

Problem Set 1

Nizhny Novgorod HSE
Masters Growth and Development
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Answers must be written neatly by hand, or by computer. Students may work in groups, but final answers turned in must be the individual's. Justify all answers.

(Problems 2&4 consider the Solow and H-augmented Solow model – you may want to wait until Tuesday's lecture to complete those problems.)

Throughout the problem set, total factor productivity A is constant.

1. Imagine the following goal of Lenin/Stalin at the beginning of the Soviet regime in Russia: to overtake and surpass the world's industrialized economies in terms of GDP per capita. To achieve this goal, the main instrument of control is the fraction of national production that is devoted to building the nation's productive capacity (i.e. investment as a share of GDP): new machines, factories, transportation equipment, and roads (i.e. capital). The rest of national production is used for consumer items like clothing and food. The country begins with relatively little capital, being mostly rural and non-industrialized. Assume each of the following:

- GDP per capita starts in USSR at \$500/year.
- The world's industrialized economies start with GDP per capita of \$5000/year.
- Population growth rates are 2% everywhere in the world.
- All capital depreciates at 8% per year.

a. At what average rate will income per capita in the USSR have to grow in order to overtake (i.e. to equal) the industrialized nations' income per capita in exactly 30 years? Assume the industrialized nations' income per capita is growing at 2% per year.

b. If the USSR sustains the growth rate of part a., how long *after* it has overtaken the industrialized nations' GDP per capita will it take for it to attain **double** the industrialized nations' GDP per capita? Again, assume the industrialized nations' GDP per capita is growing at 2% per year.

For parts c.-e., assume the basic growth framework of the AK model, and that 1 ruble's worth of capital always produces 0.5 ruble's worth of output (i.e. $A=0.5$). Also, assume inputs are used more efficiently in the industrialized countries, so that $A=0.6$ there.

c. What fraction of national output must the USSR devote to building new capital goods in order to attain the growth rate of part a.? What fraction would be left for consumer items? [Hint: another word for the fraction of output devoted to building new capital goods is the investment rate, i.e. the ratio I_t/Y_t . And, remember that savings equals investment, so the investment rate equals the savings rate.]

d. At what rate are the industrialized countries saving if they are growing at 2% per year?

e. What would you calculate the ratio of consumption per capita in the USSR to consumption per capita in the industrialized countries when the USSR overtakes the industrialized countries (i.e. when GDP per capita is equal)? Assume the savings rates of parts c. & d. What would the ratio be when the USSR reaches double the industrialized nations' GDP per capita?

2. Consider the same scenario as in problem 1, but now assume the Solow model is correct. Assume each of the following:

- GDP per capita starts in Russia at \$500/year.
- The world's industrialized economies start with GDP per capita of \$5000/year.
- Population growth rates are 2% everywhere in the world.
- All capital depreciates at 8% per year.

Further assume $y = 400k^{1/3}$ in Russia.

a. Solve for Soviet long-run GDP per capita (y^*) as a function of its savings rate.

For parts b.-d., assume long-run GDP per capita equals \$10,000 in the industrialized countries, and that they are saving 10% of income.

- b. What fraction of national output should be devoted by the Soviet Union to building new capital goods in order to overtake, i.e. **equal**, the industrialized nations' GDP per capita in the long-run? What fraction is left for consumer items?
- c. What fraction of national output should be devoted by the Soviet Union to building new capital goods in order to surpass, i.e. **double**, the industrialized nations' GDP per capita in the long-run? What fraction is left for consumer items?
- d. In the long run, what is the ratio of Soviet GDP per capita to GDP per capita in the industrialized countries, and what is the ratio of Soviet consumption per capita to consumption per capita in industrialized countries if the Soviet Union achieves the goal of part b? of part c? (That is, find four ratios.)
- e. Comparing the outcomes of part d. of this question to the same question answered using the AK model (problem 1), which model produces a more optimistic outlook for achieving, by saving and investing at high rates, the Soviet goals of overtaking and surpassing industrialized countries' living standards? Remember that living standards are best measured by consumption here.

3. Assume the framework of the AK model, with the following modification. A household's savings rate, specifically its marginal propensity to save, is low when its income is below \$1000. It then switches to a higher rate when income exceeds \$1000. Specifically:

- $n = 1\%$
- $d = 5\%$
- $A = 0.5$
- $MPS_{\text{low}} = 0.1$
- $MPS_{\text{high}} = 0.25$

Assume there is no income inequality. What are the growth dynamics in this economy? Specifically, answer the following questions.

- a. Derive the average savings rate in the economy as a function of capital per worker, k . [Hint: this is useful because the relevant savings rate in the key growth equations, e.g. the equation for g_k , is the average savings rate, not the marginal propensity to save.]
- b. What is the average income level above which sustained growth will occur, and below which it will not occur?
- c. What is the asymptotic (i.e. long-run) growth rate for an economy that starts with enough income to sustain growth? How does the income growth rate evolve over time for such an economy (i.e. does it increase or decrease)?
- d. What is the long-run income for an economy that starts poor enough?
- e. How would income inequality affect growth prospects in this model? Specifically, holding fixed the average income level, would higher inequality raise or lower the average savings rate of the economy? A discussion or diagram without a formal proof is fine.

4. Compare the effect on long-run income of a fifty percent increase in the savings rate, e.g. from 20% to 30%, in the Solow model vs. the H-augmented Solow model. In the case of the Solow model, assume $Y=AK^\alpha N^{1-\alpha}$, while in the H-augmented Solow model, $Y=AK^\alpha H^\alpha N^{1-2\alpha}$. For example, $Y=AK^{1/3}N^{2/3}$ in the Solow model, and $Y=AK^{1/3}H^{1/3}N^{1/3}$ in the H-augmented Solow model. What do you think accounts for the difference?

Optional Challenge Problems

5. Consider a mix of the AK model and the Solow model. In particular, let $Y=AK + BK^\alpha N^{1-\alpha}$, with $0 < \alpha < 1$. All other aspects of the economy are the same as in the AK and Solow models.
 - a. What is the condition for sustained long-run growth to be possible? Is this similar to the AK or Solow model?
 - b. Assuming a growing economy, how does the growth rate of k evolve over time (i.e. does it increase or decrease)? Is this similar to the AK or Solow model?
 - c. If two countries are identical except that one has a small capital stock and the other a large one, how do the growth rates (of k) of these two countries compare? Is this similar to the AK or Solow model?

d. How does an increase in the savings rate s affect the long-run growth rate of the economy? What condition does this depend upon? Is this similar to the AK or Solow model?

e. In what ways does this model capture the best predictions of both the AK and Solow models?

6. Consider the AK or Solow model, except generalize the production function to be of the constant-elasticity-of-substitution form:

$$Y = A \{ a(bK)^\psi + (1-a)[(1-b)L]^\psi \}^{1/\psi},$$

where $0 < a < 1$, $0 < b < 1$, and $\psi < 1$. As $\psi \rightarrow 1$, the production function approaches linearity, i.e. capital and labor are perfect substitutes. As $\psi \rightarrow -\infty$, the production function approaches a fixed proportion (Leontief) one, with no substitutability. As $\psi \rightarrow 0$, the production function approaches Cobb-Douglas, with elasticity of substitution equal to 1.

a. Verify the production function exhibits constant returns to scale.

b. Does $f(k)/k \rightarrow 0$ as $k \rightarrow \infty$? What does it depend upon? Does $f(k)/k \rightarrow \infty$ as $k \rightarrow 0$? What does it depend upon?

c. Under what conditions is sustained growth possible? Be as specific as possible.

d. Under what conditions is long-run income zero? Be as specific as possible.

e. If the conditions in c.&d. are not met, what is the steady state income level? Does it increase or decrease with s , A , and n , respectively?

f. Why is sustained growth sometimes possible even with diminishing returns to capital?