Consumption-based asset pricing

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Outline

Financial markets have a tremendous impact over many spheres of our lives to be smply ignored.

- While they have many distinguishing features, the basis of their functioning is the same as any other market.
- People try to make the best possible decisions, given information and circumstances
- Investment and consumption decisions are tightly linked
- The whole purpose of financial markets is to transfer wealth from one state of the world to another to enhance consumtion abilities

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Today we will look at the basic results in consumption-based asset pricing:

- main ideas behind the set of modelling techniques
- stylized facts of the data
- current strands of the literature

Salient features of financial data

International stock and bill returns

Country	Sample period	$\overline{r_e}$	$\sigma(r_e)$	$\rho(r_e)$	$\overline{r_f}$	$\sigma(r_f)$	$\rho(r_f)$
USA	1947.2–1998.4	8.085	15.645	0.083	0.896	1.748	0.508
AUL	1970.1–1999.1	3.540	22.699	0.005	2.054	2.528	0.645
CAN	1970.1–1999.2	5.431	17.279	0.072	2.713	1.855	0.667
FR	1973.2–1998.4	9.023	23.425	0.048	2.715	1.837	0.710
GER	1978.4–1997.4	9.838	20.097	0.090	3.219	1.152	0.348
ITA	1971.2–1998.2	3.168	27.039	0.079	2.371	2.847	0.691
JAP	1970.2–1999.1	4.715	21.909	0.021	1.388	2.298	0.480
NTH	1977.2–1998.4	14.070	17.228	−0.030	3.377	1.591	−0.085
SWD	1970.1–1999.3	10.648	23.839	0.022	1.995	2.835	0.260
SWT	1982.2–1999.1	13.744	21.828	−0.128	1.393	1.498	0.243
UK	1970.1–1999.2	8.155	21.190	0.084	1.301	2.957	0.478
USA	1970.1–1998.4	6.929	17.556	0.051	1.494	1.687	0.571
SWD	1920–1998	7.084	18.641	0.096	2.209	5.800	0.710
UK	1919–1998	7.713	22.170	−0.023	1.255	5.319	0.589
USA	1891–1998	7.169	18.599	0.047	2.020	8.811	0.338

Source: Cochrane (2002)

Salient features of financial data

International consumption and dividends

Country	Sample period	$\overline{\Delta c}$	$\sigma(\Delta c)$	$\rho(\Delta c)$	$\overline{\Delta d}$	$\sigma(\Delta d)$	$\rho(\Delta d)$
USA	1947.2–1998.4	1.964	1.073	0.216	2.159	28.291	−0.544
AUL	1970.1–1999.1	2.099	2.056	−0.324	0.656	34.584	−0.450
CAN	1970.1–1999.2	2.082	1.971	0.105	−0.488	5.604	0.522
FR	1973.2–1998.4	1.233	2.909	0.029	−0.255	13.108	−0.133
GER	1978.4–1997.4	1.681	2.431	−0.327	1.189	8.932	0.078
ITA	1971.2–1998.2	2.200	1.700	0.283	−3.100	19.092	0.298
JAP	1970.2–1999.1	3.205	2.554	−0.275	−2.350	4.351	0.354
NTH	1977.2–1998.4	1.841	2.619	−0.257	4.679	4.973	0.294
SWD	1970.1–1999.3	0.962	1.856	−0.266	4.977	14.050	0.386
SWT	1982.2–1999.1	0.524	2.112	−0.399	6.052	7.698	0.271
UK	1970.1–1999.2	2.203	2.507	−0.006	0.591	7.047	0.313
USA	1970.1–1998.4	1.812	0.907	0.374	0.612	16.803	−0.578
SWD	1920–1998	1.770	2.816	0.150	1.551	12.894	0.315
UK	1919–1998	1.551	2.886	0.294	1.990	7.824	0.233
USA	1891–1998	1.789	3.218	−0.116	1.516	14.019	−0.087

Source: Cochrane (2002)

Stylized facts

- Average real returns on the stocks are usually 4.5% or higher
- Short-term debt instruments tend to have a substantially lower return
- Annualized standard deviation of the stocks varies a lot (15-27%)
- Annualized volatility of the debt instruments is low (2-3%)
- Real consumption growth is low, with standard deviation rarely above 3%
- Both consumption and returns are procyclical
- Dividends are more volatile than consumption, but not as much as stocks

These features are persistent across all countries.

Hereinafter, I will make use of US data for illustrative purpose.

The basic model of asset pricing

An infinitely lived representative household maximizes his expected expected utility of consumption

$$E_t[U_t] = u(c_t) + E_t\left[\beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + ...\right] =$$
$$= u(c_t) + E_t\left[\sum_{i=1}^{i=\infty} \beta^i u(c_{t+i})\right]$$

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Each period he gets some proceeds from the old savings, and divides them between consumption and investment

$$W_{t+1} = (1 + R_{w,t+1})(W_t - c_t)$$

where W_t is the stock aggregate wealth, and $R_{w,t+1}$ is its net return.

Fundamental pricing relation

FOC: maximize utility w.r.t. each c_t (note the use of 2 budject constraints)

$$-u'(c_t) + \beta E_t \left[R_{w,t+1} u'(c_{t+1}) \right] = 0$$

utility loss from investing another dollar in the stock should be offset by the expected gain next period, or

$$1 = E_t \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} R_{w,t+1} \right]$$

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In general, we can write this as

$$1 = E_t \left(M_{t+1} R_{w,t+1} \right)$$

where M_{t+1} is a *stochastic discount factor*, a particular function of consumption and other variables, depending on the utility function and other features of the economy.

Stochastic discount factor

The fundamental pricing relation should work for all the assets, including the risk-free bond and the market return:

$$1 + R_f = \frac{1}{E_t\left[M_t\right]} 1 = E_t\left[M_{t+1} R_{m,t+1}\right]$$

Equivalently one can formulate for excess returns:

$$0 = E_t\left[M_{t+1} R^{ex}_{t+1}\right]$$

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Recalling that $E[MX] = cov(M, X) + E[M]E[X]$:

$$0 = Cov_t\left(M_{t+1}, R^{ex}_{t+1}\right) + \frac{E_t\left[R^{ex}_{t+1}\right]}{R_f}$$

$$E_t\left[R_{t+1}\right] - R_f = -R_f \, Cov_t\left[M_{t+1}, R_{t+1}\right]$$

In the equilibrium return of the asset is explained by its co-movement with the consumption. This is testable, once we specify utility function.

Mehra, Prescott (1985) *The equity premium: a puzzle*

Take a particular utility function, Constant Relative Risk Aversion (CRRA)

$$u(c_t) = \left(\frac{c_t^{1-\gamma}}{1-\gamma}\right) \;\Rightarrow\; 1 = E_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} R_{m,t+1}\right]$$

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β and γ can be estimated with GMM or a nonlinear regression straight away.
Note, that $M_{t+1} \approx \beta[1-\gamma\Delta c_{t+1}]$.

Other things being equal, the higher is risk-aversion, the larger is the covariance between R_{t+1} and M_{t+1}.

How big should be RRA to fit the real data?

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- to sum up, we need the real growth rate of nondurable consumption per capita and real returns on market and a short-term government bond to find their means and standard deviations.
- what kind of time stamp to take? The more, the better $\Rightarrow$ use quarterly.

Data

Where to get all this data? Two main free sources:

- Bureau of Economic Analysis with NIPA tables (www.bea.gov)
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Findings:

- pure variation in the consumption growth rate is too low to drive the risk premium through covariance with the stocks
- to match the average market return, you need risk aversion of > 90
- normal values of RRA generate the equity premium of only 1-2% pa, not required 6% $\Rightarrow$ *equity premium puzzle*
- normal values of RRA also generate too high risk-free rate, 12% pa, instead of the usual 2% $\Rightarrow$ *risk free rate puzzle*

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$$\frac{10^{1-\gamma}}{1-\gamma} = \frac{1}{2}\frac{X^{1-\gamma}}{1-\gamma} + \frac{1}{2}\frac{0}{1-\gamma}$$

$$\gamma = 1 + \frac{ln\frac{1}{2}}{ln\frac{X}{10}}$$

To sum up

The fundamental relation between macroeconomic variables and asset pricing

$$1 = E\left[M_{t+1} R_{t+1}\right]$$

where M_{t+1} is a stochastic discount factor, usually a function of marginal utility, consumption, and other things. Returns are driven by their covariance with the SDF.

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where M_{t+1} is a stochastic discount factor, usually a function of marginal utility, consumption, and other things. Returns are driven by their covariance with the SDF. The basic specification:

- consumption varies too slowly compared to the stock market;
- CRRA utility is not enough to amplify this co-movement to match the stylized facts.

Potential solutions

Good overviews:

- Mehra, Prescott (2003) *The equity premium in retrospect*
- Ludvigson(2012) *Advances in consumption-based asset pricing: empirical tests*

Overall, the literature focuses on

- measurement error in data
- changing preferences (utility channel)
- introducing market frictions (budget constraint channel)
- introducing a production sector in the economy

Savov(2011)

- Consumption data is difficult to accurately measure
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- Garbage growth rates are more procyclical $\Rightarrow$ don't need very high risk-aversion
- Estimated RRA goes does down from 89 to 19
- Good, but still too high... Measurement error in consumption cannot explain all of it (and many other features).

Consumption vs garbage

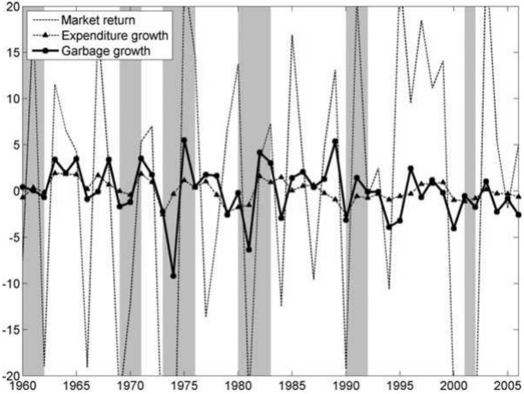

Figure 1. The time series of garbage growth and expenditure growth. This figure shows the time series of realized annual garbage growth and NIPA nondurables and services expenditure growth over the sample 1960 through 2007 against the contemporaneous excess return on the CRSP value-weighted portfolio. Gray bands indicate NBER recessions. All three series are demeaned for ease of comparison. The growth of consumption measure c is c_{t+1}/c_t, using the beginning-of-period convention.

Source: Savov (2011)

Recursive preferences: Epstein and Zin (1989, 1991)

$$V_t = \left[(1-\beta)C_t^{1-\rho} + \beta B_t(V_{t+1})^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

$$B_t(V_{t+1}) = \left(E_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}$$

where V_{t+1} is the continuation value of the future consumption plan, γ is the RRA, $1/\rho$ is the elasticity of intertemporal substitution.

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- When $\theta = \gamma$, we get the usual CRRA utility.
- Disentanges 1 period risk attitude with intertemporal substitution
- Any shock to current consumption will have a more prominent impact on utility, due to a stronger desire to smooth consumption over time
- Thus, there is no need to have a too high one-period risk aversion
- No analytical solution, more complicated econometrics, but explaines lots of stuff!
- **A standard building block in any consumption-related model.**

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- Matches many empirical findings surprisingly well
- Downside 1: even more complicated algebra and econometrics (calibration)
- Downside 2: relies on investors being able to correctly identify this slow-moving component (Croce, Lettau, Ludvigson, 2010)

So far, this is **the best framework to buid on**. It is flexible enough to accomodate many other feature, but also sturdy to successfully work on its own in many other aspects.

Asset pricing with habits

Maybe what matters, is not the consumption/utility level per se, but how it relates to the benchmark X_t

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A shock to consumption propagates further by getting you away from the desired "habit" level, hence, higher impact on utility, lower risk-aversion, etc.

Limited market participation

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Typically, only a small proportion of the population invests in financial markets. Constantinides and Duffie (1996), Mankiw and Zeldes (1991), Guvenen (2009)

Asset prices will be mostly driven by the dynamics and shocks of the people, actively participating in trading. The aggregate consumption can be slow-moving.

Credit channel to non-participants, spillover effects.

Successfully explains the equity premium puzzle and many other features.

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- Alongside with long-run risk models explains many salient features of the data
- Wachter(2010): time-varying probability of disaster, matching the yield curve
- Bansal, Shaliastovich (2010, 2011, 2013): implications for time-varying volatility, predictability, etc