Caveats/critiques:

Different measurements of human capital reduce explanatory power to 50-60%

(Klenow, Rodriguez-Clare 1997) – leaving bigger role for A

Is the causation really in this direction?

What are the underlying causes of s, q and n?

<u>Endogenizing s (Solow model):</u>

We've simply assumed a fixed savings rate – we can endogenize it by utility maximization.

Ignore population growth for the moment. Switch to discrete time.

$\max_{\{c0,c1,c2,\ldots;\, k1,k2,\ldots\}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$

s.t. $c_t + k_{t+1} \leq Ak_t^{\alpha} + (1-d)k_t$, for t=0,1,2,… [$\lambda_t$ multiplier]

k_0 given

First-order conditions:

$[c_t]$: $\beta^t c_t^{-\sigma} = \lambda_t$

$[k_{t+1}]$: $\lambda_{t+1}[\alpha A k_{t+1}^{\alpha-1} + 1 - d] = \lambda_t$

Combining, we get the "Euler equation":

$c_t^{-\sigma} = \beta c_{t+1}^{-\sigma}[\alpha A k_{t+1}^{\alpha-1} + 1 - d]$

LHS: Marginal utility from consuming output today

RHS: Marginal utility from saving output today, investing it in tomorrow's capital stock, which raises output tomorrow by the MPK (+ whatever capital does not depreciate), and thus allows for consuming more output tomorrow

Look for steady state, where c, k, y are all constant.

$1=\beta(\alpha A/k^{*1-\alpha} + 1-d)$

$k^* = [\alpha A/(1/\beta - 1+d)]^{1/(1-\alpha)}$

$y^* = Ak^{*\alpha} = A^{1/(1-\alpha)} [\alpha/(1/\beta - 1+d)]^{\alpha/(1-\alpha)}$

Note that every year, y gets split between consumption and savings/investment.

From budget constraint above: $c_t + k_{t+1} \leq Ak_t^{\alpha} + (1-d)k_t$

i.e. in steady state: $c^* + k^* = Ak^{*\alpha} + (1-d)k^*$

or $c^* + dk^* = y^*$

Consumption = $(1-s)y^* = c^*$, so $s = (y^*-c^*)/y^* = dk^*/y^*$

(savings = investment ⇔ $sy^* = dk^*$ (need to replace depreciated capital).)

So, $s = dk^*/y^*$ = $\mathbf{d\,\alpha/(1/\beta - 1+d)}$.

Increases in α, β, and d. Independent (in SS) of A, σ.

Only Solow technology

$$Y_{St} = A_{St} K_t^{\theta} N_t^{1-\theta}$$

$$C_t + K_{t+1} = Y_{St} \quad (1) \quad (d=1) \text{ full depreciation}$$

$$y_{St} = A_{St}\left(\frac{K_t}{N_t}\right)^{\theta} = A_{St} k_t^{\theta}$$

[Here, let $g_X \equiv$ gross growth rate of X (growth factor for X) = 1 + growth rate of X

Ex. $X' = g_X X$ (not $X' = (1+g_X)X$)]

What does Balanced Growth Path ("BGP") look like (given $g_{A_S} = \gamma_S > 1$)?

From (1): $g_C = g_K = g_{Y_S}$ on BGP

Clearly also $g_c = g_k = g_{y_S}$

$$g_{y_S} = \gamma_S g_k^{\theta} = \gamma_S g_{y_S}^{\theta}$$

$$g_{y_S}^{1-\theta} = \gamma_S \Rightarrow \boxed{g_{y_S} = \gamma_S^{\frac{1}{1-\theta}}}$$

Only depends on rate of techn. progess

Only Malthusian technology

$$Y_{Mt} = A_{Mt} K_t^{\phi} N_t^{\mu} L_t^{1-\phi-\mu}$$

L_t = land = 1 (fixed factor)

$$y_{Mt} = A_{Mt} k_t^{\phi} \ell_t^{1-\phi-\mu} = \frac{A_{Mt} k_t^{\phi}}{N_t^{1-\phi-\mu}}$$

First, assume $g_N = n$, a constant.

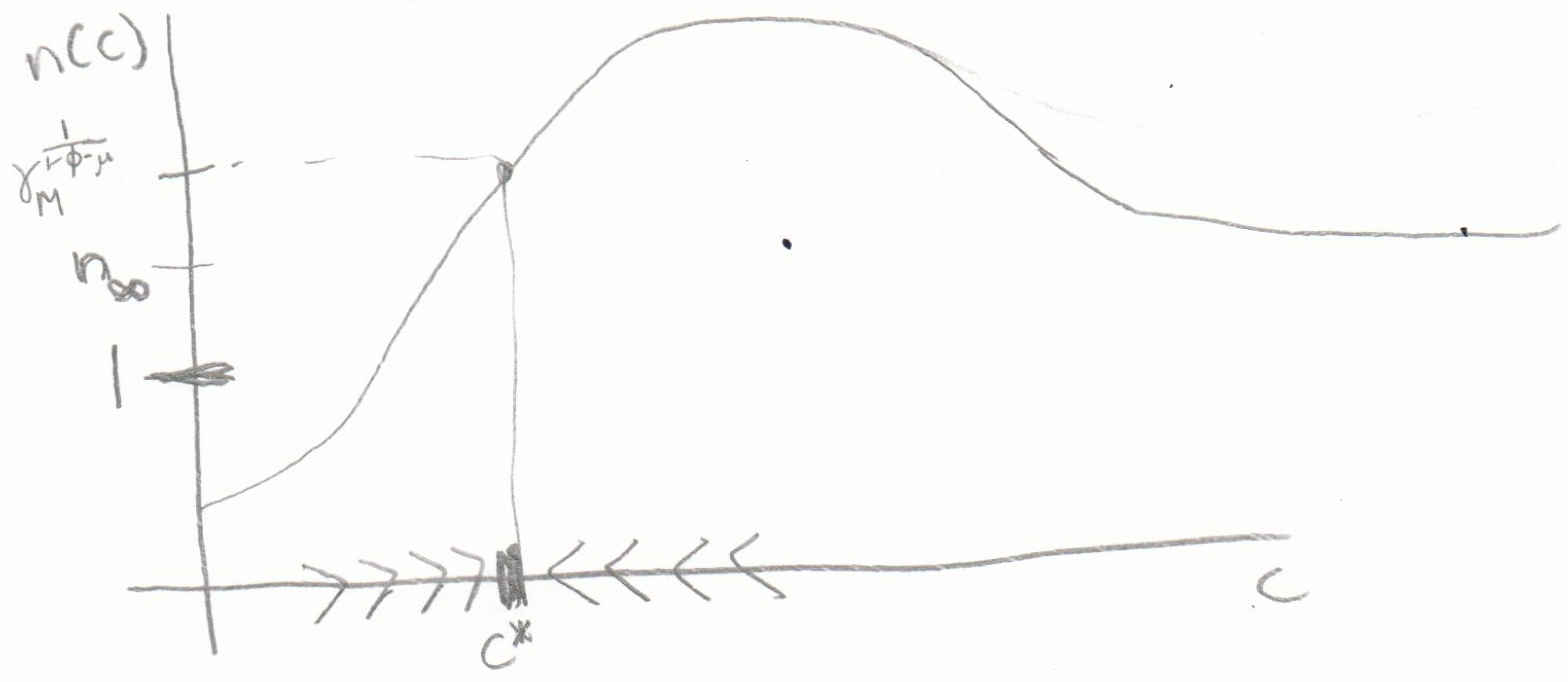
$$g_{y_M} = \frac{\gamma_M g_k^{\phi}}{n^{1-\phi-\mu}} = \frac{\gamma_M g_{y_M}^{\phi}}{n^{1-\phi-\mu}} \Rightarrow \boxed{g_{y_M} = \left(\frac{\gamma_M}{n^{1-\phi-\mu}}\right)^{\frac{1}{1-\phi}}}$$

Population growth puts a drag on income growth, because of the fixed factor land.

Sustained growth is still possible, but not inevitable, if population is growing – it becomes a race btw technological progress and population growth (which lower's land per person).

Malthusian Trap

Now assume $g_N = n(c)$ (as Hansen/Prescott do)



Now there is a steady state (ie. trap, no growth in income)

Recall, in BGP/SS: $g_{Y_m}^{1-\phi} \, n(c)^{1-\phi-\mu} = \gamma_M$

Define c^* st. $n(c^*) = \gamma_M^{\frac{1}{1-\phi-\mu}}$. Then $c = c^*$, $g_{Y_m} = 1$ is a SS.

This is a stable trap involving stagnation in c & y.

If y & c grow a little, n rises & g_y must go below one.

If y & c drop a little, n falls & g_y goes above one.

(No trap with Solow technology - does not depend on n in steady state.)

What is happening in the trap?

$$y_{Mt} = A_{Mt} \, k_{mt}^{\phi} \, l_{Mt}^{1-\phi-\mu}$$

$$\underset{\gamma_M}{\uparrow} \qquad \underset{\gamma_M}{\downarrow}$$

Technology advancing, exactly balanced by decline in land per person.

Malthus to Solow

Imagine both technologies are available through all history.

$\gamma_S > 1$, $\gamma_M > 1$. Uninformative approach: assume $\gamma_S > \gamma_M$ (or similar). $A_{M0} \gg A_{S0}$

Instead, Assume Malthusian technology initially less costly.

$\Rightarrow$ early history characterized by Malthusian trap.

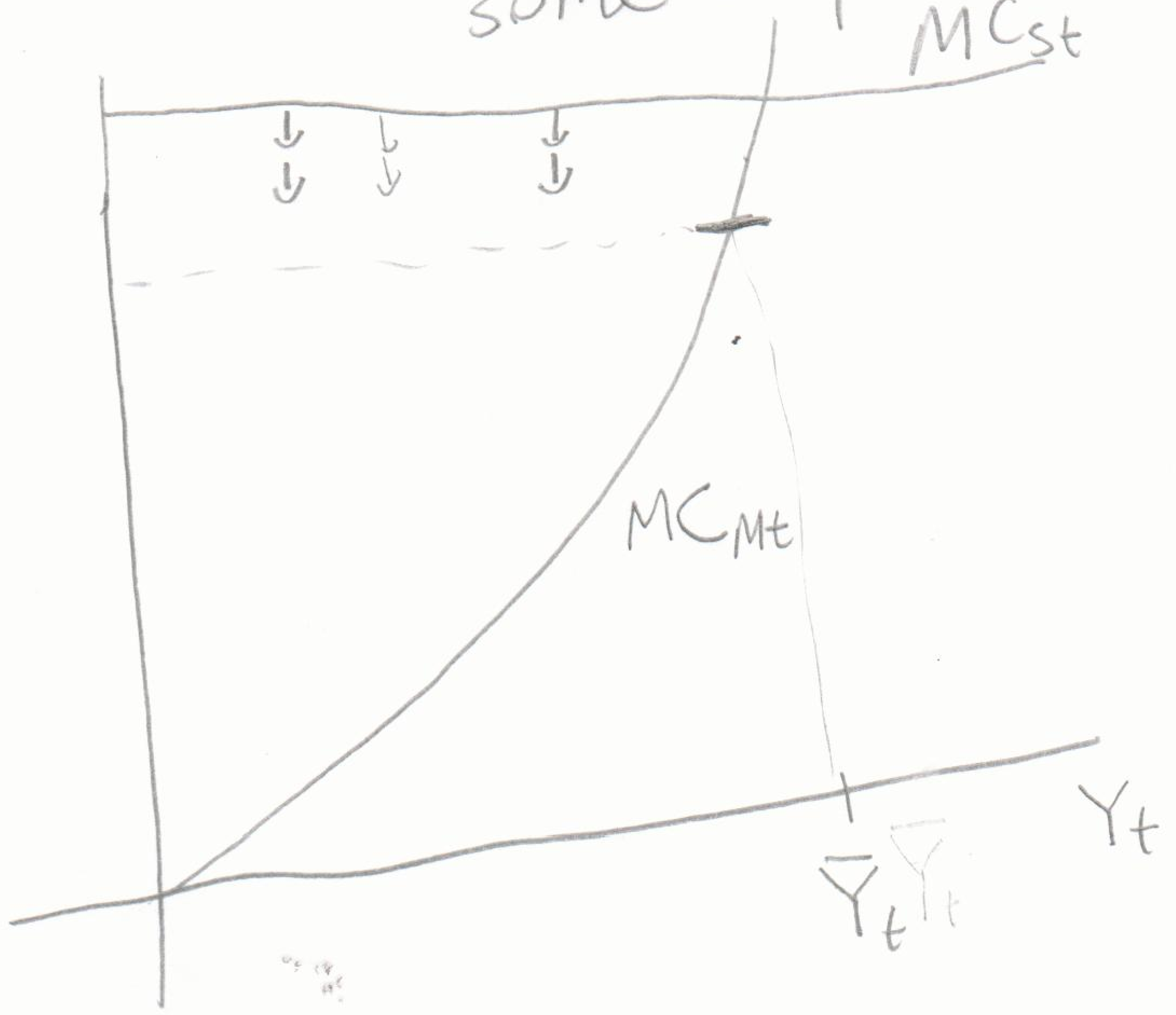
Can also show: Marginal Cost of Output using Malthusian technology also stagnates

$A_{Mt}\uparrow \Rightarrow$ cost $\downarrow$ } Cancel out
$l_t \downarrow \Rightarrow$ cost $\uparrow$

Marginal Cost of Solow technology $\searrow$

$A_{St}\uparrow \Rightarrow$ cost $\downarrow$ } $\downarrow$
no fixed factor

So, eventually Solow technology becomes cheaper for some output, regardless of γ_S vs. γ_M.



Since Solow technology does not have the population growth drag, it can get the economy past demographic hump, to modern economic growth.

Malthus to Solow, Hansen & Prescott, AER 9/2002

Explain within a single model without changing parameter values a long human history of stagnation in per capita living standards followed by sustained exponential growth in living standards, with declining share of land in production.

The basic idea is that there are **two technologies** (though one sector), both CRS and requiring labor and capital; but one additionally requires the fixed factor land (the Malthusian technology) while the other (the Solow technology) does not. TFP evolves through history separately for both technologies, and exogenously. The other main building block is a Malthusian dependency, namely that the population growth rate is a hump-shaped function of per capita living standards. With the Malthus technology, one can get stuck on the left side of the peak of this Malthusian hump.

One can show that, at a given r and w, the unit cost of producing output using the Malthusian technology is increasing and begins at zero; the unit cost of the Solow technology is constant. Thus, the Malthusian technology will always be used, and if Solow TFP is relatively low enough, it will be used exclusively.

Fix land at 1. Note that for the Malthus technology, if capital's and labor's shares are φ and μ, resp., then $y = Ak^{\varphi}/N^{1-\varphi-\mu}$. One can show that in a BGP, c, k, w, and y all have to grow at the same rate, factor g say, so if N grows at factor n, and A at factor γ, then $g = [\gamma/n^{1-\varphi-\mu}]^{1/(1-\varphi)}$. So, if $n<\gamma^{1/(1-\varphi-\mu)}$, c is growing and n is increasing; and vice versa (as long as this all takes place to the left of the Malthusian hump; otherwise, there can be sustained per capita growth even with the Malthusian technology). Thus, n will gravitate toward $\gamma^{1/(1-\varphi-\mu)}$, at which point g=1, i.e. c, w, k, and y all stagnate, while n grows at $\gamma^{1/(1-\varphi-\mu)}$. This is in line with the Malthusian idea of all TFP growth fueling population growth while living standards stagnate.

Since w (and r) stagnate, the marginal cost of producing the final unit of output using the Malthusian technology remains fixed over time. Thus, as long as TFP in the Solow technology is increasing and not asymptoting, it will eventually lower Solow unit costs below this fixed Malthusian marginal cost of producing the final unit of output. Remarkably, this does not depend on a horserace – as long as Solow TFP eventually crosses a fixed threshold, it will eventually start to be used, since the Malthusian costs are stagnating – even though Malthusian TFP is growing, lowering costs, land per population is declining, and this raises costs, balancing to stagnation.

Thus, Solow is used when Solow TFP crosses a threshold. At this point, Solow technology thus relieves some of the cost pressures of the Malthusian technology (think of an increasing cost curve that flattens at the end). This allows for a rise in income and consumption per capita. This causes population to grow faster (if the rise in consumption keeps n above where it was in the Malthusian steady state), which essentially raises Malthusian costs: population growth rate pressures and M-TFP growth rate had balanced, but with the former rising, costs rise. (Note that the Solow technology has no fixed factor and thus no population-related drag on growth. Indeed, $y = Ak^{\theta}$, so on a BGP $g = \gamma^{1/(1-\theta)}$, which is independent of n.) This leads to even more substitution away from Malthusian technology toward Solow. As long as consumption keeps rising [it could actually fall below its Malthusian steady state level if the population growth rate is super-responsive to the increase in consumption, so as to cause consumption then to fall – this can create boom-bust cycles of consumption and output] or more specifically, stays in the range so that population growth is above its Malthusian steady state level, population cost pressures will rise faster than TFP in the Malthusian technology, and it will become more and more costly. Thus, the economy will substitute toward Solow technology [and this even if the Solow technology had just experienced a one-time increase].

The basic reason is that the Solow technology does not have population-related cost pressures on growth.

If the Solow TFP increase is big enough (one-time or sustained), it will bring consumption past the Malthusian hump-point and the economy will be past the Malthusian trap. It will enjoy sustained growth from either technology based on TFP growth (for Solow, population growth does not matter; for Malthus, it would probably asymptote to a low enough level, below $\gamma_M^{1/(1-\varphi-\mu)}$, that sustained growth is possible). Ultimately, if both TFP's grow at a fixed rate, in the model without capital Solow will end up dominant in the long run if $\gamma_S >= \gamma_M/n_\infty^{1-\varphi-\mu}$, and Malthus will dominate in the long run otherwise. (The reason is that when consumption gets high enough, the asymptotic population growth rate n_∞ may now be below its Malthusian rate, so that the Malthusian technology is getting cheaper again: TFP growth is outstripping population cost pressures on land. Malthusian technology will eventually be cheaper if the above inequality does not hold.)

Unified Growth Theory, Handbook chapter, Galor, draft April 1, 2005

Reviews data on Malthusian period, transition, etc.

Reviews first unified growth model, Galor & Weil AER 2000.
They have log preferences over own consumption and a mix of quantity/quality of children, with a subsistence constraint for own consumption. They have technological progress that is a positive function of population size and average human capital. Meanwhile, human capital of next generation is a positive function of parental time and a decreasing function of the aggregate growth rate, with a positive cross-partial, meaning parental input is more valuable in producing human capital when growth is faster. Output per capita is produced by a Cobb-Douglas in effective resources (think land*A) per person and human capital per capita.

The Malthusian phase comes when productivity is low, so consumption is low, and people are at a corner solution and not able to have many children due to the subsistence constraint – they would have less consumption and more kids were it not for the constraint. Population is also low, so productivity growth is low. Because of low productivity growth, it does not pay to invest in human capital, so all quantity/no quality. Population also grows very slowly, since people can't have many children. So, consumption isn't changing, all of the (slow) productivity growth feeds into population growth with no investment in human capital.

But this trap cannot persist forever. If population were stagnant, then effective resources would be growing since A is growing (even if slowly), and this means people are able to have more kids, a contradiction. Further, population cannot be growing at the same rate as A, as in a BGP, for the growing population means A is accelerating. Thus, in the Malthusian phase, consumption is stagnant, productivity growth is giving rise to greater (realized) fertility rates and higher population and population growth, and this is causing A to accelerate, fueling further population growth, etc.

This by itself would be enough for a gradual transition to faster growth and growing living standards, because of one "tripwire" in the model: when productivity gets high enough so that consumption is no longer at a corner solution, because fertility rates have been freed up enough to no longer be too low, both kids (quantity/quality mix) and consumption are at unconstrained maxima. Now, (realized) fertility is at its maximum, since the unconstrained optimal fraction of time is devoted to child-rearing (no income effect on fraction of time devoted to kids, due to log preferences), and with no human capital investment, it is devoted all to quantity of children. Thus the population growth rate is at its maximum. Here, productivity growth gives rise to more consumption, as it raises output per person, based on a fixed fraction of time that goes to production (the complement going to having kids). Population is growing maximally, accelerating productivity growth and the growth in consumption.

The second "tripwire" kicks in when productivity growth exceeds some bound. This raises the returns to human capital, and it becomes optimal to substitute away from quantity toward quality of children. Hence, fertility rates decline and human capital rises. The rise in human capital feeds into greater productivity growth, which further raises returns to human capital. People thus spend the same amount of time on kids, but more of it on quality (human capital production) rather than quantity. In this way, we have an acceleration of productivity growth and human capital, but also this model also produces a demographic transition, where population growth slows down as people substitute toward quality away from quantity of kids.

[The paper says that initially as human capital starts to be invested in, this results in growing quantity and quality of kids, but I don't see this – quantity was at its maximum, so any increase in quality results in a decrease in quantity of kids, i.e. decelerating population.]

To sum up, the first tripwire (breaking the subsistence constraint) corresponds to post-Malthusian transition and the first phase of the Industrial Revolution, where human capital is not yet invested in and growing income feeds into growing population and growing consumption. [Again, the population growth rate is fixed and maximal during this era in the model.] Then, the second tripwire initiated the transition to modern economic growth and the demographic transition, as people started investing in human capital and substituting away from quantity of kids toward quality.

In short, they model an endogenous demographic transition as well as transition from Malthusian stagnation to modern economic growth, focusing on the growing role of human capital, and assuming certain determinants of productivity growth (human capital and population size) and a subsistence consumption constraint.