4) What if MPK (&APK) varies with y?

<u>Solow Model:</u>

Same as AK model, except production function changes:

1') $Y_t = AF(K_t,N_t)$

$y_t = Y_t/N_t = AF(k_t,1) \equiv Af(k_t)$

GRAPH f(k)

f(k) exhibits decreasing returns to scale in k. Why?

Also, assume Inada conditions are satisfied:

$f'(k)\to 0$ as $k\to\infty$ & $f'(k)\to\infty$ as $k\to 0$

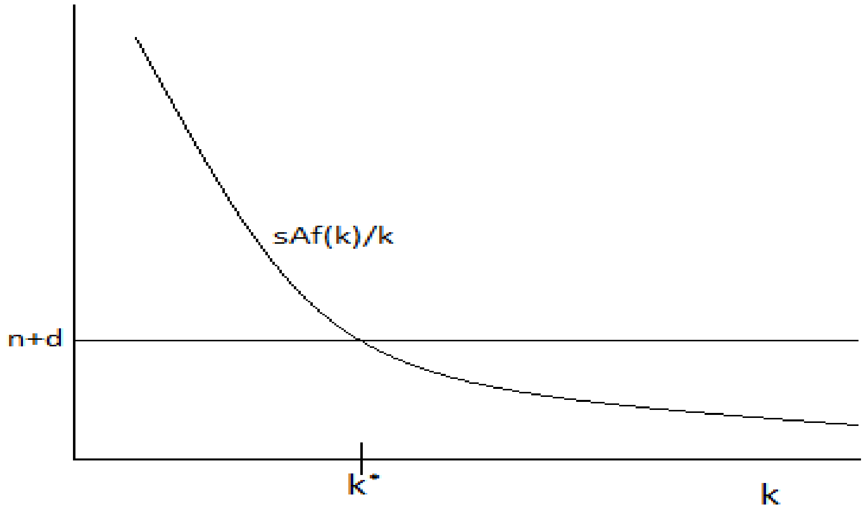
2–5) remain the same

$g_k = sAf(k_t)/k_t - (n+d)$

$y_t = Af(k_t)$

$c_t = (1-s)y_t$

SOLOW GRAPH



IMPLICATIONS

1- Capital per person k_t and output per person approach constant, "steady-state" levels k^* and y^*, determined by parameters s,A,d,n (See graph)

k^* from setting $g_k=0$; $y^*=Af(k^*)$

2- A *temporary* increase in the GDP/capita growth rate, and a *permanent* increase in long-run GDP/capita level, results from an increase in s, A, or a decrease in n

3- GDP per capita can only grow in a sustained way if A grows in a sustained way

s, n cannot provide sustained growth $\Rightarrow$ productivity growth, factor accumulation not "created equal"

4- Countries converge in GDP/capita (provided A, s, n, and d are similar across countries)

?? Where does the difference come from between Solow and H-D?

DRS in capital.

Policy Implications of this model:

Same as H-D: s,A up, n down for increased growth

But, the increased growth rate is only temporary here, not permanent as in H-D

(while the increase in the long-run income level is permanent)

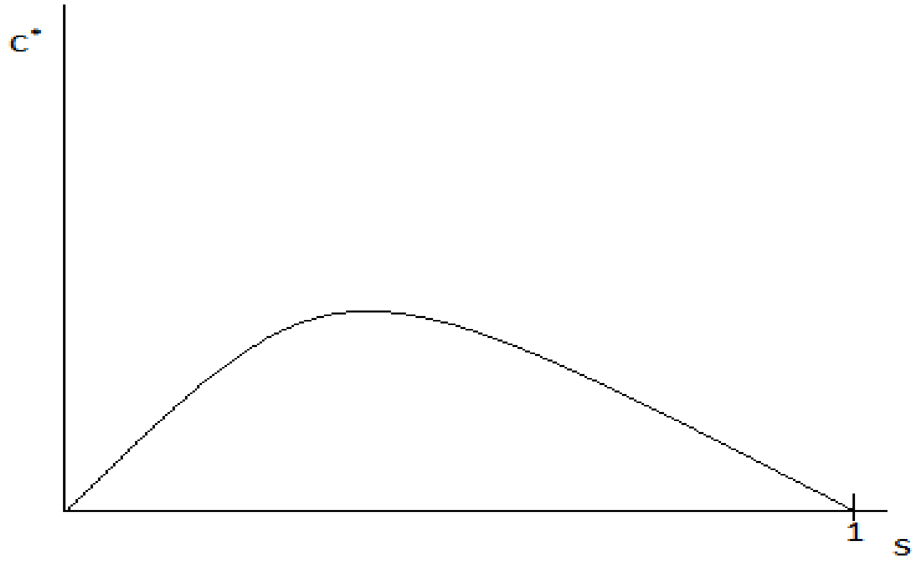
IF THE GOAL IS A PERMANENT INCREASE IN GROWTH of GDP/capita – only policies that lead to FASTER sustained growth in A

?? What savings rate maximizes long-run GDP per capita?

s=1 (show graphically)

But then $c_t = 0$, since $c_t = (1–s)y_t =0$.

HEURISTIC GRAPH OF c^* vs. s



HUMAN CAPITAL

Physical capital = machines, buildings, etc.

Human capital = the accumulation of investments *in people* that affect their productivity

Ex. Education, health/nutrition

Why call it capital?

1) Durable – depreciates, but not immediately
2) Grows via investment – diverting resources away from production
3) Increases worker's productivity

The more educated the workforce, the more productive they are with each unit of capital

?? How is it different from productivity A?

The economic term is:

H_t – National human capital (usually education)

$h_t \equiv H_t/N_t$ = average human capital (e.g. avg education level)

<u>H-Augmented Solow Model</u>

1'') $Y_t = AF(K_t, H_t, N_t)$

$y_t = Y_t/N_t = AF(k_t, h_t, 1) \equiv Af(k_t, h_t)$

f displays decreasing returns to scale in k and h together

+ Inada conditions on k and h

2') $\dot{\mathbf{K}}_{\mathbf{t}} = I_{k,t} - dK_t$

$\dot{\mathbf{H}}_{\mathbf{t}} = I_{h,t} - dH_t$

$I_{k,t}$ - investment in physical capital K at t

$I_{h,t}$ - investment in human capital H at t

Now decision is not only how much to invest, but how to divide investment between human and physical capital

Note: depreciation rates are probably different, but let's keep them the same for simplicity

3', 4') $sY_t = I_{k,t}$

$qY_t = I_{h,t}$

s - rate of investment in physical capital

q - rate of investment in human capital

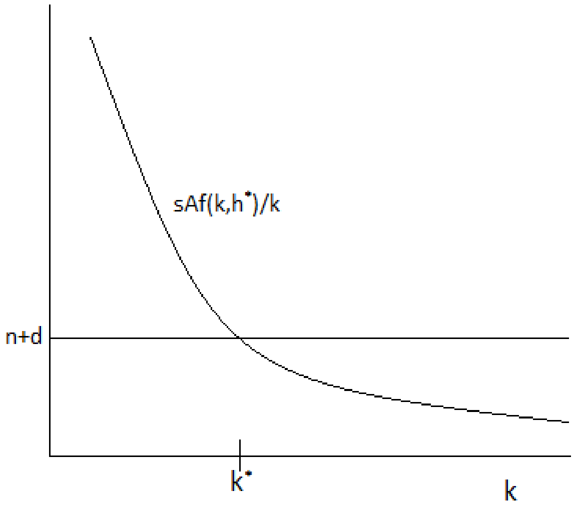
5) as before

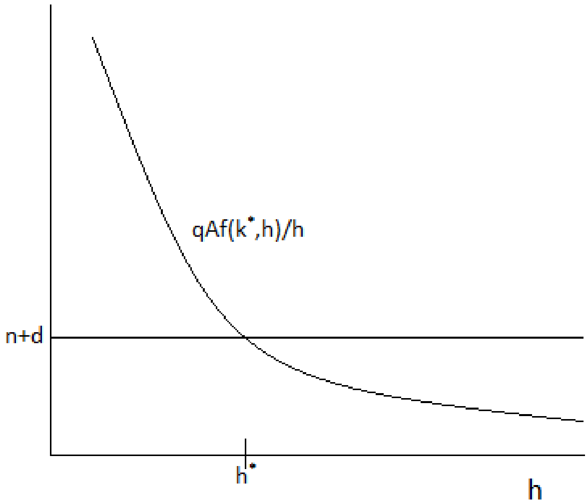
H-augmented Solow key equations:

$g_k = sAf(k_t,h_t)/k_t - (n+d)$
$g_h = qAf(k_t,h_t)/h_t - (n+d)$

Also, $y_t = Af(k_t,h_t)$ and $c_t = (1-s-q)y_t$

SOLOW-STYLE GRAPHS





<u>Implications of the Model</u>

1- Physical capital per person k_t, human capital per person h_t, and output per person approach constant, "steady-state" levels k^*, h^* and y^*, determined by parameters s,q,A,d,n

Find k^* and h^* by simultaneously setting $g_k = g_h = 0$

2- A *temporary* increase in the GDP/capita growth rate, and a *permanent* increase in the long-run GDP/capita level, results from an increase in s, A, q (← key difference #1) or a decrease in n

The growth effect is bigger and lasts longer, compared to the Solow model (← key difference #2)

$y^* = Af(k^*, h^*)$

3- GDP per capita can only grow in a sustained way if A grows in a sustained way

4- Countries converge in GDP/capita (provided A, s, q, n, and d are similar across countries)

Theory <> Evidence ⇒ Policy

<u>Testing the models</u>

Mankiw, Romer, Weil (1992) take Solow literally

$y= Ak^\alpha$

Find y*: … Setting $g_k=0$ gives $k^* = [sA/(n+d)]^{1/(1-\alpha)}$

$y^* = Af(k^*) = A\,[sA/(n+d)]^{\alpha/(1-\alpha)} = A^{1/(1-\alpha)}\, s^{\alpha/(1-\alpha)} / (n+d)^{\alpha/(1-\alpha)}$

Linearize by taking logs:

$\ln y^* = 1/(1-\alpha) \ln A + \alpha/(1-\alpha) \ln s - \alpha/(1-\alpha) \ln (n+d)$

d set to 5% [Actually, d set to 3%, g set to 2%, so n+d+g = n+5%].

s, n from data on I/Y ratio average for 1960-1985, population growth rates, respectively.

⇒ Empirical specification:

$\ln y_i = \beta_0 + \beta_1 \ln s_i + \beta_2 \ln (n_i+0.05) + \varepsilon_i$

What do we expect?

• $\beta_1 = -\beta_2 = 1/2$ (if $\alpha=1/3$)

What did they get?

• $R^2 = 0.59 \Rightarrow$ 59% of variation in GDP per capita explained by s and n! (high)

• β_1: 1.42; on β_2: −1.97 $\Rightarrow$ right signs, but too big!

The data indicate growth is more responsive to s, n than basic Solow theory predicts

Using the H-Solow model:

Can solve for y^*: if $y = Ak^\alpha h^\beta$, then $y^* = s^{\frac{\alpha}{1-\alpha-\beta}} q^{\frac{\beta}{1-\alpha-\beta}} A^{\frac{1}{1-\alpha-\beta}} \left(\frac{1}{n+d}\right)^{\frac{\alpha+\beta}{1-\alpha-\beta}}$

$\ln y^* = 1/(1-\alpha-\beta) \ln A + \alpha/(1-\alpha-\beta) \ln s + \beta/(1-\alpha-\beta) \ln q - (\alpha+\beta)/(1-\alpha-\beta) \ln (n+d)$

d set to 5%.

$\Rightarrow$ Empirical specification:

$\ln y_i = \beta_0 + \beta_1 \ln s_i + \beta_2 \ln (n_i+0.05) + \beta_3 \ln q_i + \varepsilon_i$

q measured by secondary enrollment rates (due to data limitations)

What did they get?

- $\beta_1 = .69$, $\beta_2 = .66$, $\beta_3 = -1.73$; (implying $\alpha, \beta \approx 0.3$)
- $R^2 = 0.78 \Rightarrow$ 78% of variation in GDP per capita explained by s, q, and n! (not A)

1) H-Solow model fits data well
2) Elasticities are right/reasonable

$\Rightarrow$ differences in rates of human and physical capital accumulation and population growth rates explain a lot! Differences in A (unmeasured here) account for less than 20% of differences