

Economic Growth

Basic Theories of (Trend) Growth

?? What might be behind differences in growth and income?

Culture?

Institutions?

Economic setup?

Output == fn(Inputs, Efficiency)

Growth in output = fn(Growth in inputs, growth in efficiency)

Growth in production then comes from

- 1) growth in inputs – “factor accumulation”
- 2) growth in efficiency – “technological progress”, etc.

Among inputs, economists have focused on capital.

Capital = tools, objects, goods that extend our ability to produce other goods and services

e.g. machines, buildings, infrastructure, vehicles, computers, software

[from Jones, p. 49]

Data show that in 2000, the average worker in _____ worked with _____ of capital :

U.S.	\$148,091
Mexico	\$42,991
India	\$6,720

National Level:

Y_t – national income (GDP) at time t

K_t – national capital stock at time t [equipment]

X_{1t}, \dots, X_{Jt} -- other inputs (e.g. human capital)

A – total factor productivity (TFP) (constant for now) [techniques]

A embodies knowledge, ideas, and methods of organizing inputs into output

N_t – population at time t

$y_t = Y_t/N_t$ – income per capita at time t

$k_t = K_t/N_t$ – capital per capita at time t

In general, lower case denotes per capita values

Production Function

$$Y_t = AF(K_t, X_{1t}, X_{2t}, \dots, X_{Jt}, N_t)$$

Assume F exhibits CRS in all factors of production together

$$F(zK, zX_1, \dots, zX_J, zN) = zF(K, X_1, \dots, X_J, N), z > 0$$

$$\begin{aligned} y_t &= Y_t/N_t = AF(K_t, X_{1t}, \dots, X_{Jt}, N_t)/N_t \\ &= AF(K_t/N_t, X_{1t}/N_t, \dots, X_{Jt}/N_t, 1) \\ &= AF(k_t, x_{1t}, \dots, x_{Jt}, 1) \equiv Af(k_t, x_{1t}, \dots, x_{Jt}) \end{aligned}$$

CRS implies per capita output depends only on per capita amounts of inputs

AK Model:

Key Elements:

1) Production Function – CRS in K

$$Y_t = AF(K_t) = AK_t$$

A, total factor productivity, here assumed constant over time

$$y_t = AK_t/N_t = Ak_t = Af(k_t)$$

2) Evolution of Capital

$$\dot{K}_t = I_t - dK_t$$

I_t – national investment at time t

d – rate at which capital depreciates (assumed constant over time)

3) Macroeconomic Balance: Savings = Investment

$$S_t = I_t$$

[Note from macro, production $Y_t = C_t + I_t (+ G_t + NX_t)$; income $Y_t = C_t + S_t$. So $S_t = I_t$.]

4) Savings Behavior – constant savings rate

$$S_t = sY_t$$

S_t – national savings at time t

s – savings rate (assumed constant over time)

5) Population Growth

$$\dot{N}_t/N_t = n$$

Overall goal: How fast will this economy grow, i.e. what is g_y ?

First show $\frac{\dot{y}_t}{y_t} = \frac{\dot{k}_t}{k_t}$.

Next show $\frac{\dot{k}_t}{k_t} = \frac{\dot{K}_t}{K_t} - \frac{\dot{N}_t}{N_t}$

Let $g_x \equiv \frac{\dot{x}_t}{x_t}$, $g \equiv \frac{\dot{y}_t}{y_t}$

Rearrange to show $g_y = sA - d - n$:

$$g_k = \frac{\dot{K}_t}{K_t} - \frac{\dot{N}_t}{N_t} = (I_t - dK_t)/K_t - n \quad (\text{Using 2\&5})$$

$$= S_t/K_t - d - n \quad (\text{Using 3})$$

$$= sY_t/K_t - (n+d) \quad (\text{Using 4})$$

$$= sy_t/k_t - (n+d)$$

$$= sAf(k_t)/k_t - (n+d) \quad (\text{Key Equation in general})$$

$$= sA - (n+d) \quad (\text{Using 1})$$

IMPLICATIONS:

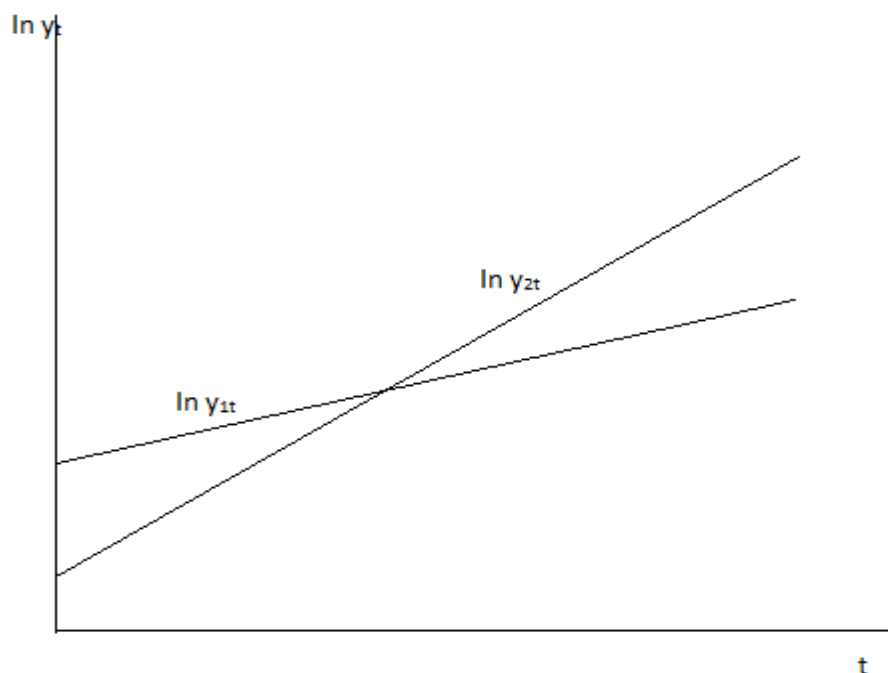
- 1- Growth continues indefinitely at a constant rate depending on s , A , d , and n
- 2- g increases in the savings rate, s , and the level of productivity, A
 g decreases in population growth, n , and rate of depreciation of capital, d
- 3- changes in s , A , n affect the growth rate permanently
- 4- No force for convergence or catch-up in GDP/capita

Ex. $A = 0.5$, $n = 2\%$, $d = 8\%$; $s_1 = 22\%$, $s_2 = 28\%$.

$$\Rightarrow g_1 = 0.5 * 0.22 - 0.08 - 0.02 = 1\% \quad (\text{in 50 years, } e^{0.01 * 50} = 1.6x \text{ as well off})$$

$$g_2 = 0.5 * 0.28 - 0.08 - 0.02 = 4\% \quad (\text{in 50 years, } e^{0.04 * 50} = 7.4x \text{ as well off})$$

GRAPH COUNTRIES – they diverge over time



What is optimal savings rate?

$s=1$ maximizes growth, since $g = sA - d - n$

C_t – national consumption at time t

$$C_t = Y_t - S_t = Y_t - sY_t = (1-s)Y_t$$

$c_t \equiv C_t/N_t$ – consumption per person at t

$$*** c_T = (1-s) y_T = (1-s) y_t e^{g(T-t)}$$

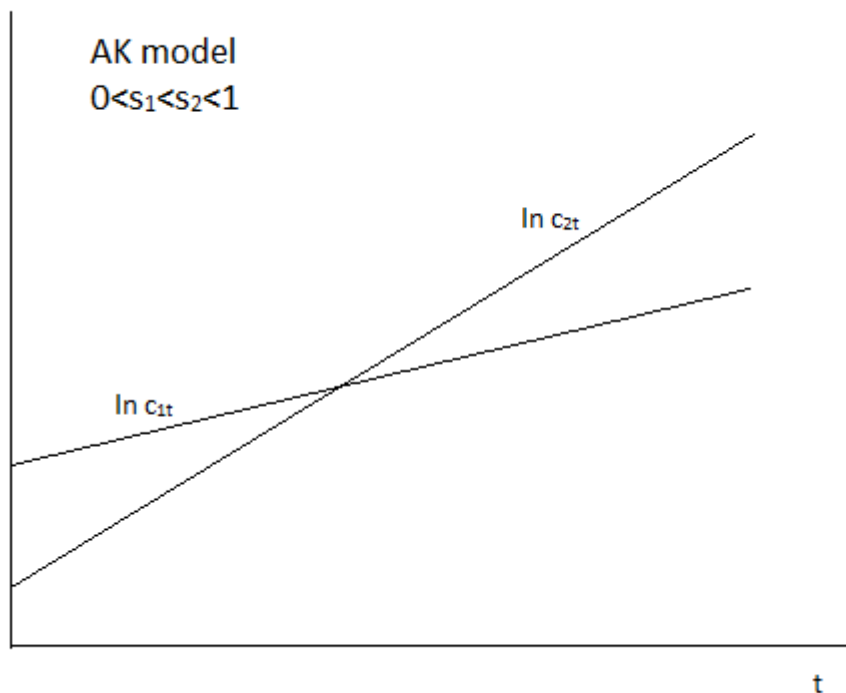
But then $c_T = 0$ if $s=1$.

So growth maximization may not be the best goal – need some intermediate savings rate.

What is the optimal savings rate?

A higher savings rate always lowers consumption now but raises it in the future (provided $s < 1$)

GRAPH of $\ln c_t$ vs. time for two different savings rates



The more patient (or future-regarding) you are, the higher your optimal savings rate

?? Policy Implications of this model:

IF THE GOAL IS TO INCREASE GROWTH RATE

Increase the savings rate.

Government saving – tax, and invest in firms or public capital (e.g. Soviet Union)

Reduce taxes on saving/investing (assuming elastic)

e.g. capital gains tax, tax on interest income, corporate tax, estate tax

Firm-level investment subsidies

Increase productivity.

Technology assimilation programs

Protect intellectual property rights

Govt.-funded research

Create a business-friendly, competition-friendly environment (not over-regulated)

Lower population growth

Coercion, tax incentives, women's education, etc.

KEY EQUATION, USEFUL ACROSS MODELS:

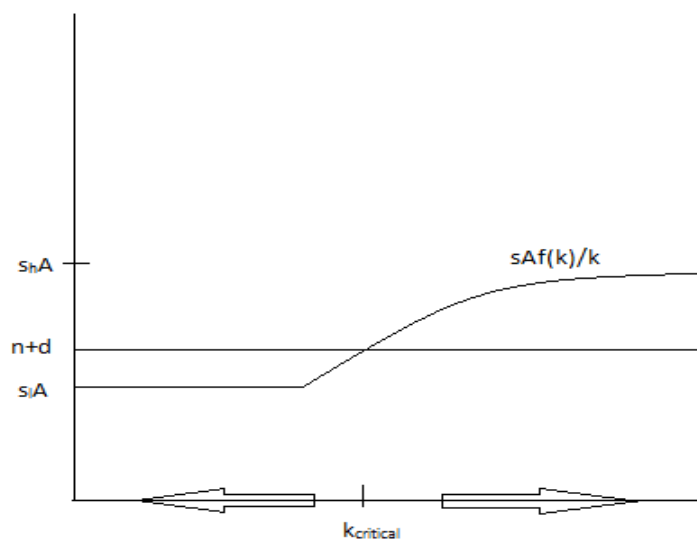
$$**g_k = sAf(k_t)/k_t - (n+d)$$

1) What if s varies with y ?

In general, **Poverty trap**: low income \Rightarrow low growth \Rightarrow low future income ...

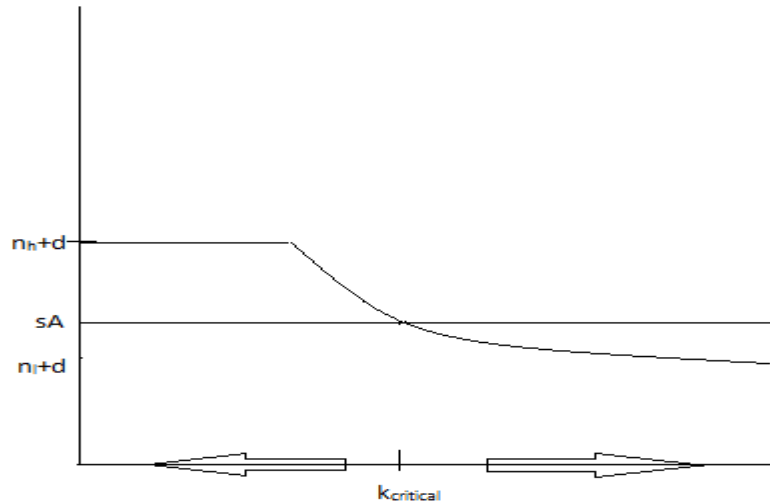
Savings-based poverty trap: low income \Rightarrow low savings/investment per worker \Rightarrow cannot maintain or grow the capital stock \Rightarrow low growth \Rightarrow low income ...

Assume AK model, except that marginal propensity to save is s_l below some income level, and s_h above that level. Graphically:



2) What if n varies with y ? Assume n is high when income is low, n is low when income is high

Population-growth based poverty trap: low income \Rightarrow high population growth \Rightarrow higher investment requirements to maintain high capital/worker \Rightarrow cannot maintain or grow the capital stock \Rightarrow low growth \Rightarrow low income ...



(3) Health-based poverty trap: low income \Rightarrow poor nutrition, health \Rightarrow low productivity \Rightarrow low income per worker \Rightarrow savings/investment per worker \Rightarrow cannot maintain or grow the capital stock \Rightarrow low growth \Rightarrow low income ...

