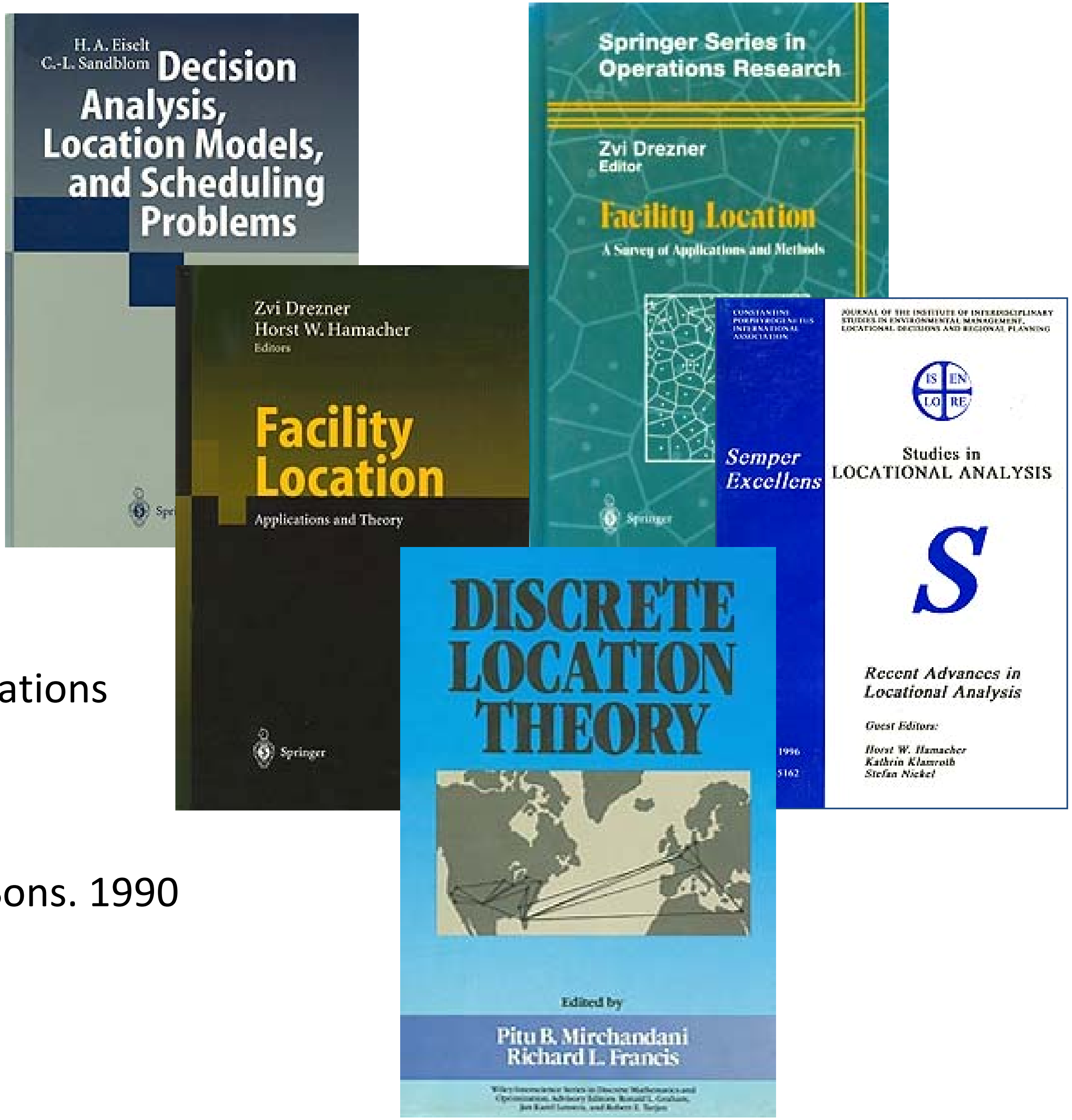
H.A. Eiselt, C.-L. Sandblom.
Decision Analysis, Location Models, and Scheduling Problems
Springer. 2004.

Z. Drezner, H. Hamaher (Eds.)
Facility Location. Applications and Theory. Springer. 2004.

Z. Drezner (Ed.)
Facility Location. A Survey of Applications and Methods. Springer. 1995.

P.B. Mirchandani, R.L. Francis (Eds.)
Discrete Location Theory. Wiley & Sons. 1990

Studies and Locational Analysis.
ISENLORE. Editor-in-Chief F.Plastria



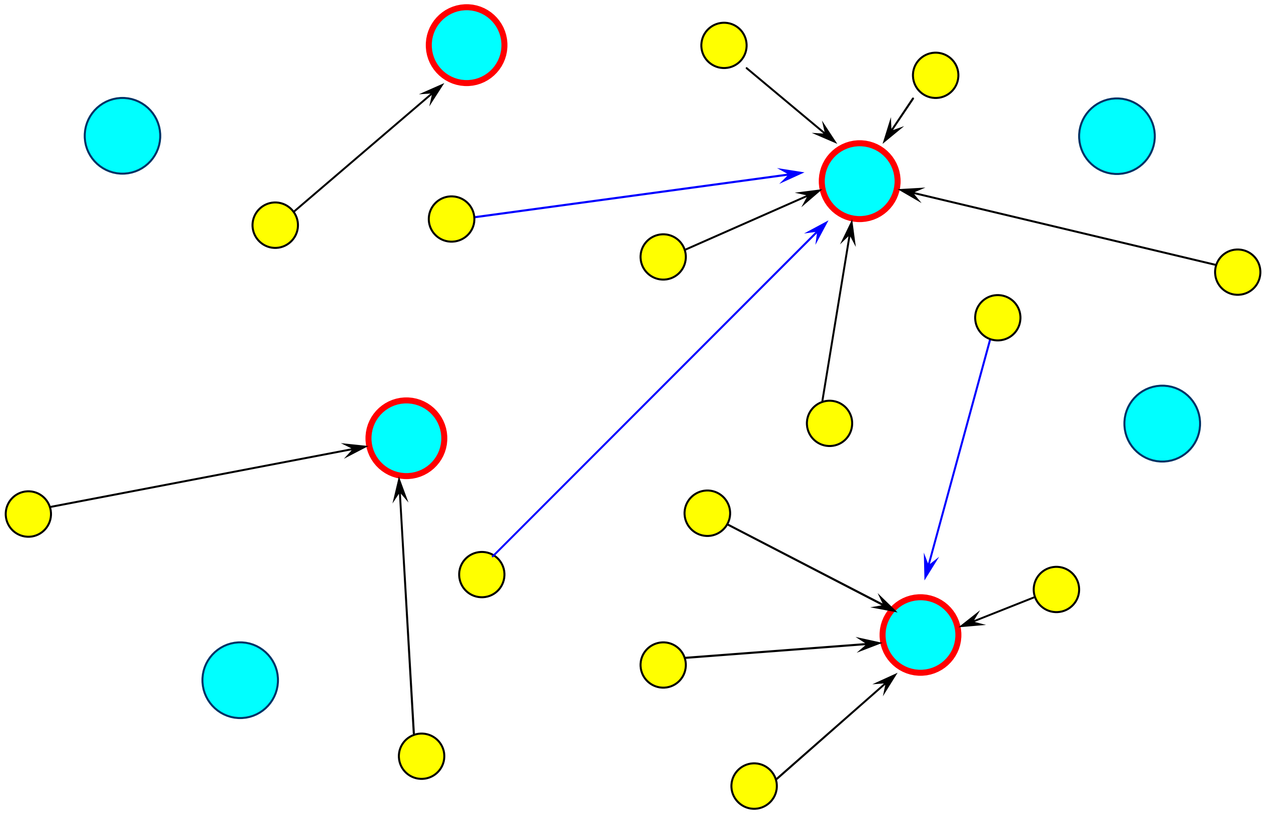
Theoretical Results

- The UFL problem is strongly NP-hard even for metric case.
- An 1,463–factor approximation algorithm for the metric UFL problem would imply P = NP (Guna, Khuller 1999; Sviridenko).
- An 1,52–factor approximation algorithm for the metric UFL problem (Mahdian, Ye, and Zhang 2002).
- An ε–factor approximation algorithm for any $\varepsilon > 0$ in the special case when facilities and users are points in the plane and service costs are geometrical distances (Arora, Raghavan, Rao 1998; Kolliopoulos, Rao 1999).
- There is no constant–factor approximation algorithm for general UFL problem if P $\neq$ NP.

Empirical Results

- Efficient branch and bound methods based on the fast heuristics for dual problem (Lebedev, Kovalevskaya 1974; Trubin 1973; Beresnev 1974; Bilde, Krarup 1977; Erlenkotter 1978).
- Fast randomized heuristics for large scale metric instances (Chudak, Barahona 2000).
- Improved branch and bound method for large scale instances (Hansen, Mladenovic 2003).
- Effective and efficient local search methods for large scale instances (Resende, Werneck 2002, 2004; Hansen, Mladenović 1997)
- Benchmark library “Discrete Location Problems” (Alekseeva, Kochetov, Kochetova, et al.)

UFL Problem with User Preferences (UFLPUP)



I = {1,..., 8} is potential facility locations;

J = {1,..., 15} is set of users

User is serviced by the most desirable facility.

- Input: J is the set of users;
  I is the set of potential facilities;
  f_i is the fixed cost of opening facility i;
  c_{ij} is the cost for servicing user j from facility i;
  d_{ij} is user preferences: facility i_1 is more desirable then i_2 for user j if $d_{i_1 j} < d_{i_2 j}$.
- Output: a set $S \subseteq I$ of opening facilities;
- Goal: minimize the total cost of opening facilities and servicing users:

$$F(S) = \sum_{i \in S} f_i + \sum_{j \in J} \min_{i \in S} c_{i(s_j)j},$$

where $$i(s_j) = \arg\min_{i \in S} d_{ij}, \ j \in J.$$

Integer Programming Formulation

Variables:

$$x_i = \begin{cases} 1 & \text{if facility } i \text{ is opened} \\ 0 & \text{otherwise} \end{cases} \qquad y_{ij} = \begin{cases} 1 & \text{if user } j \text{ is served by facility } i \\ 0 & \text{otherwise} \end{cases}$$

Mathematical model:

Company:

$$\min_x \{ \sum_{i \in I} f_i x_i + \sum_{i \in I} \sum_{j \in J} c_{ij} y_{ij}^* \}$$

$$\text{s.t.} \qquad x_i \in \{0,1\}, \qquad i \in I;$$

where $y_{ij}^*(x)$ is optimal solution of the user problem:

Users:

$$\min_y \sum_{j \in J} \sum_{i \in I} d_{ij} y_{ij}$$

$$s.t. \qquad \sum_{i \in I} y_{ij} = 1, \qquad j \in J;$$

$$x_i \geq y_{ij}, \qquad x_i, y_{ij} \in \{0,1\}, \qquad i \in I, j \in J.$$

The Single Level Reformulations

$$\min_{x,y}\{\sum_{i\in I} f_i x_i + \sum_{i\in I}\sum_{j\in J} c_{ij} y_{ij}\}$$

$$\text{s.t.} \qquad \sum_{i\in I} y_{ij} = 1, \qquad j \in J;$$

$$x_i \geq y_{ij}, \qquad i \in I, j \in J;$$

$$y_{ij} + x_l \leq 1, \qquad l \in S_{ij}, i \in I, j \in J;$$

$$x_i, y_{ij} \in \{0,1), \qquad i \in I, j \in J;$$

$$\text{where} \qquad S_{ij} = \{k \in I \mid d_{kj} < d_{ij}\}, \ i \in I, j \in J.$$

Main Results

- UFLPUP is NP-hard problem in the strong sense and coincide to UFLP if $c_{ij} = g_{ij}$ for all ij;
- It is polynomially solvable if $c_{ij} = - g_{ij}$ for all ij;
- Strengthened reformulations (M. Labbe, L. Canovas, S. Garsia, A. Martin);
- Reduction to pseudo-Boolean functions (L. Gorbachevskaya, V. Dementiev, Yu. Shamardin);
- Improved lower bounds (Yu. Kochetov, K. Klimentova, I.Vasiliev);
- The branch and cut method (K. Klimentova, I.Vasiliev);
- Metaheuristics (M. Marich, Z. Stanimirovich, N. Milenkovich, A. Djenic)

The Leader-Follower Location Problem

- Input: J is the set of users;
  I is the set of potential facilities;
  w_j is the demand of user j;
  c_{ij} is the distance from user j to facility i;
  p is the number of leader facilities;
  r is the number of follower facilities.
- Output: a set $S \subset I$, $|S| = p$ of opening facilities by the leader.
- Goal: maximize the market share of the leader anticipating that the follower will react to the decision by opening his own r facilities.

The Bi-Level 0-1 Linear Model

$$\max_{x} \sum_{j \in J} w_j z_j^*(x)$$

s.t.

$$\sum_{i \in I} x_i = p, \qquad x_i \in \{0, 1\}, i \in I;$$

where z_j^* is optimal solution of the Follower problem

$$\max_{z, y} \sum_{j \in J} w_j (1 - z_j)$$

s.t.

$$1 - z_j \le \sum_{i \in I_j(x)} y_i , \qquad j \in J;$$

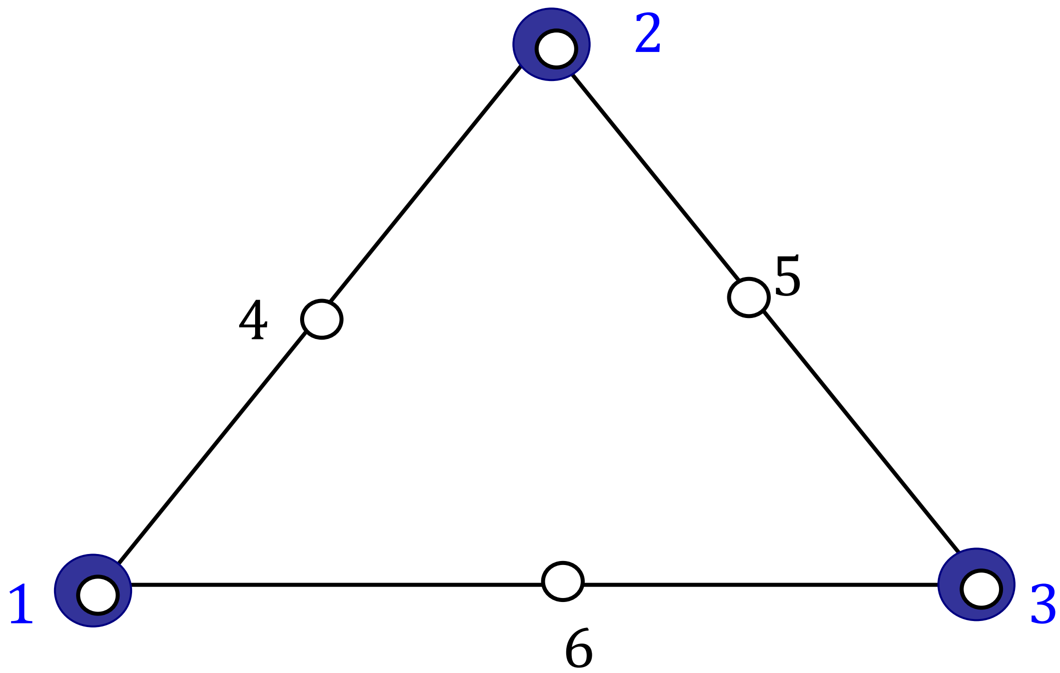
$$\sum_{i \in I} y_i = r, \qquad x_i + y_i \le 1, i \in I, \qquad y_i, z_i \in \{0, 1\};$$

Example

$I = \{1, 2, 3, 4, 5, 6\}$

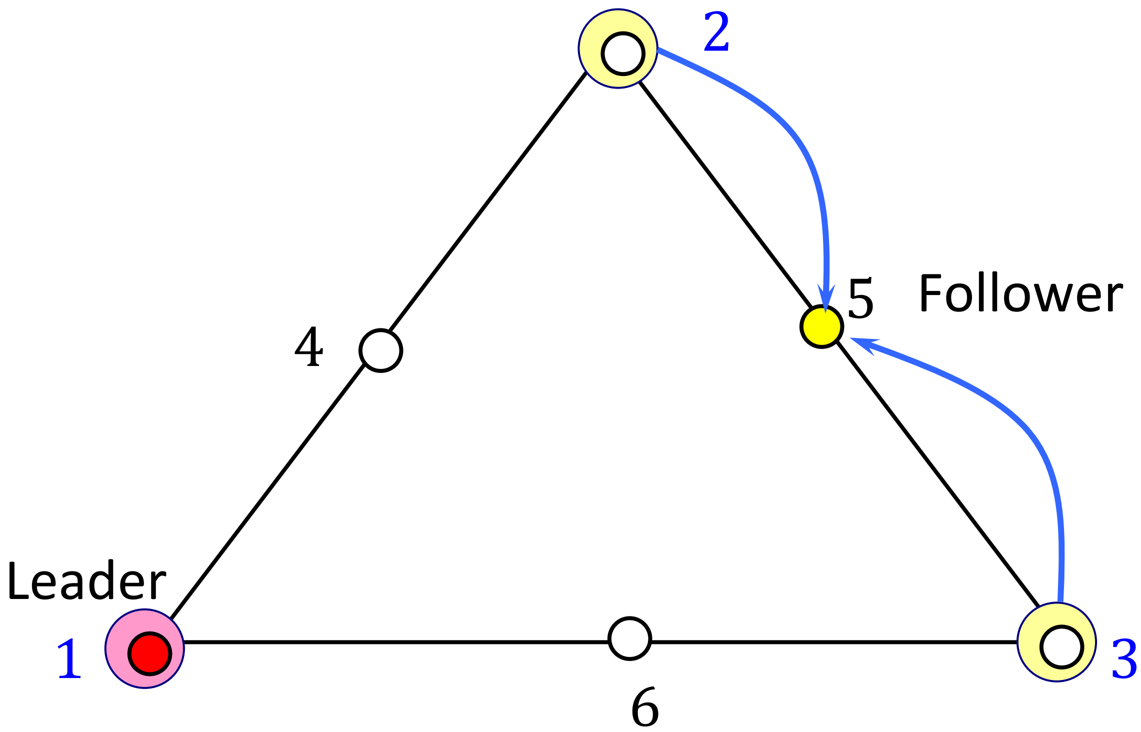
$J = \{1, 2, 3\}$

$p = r = 1$



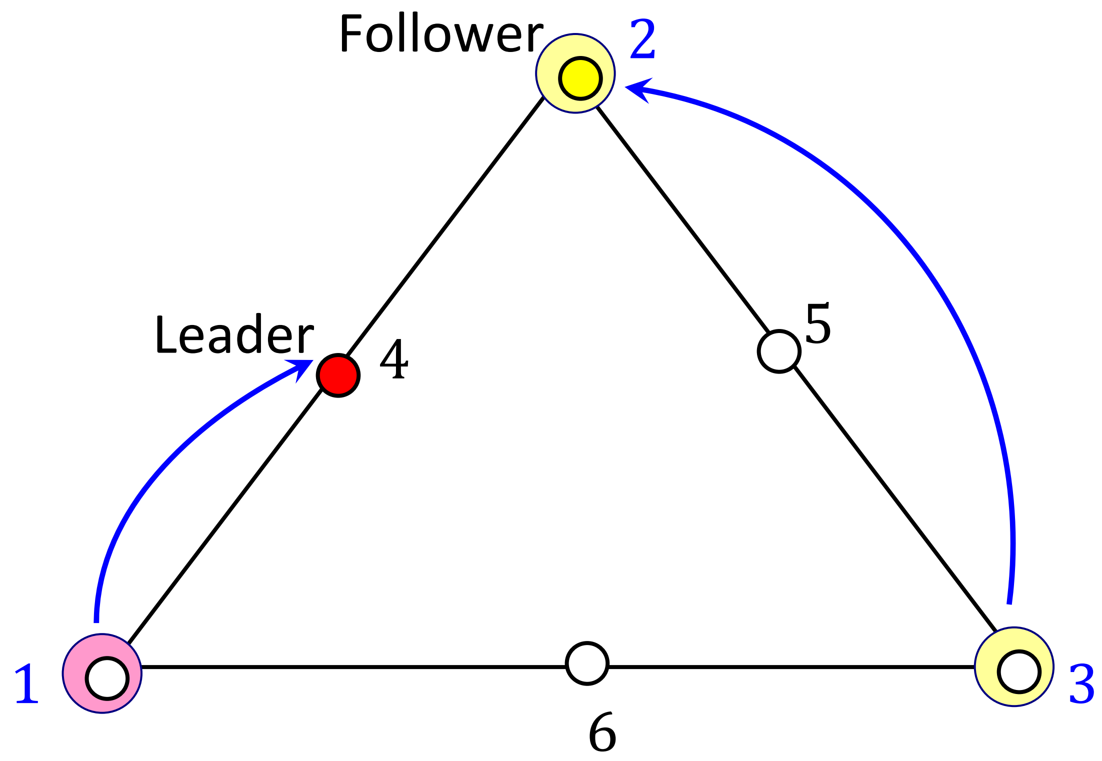
If the leader opens own facility in point 1,

then the follower opens facility in point 5.

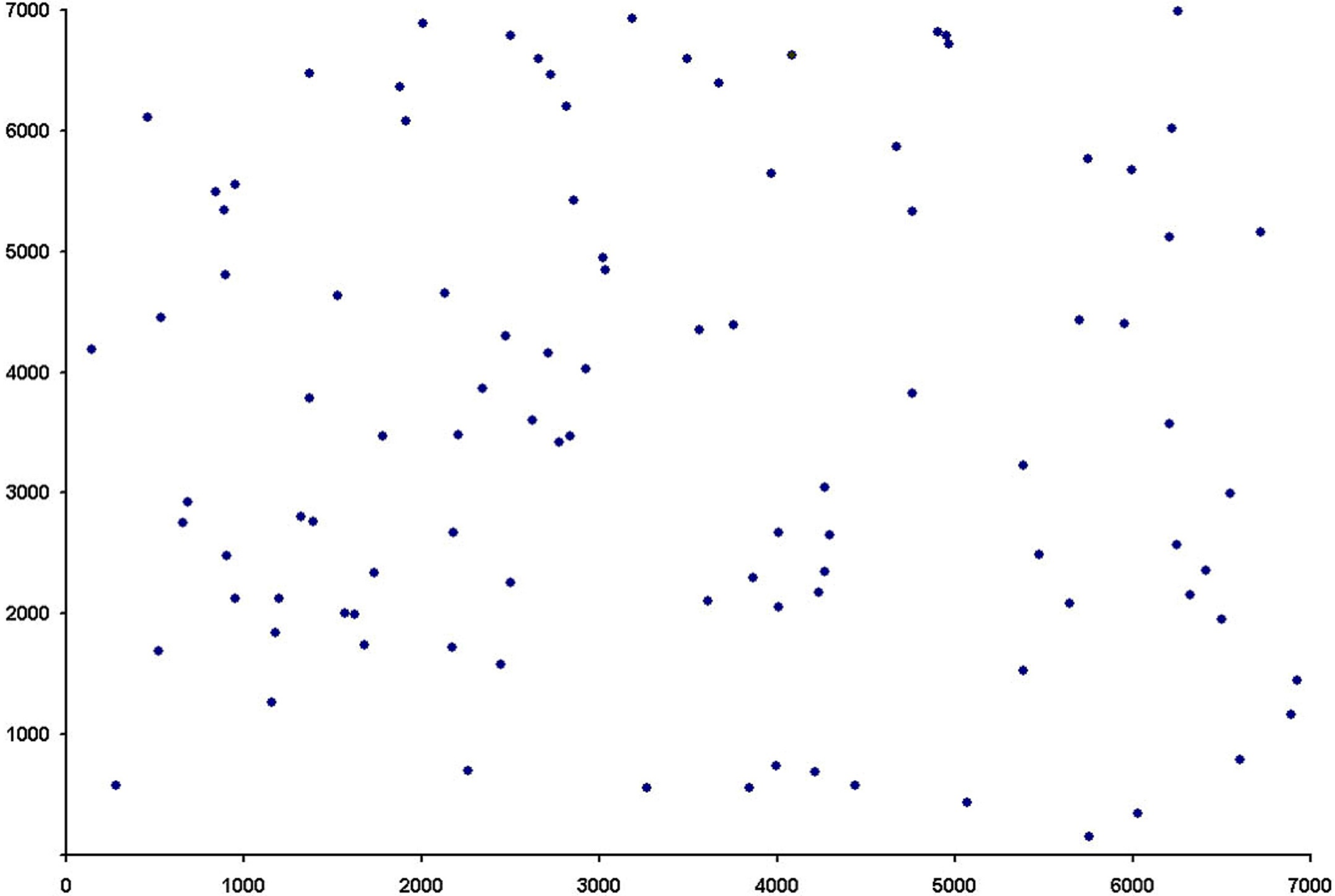


If the leader opens own facility in point 4,

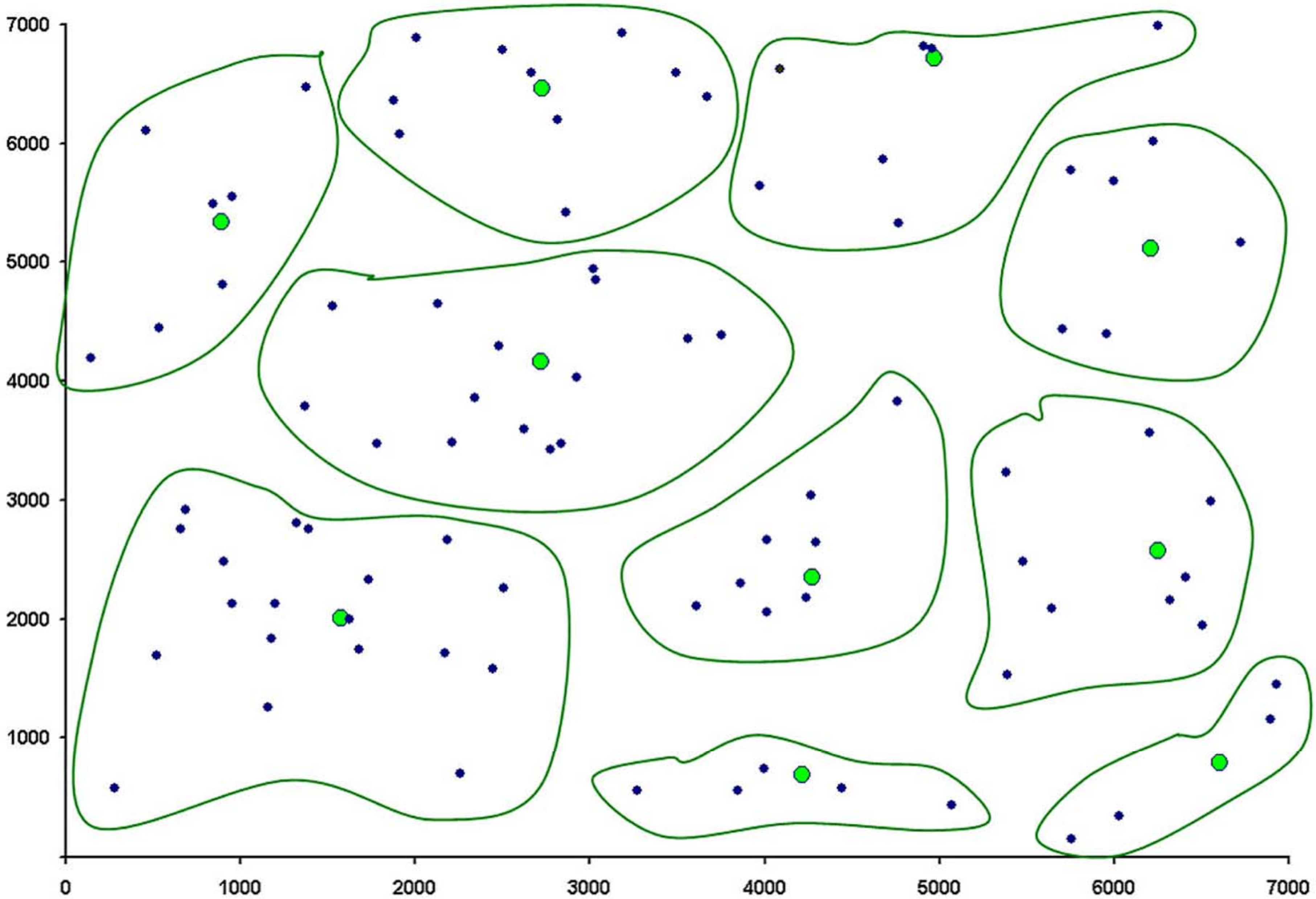
then the follower opens facility in point 2.



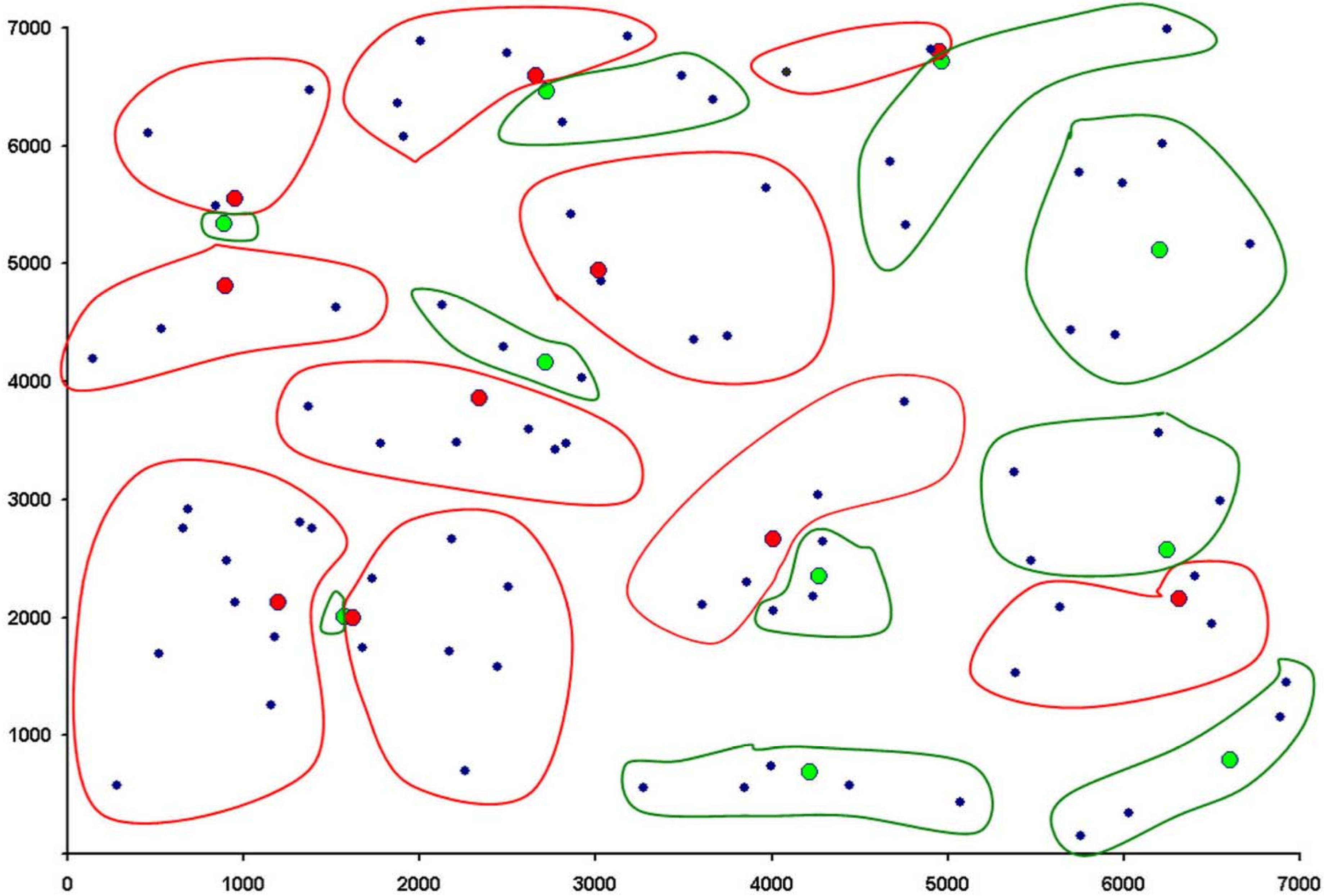
Euclidean instance, $|I| = 100$, $|J| = 100$



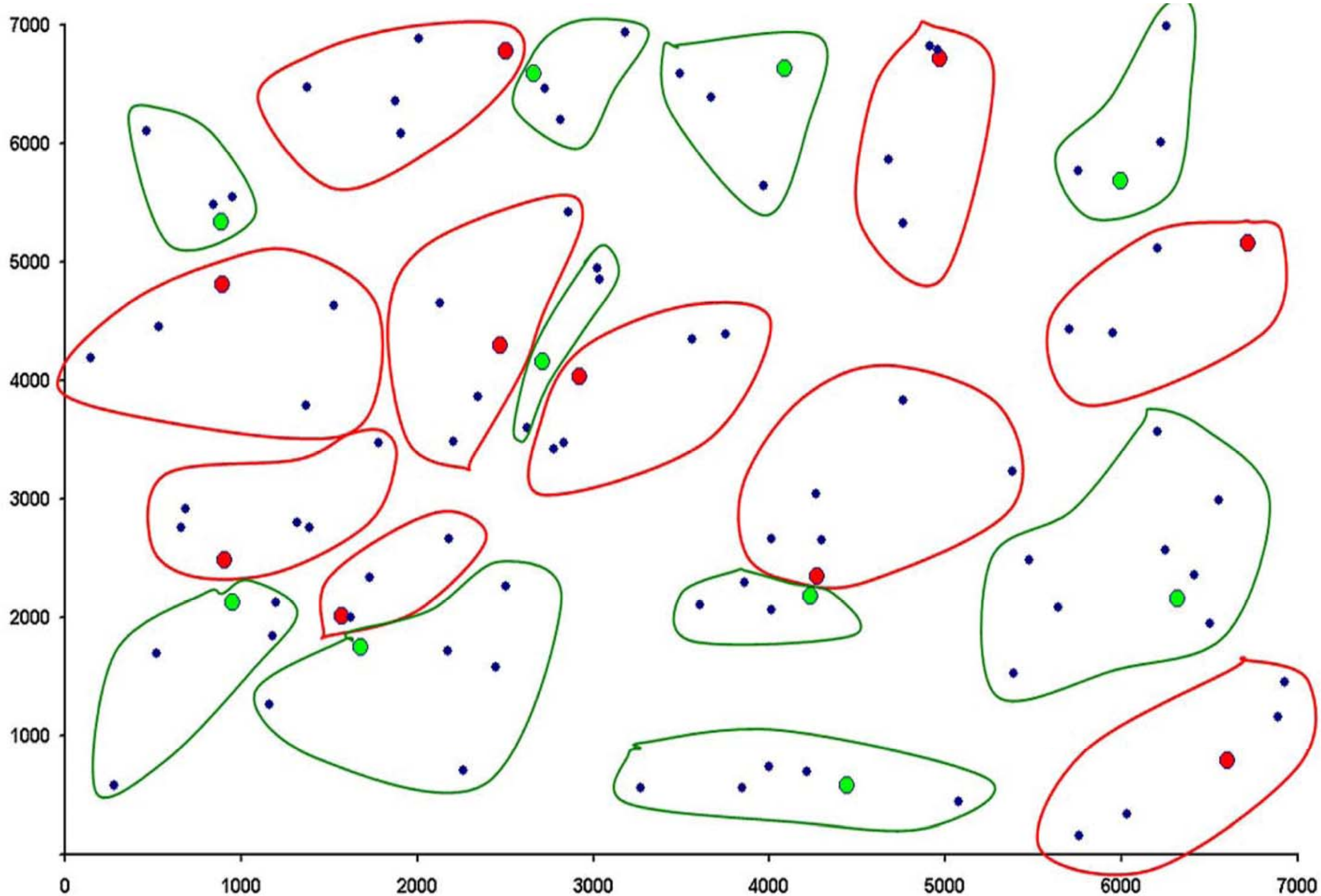
The leader ignores the follower



Optimal solution of the follower. Market share of the leader is 41 %



Optimal solution of the leader. Market share of the leader is 50 %



Conclusions and Further Research

We have considered some facility location models and presented theoretical and empirical results.

For further research

- More adequate behavior of the users
- Attractiveness of the facilities
- Profit instead of market share
- Multi-objective bi-level models
- Continuous locations on Euclidean plane,…