FIVE MINUTES SOLAR ATMOSPHERE OSCILLATIONS DUE TO THE INSTABILITY OF ACOUSTIC-GRAVITY WAVES

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1. Basic equations for acoustic-gravity waves in the nonisothermal atmosphere

The linearized system of equations of gas dynamics for the pressure disturbances p, the horizontal velocity u and the vertical velocity w is well known. We select axis z in the vertical direction against the gravity acceleration $\vec{g}$ and axis x in the horizontal direction, ρ_0 is the basic state density, c_s is a sound velocity.

We propose a model for the well-known five-minute vertical oscillations of the solar atmosphere. The existence of such oscillations is usually related to acoustic gravity waves. The properties of these waves are useful for explaining processes in the chromosphere and the lower corona, because the magnetic-field forces in these regions of the solar atmosphere can be relatively small on the corresponding space and time scales. However, the frequencies of the observed oscillations correspond to imaginary vertical wave numbers. This fact obstacles to an interpretation of the observed oscillations. S. Kaplan in 1977 had an opinion that nonisothermity of solar atmosphere is very important for this problem.

$$\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x},$$

$$\rho_0 \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} - \rho g,$$

$$\frac{\partial \rho}{\partial t} + w \frac{d\rho_0}{dz} + \rho_0 \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) = 0,$$

$$\frac{\partial p}{\partial t} + w \frac{dp_0}{dz} = c_s^2 \left(\frac{\partial \rho}{\partial t} + w \frac{d\rho_0}{dz} \right)$$

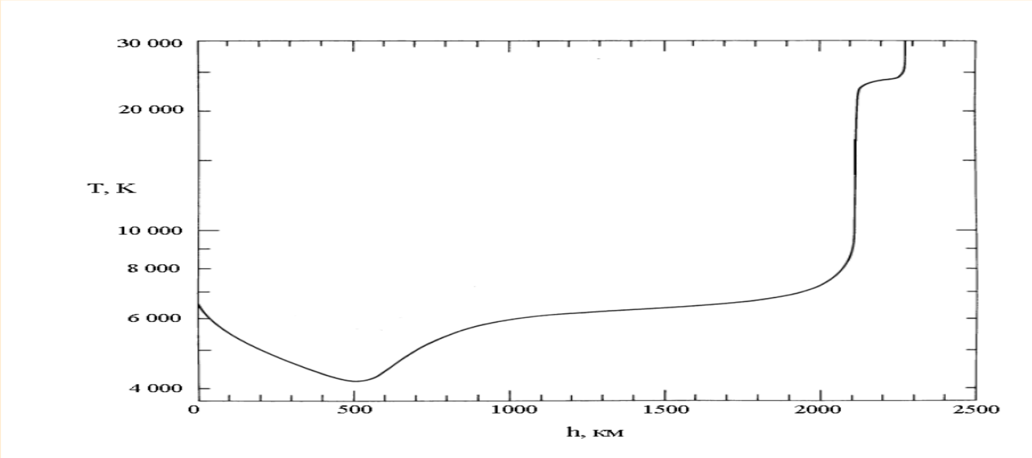
Really, the background temperature T depends on vertical coordinate. The regular pressure p_0 depends on an altitude according to a condition of an thermodynamic equilibrium

$$p_0(z) = p_{00} \exp\left[-\int_0^z \frac{dz'}{H(z')} \right]$$

The regular density is described by a relation

$$\rho_0(z) = \frac{p_0(z)}{gH(z)},$$

Real temperature dependence has a rather complicated form



The following equation for the vertical velocity w component can be obtained from the system in the foregoing slide.

$$c^2\left(\frac{\partial^2}{\partial t^2}-c^2\frac{\partial^2}{\partial x^2}-c^2\frac{\partial^2}{\partial y^2}\right)\frac{\partial^4 w}{\partial t^2\partial z^2}+\gamma g\left[c^2\left(\frac{dH}{dz}+1\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)-\frac{\partial^2}{\partial t^2}\right]\frac{\partial^3 w}{\partial t^2\partial z}-$$

$$-[\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2}{\partial t^2}-c^2\frac{\partial^2}{\partial x^2}-c^2\frac{\partial^2}{\partial y^2}\right)^2+c^2\gamma g^2\frac{dH}{dz}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2+$$

$$+\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)g^2(1-\gamma)\left(\frac{\partial^2}{\partial t^2}-c^2\frac{\partial^2}{\partial x^2}-c^2\frac{\partial^2}{\partial y^2}\right)w=0$$

2. Characteristic frequencies of acoustic-gravity waves

The Solar atmosphere has two resonance eigenfrequencies

Brunt-Vaisala frequency

The value of this frequency we can obtain from the equation of a harmonic oscillator, if horizontal wave number $k \to \infty$

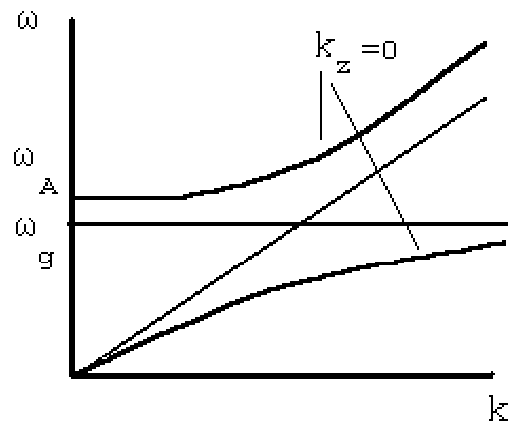
$$\frac{\partial^2 w}{\partial t^2} + \omega_g(z)^2 \frac{\partial^2 w}{\partial z^2} = 0$$

in such solution

$$u(t,x,z) = 0; \qquad p(t,x,z) = 0$$

In the real atmosphere eigenfrequency of same oscillations depends on an altitude

$\omega_g(z)^2 = g\left(-\frac{d\rho_0}{dz}\frac{1}{\rho_0} - \frac{1}{c_s^2}\right)$ is Brunt-Väisäla frequency



Another important eigenfrequency is

The lowest frequency for a fast branch

of acoustic gravity wave

$\omega_A = -\frac{c_s}{2}\frac{d \ln \rho_0}{dz}$ (E.E. Gossard, W.H. Hooke, 1975)

3. The lowest mode of acoustic-gravity wave in a nonisothermal atmosphere

In this section we discuss the frequency and characteristics of the lowest mode of acoustic-gravity wave in a nonisothermal atmosphere

3.1. The simplest model problem with an exact solution

The lowest mode in isothermal atmosphere with sharp boundary (z>0)

This task is reduced to the equation for the lowest mode

$$\frac{\partial^2 w}{\partial t^2} + \gamma g \frac{\partial w}{\partial z} - c_s{}^2 \frac{\partial^2 w}{\partial z^2} = 0 \, .$$

The solution of this equation with boundary conditions, when

$w(z=0)=0$ is following

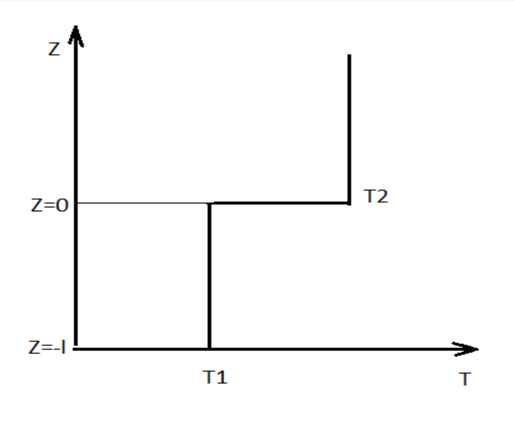
$$W(z) = A \cdot z \cdot \exp\left(z/2H\right) \exp\left(i\omega t\right) ,$$

$$\omega^2 = \frac{\gamma g}{4H} = \omega_A^2 ,$$

here ω_A is the lowest frequency for the fast branch of acoustic-gravity waves in the isothermal atmosphere.

Consider now a model of wave propagation in an atmosphere with step-like temperature profile. Let the upper half-space (z>0) with ideal gas of temperature T_1 and the lower half-space (z<0) with ideal gas of temperature T_2.

The lowest mode in atmosphere with temperature jump



The solution for eigenmode has a folloing form:

$$P_n = A_n(\omega, k_\perp)\exp\left[(-1)^n \kappa_n z\right]$$

$$W_n = B_n(\omega, k)\exp\left[(-1)^n \kappa_n z\right]$$

and satisfied with the boundary condition:

1.On the photosphere

$$w(z=0)=0.$$

2. On the transition region (boundary condition as on a free surface)

$$W_1 = W_2;\, p_1 + \rho_0 g W_1/i\omega = p_2 + \rho_0 g W_2/i\omega, \text{ where } p = \frac{i\rho_0}{\omega}\left[gw - c^2\frac{\partial w}{\partial z}\right],$$

$$\kappa_n = \sqrt{(\omega_{An}^2 - \omega^2)/c_n^2 + k^2(\omega^2 - \omega_{gn}^2)/\omega^2}$$

The characteristic equation for such waves has a form

$$\frac{\rho_{01}\left(\gamma H_1 \kappa_1 \mathrm{ctg}(\kappa_1 l) + \frac{\gamma}{2} - 1\right)}{\left(\omega^2 - c_1^2 k^2\right)} = \frac{\rho_{02}\left(i\kappa_2 \gamma H_2 + \frac{\gamma}{2} - 1\right)}{\left(\omega^2 - c_2^2 k^2\right)} + \frac{\rho_{02} - \rho_{01}}{\omega^2}$$

For the lowest mode $k=0$, then κ_n has an imaginary value, the frequency for the lowest eigenmode of atmosphere oscillation is $\omega \approx \omega_{A2}$

3.2. The exact solution for the lowest mode

The equation for the lowest mode of acoustic-gravity waves in a medium without viscosity and heat conductivity

$$\frac{\partial^2 w}{\partial t^2}+\gamma g\frac{\partial w}{\partial z}-c(z)^2\frac{\partial^2 w}{\partial z^2}=0$$

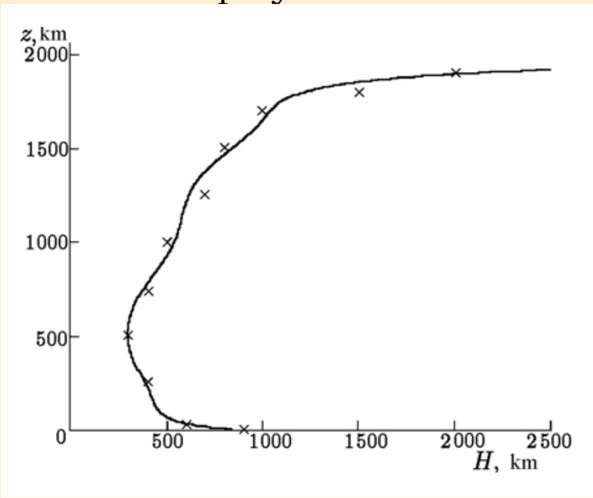
.In this equation w is the perturbation of the vertical component of a fluid velocity, $c(z)$ is the sound velocity.

Model of the Temperature Vertical Profile.

Let us to specify an analytical model of the temperature vertical profile. We seek a function describing the altitud dependence of the atmosphere scale height H(z) in the form of two polynomial ratio

$$H(z)=\frac{\sum_{k=0}^{n}B_k z^k}{\sum_{k=0}^{n}C_k z^k}$$

Such a definition of function *H(z)*, with an appropriate choice of coefficients B_k and C_k allows us to obtain H(z) vertical profile close to the experimentally observed profile.

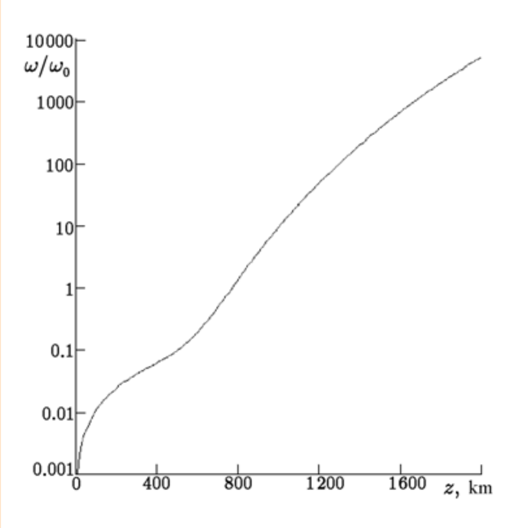


Calculated profile of the scale height

The exact solution of wave equation is proportional to another polynomial

$$w = \left(\sum_{k=0}^{n} A_k z^{k+1} \right) \exp(i\omega_A t) \exp\left(z/2\mathrm{H_T}\right).$$

The coefficients A_k are selected so, that for a polynomial, defining precise solution of the equation, there is no zero value of w in region above low boundary $z > 0$. Therefore the obtained solution is the lowest mode of the problem about eigenfunctions and eigenvalues. This last follows from the oscillation theorem for the Schturm-Liuville problem.



Relative vertical component w/w_0 of the wave-perturbation velocity as a function of the altitudet.

Analysis of the solution yields the important conclusion that the frequency

$$\omega = \sqrt{\frac{\gamma g}{4H_c}} = \omega_{Ac}$$

of eigenoscillations of the atmosphere is determined by the lower corona scale height H_c

4. An acoustic-gravity waves instability in the nonisothermal atmosphere

Inithial idea of the acoustic gravity wave instability, which develops in nonisothermal atmosphere, if the cut frequency ω_A becomes smaller, than Brunt-Vaisala frequency ω_g was suggested by T. W.Jhonston for the Earth atmosphere(*J. Geophys. Res.*, **72**, 2972 (1969)). We came to conclusion that the inequality $\omega_A < \omega_g$ can be valid in the Solar cromosphere as a result of the sharp temperature gradient in the upper cromosphere region.

The dispersion equation for acoustic-gravity waves in a local approximation

has a form

$$\left(\omega^2-\omega_A^2\right)\left(\omega^2-c^2k^2\right)+c^2k^2\left(\omega_g^2-\omega_A^2\right)=0$$

The formal solution of this equation is

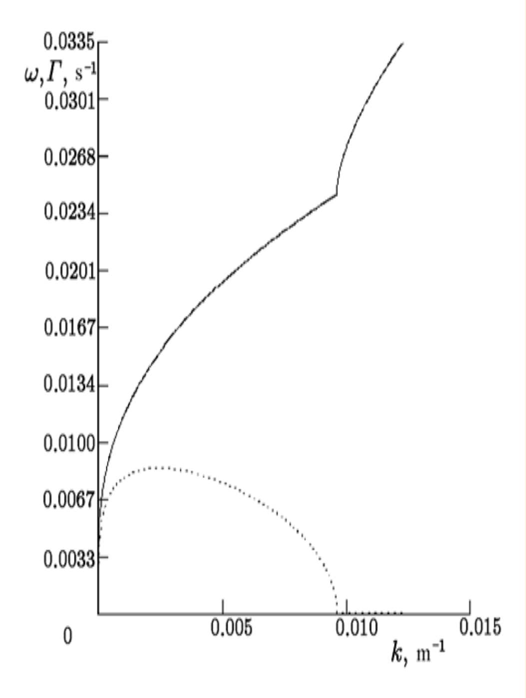
$$\omega_{1,2}^2=\frac{k^2c^2+\omega_A^2}{2}\pm\left[\frac{\left(k^2c^2+\omega_A^2\right)^2}{4}-c^2k^2\left(\omega_g^2-\omega_A^2\right)\right]^{\frac{1}{2}}$$

So, we have the following instability conditions

$$2\omega_g^2-\omega_A^2-2\omega_g\left(\omega_g^2-\omega_A^2\right)<c^2k^2<2\omega_g^2-\omega_A^2+2\omega_g\left(\omega_g^2-\omega_A^2\right)$$

Estimations of the instability growth rate γ show that:

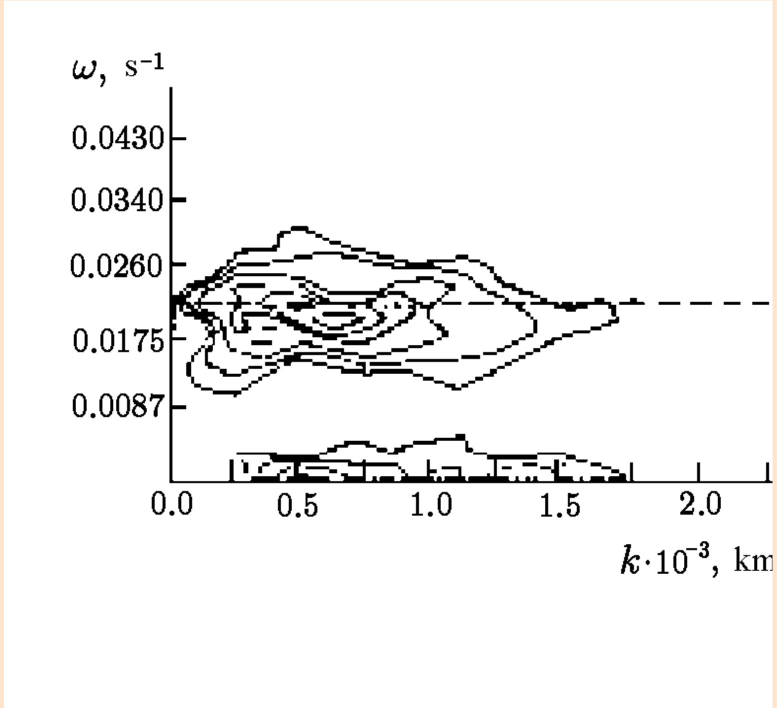
$\omega \cong \omega_g$, an expression for k is following $k \cong 2\omega_g / c^2$, $\gamma \cong \omega_g$



The dependences of the real and imaginary parts of the frequency for parameters typical of the solar atmosphere are shown in Fig. by solid and dotted lines, respectively. In the calculations, the Sun's temperature was taken equal to 5000 K.

5. Instability oscillation parameters in the Solar atmosphere

Some known experimental features of the five minutes oscillations:



- The vertical oscillations are observed almost always.
- The most intense oscillations do not appear suddenly
- They have the form of nonstationary wave packet with slowly varying amplitudes.
- The phase velocity of these waves is of the order of 100 km/sec.
- The amplitude velocity amplitude is about 500 m/sec.
- The average period of oscillations is close to 5 min, but shorter periods are also observed in the upper layers.
- Oscillations are wave packets with durations of a few wave periods, and the horizontal size of a train can be comparable with the of supergranules size.
- Horizontal wavelengths amount to several thousands of kilometers.

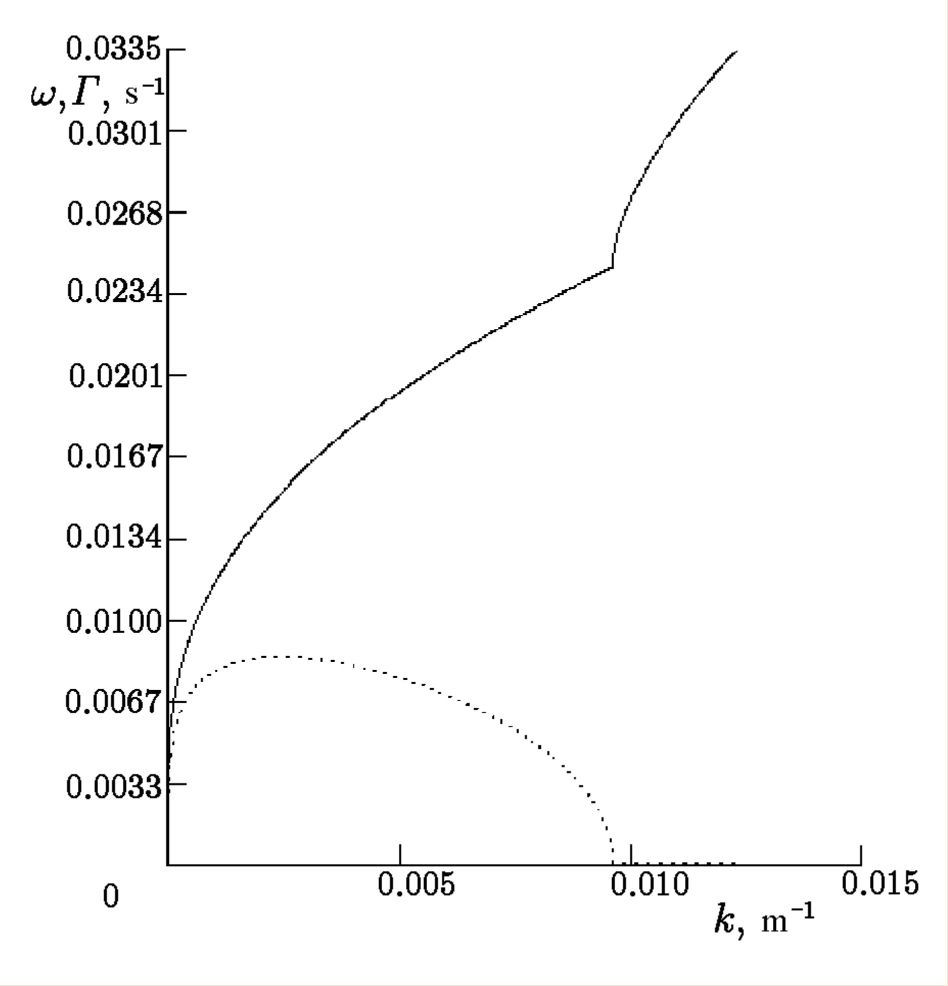
Estimations for instability in question show that (ON Savina, Radiophysics and Quantum Electronics, Vol. 44, No. 9, 2001)

- The instability growth rate is maximum at a frequency close to the Brunt-Vaisala frequency, which corresponds to the five-minute period of oscillations.

- The horizontal scale of the oscillations is about 2000 km for maximum growth rate is maximum.

- The instability is a periodic, then the oscillation amplitude increases significantly during one period.

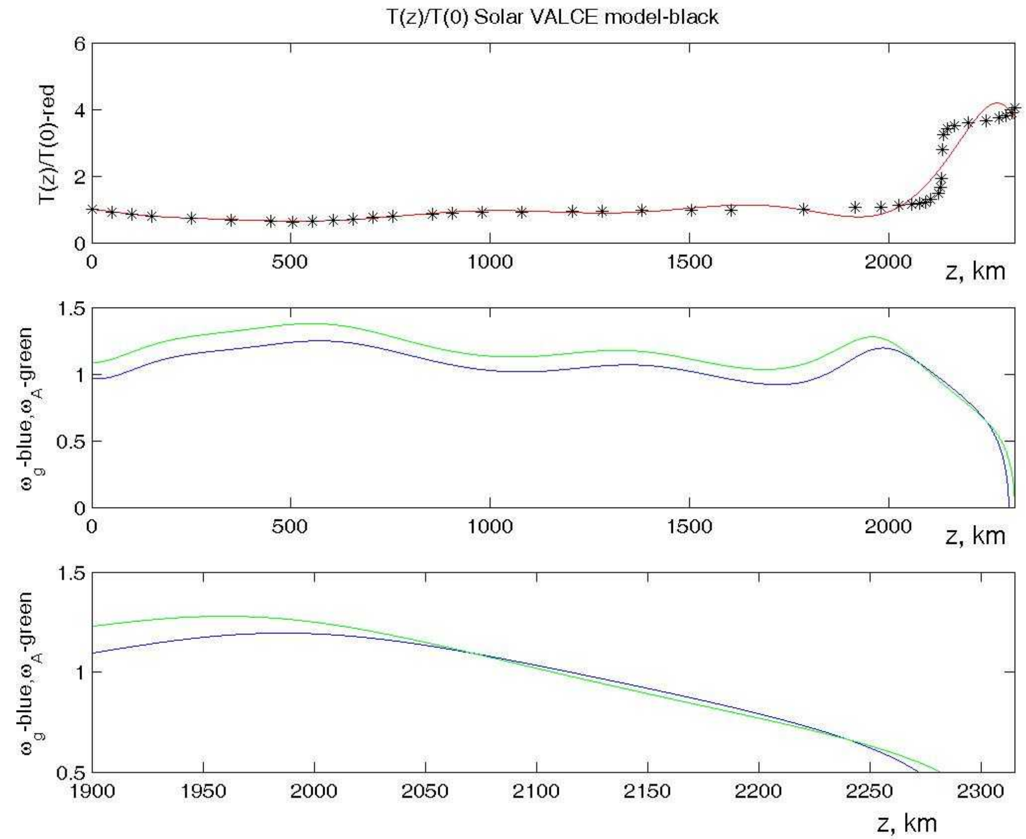
- The phase velocity of such oscillations exceeds 50 km/sec.



In the Solar atmosphere Brunt-Vaisala frequency and eigenfrequency depend on the altitude, the instability parameters should also depend on the altitude.

In framework of this hypothesis local unstable region is determined by vertical Solar temperature profile.

In the altitude region with the sharp positive temperature gradient, where the local inequality $\omega_A < \omega_g$ is satisfied, the instability in question can take place.



6. Conclusion

The instability in quation develops at frequencies and spatial scales typical to the five-minutes vertical oscillations of the Solar atmosphere.

If the considered mechanism of the instability of acoustic-gravity waves is realized in the solar aetmosphere, then the oscillation frequencies are close to the Brunt Vaisala frequency, while the horizontal scales of oscillations are about several thousand kilometers.

For the appearance of this type of oscillations, no additional sources are required besides those maintaining the given temperature profile.

A detail comparison of the experimental data with theoretical results certainly it is necessary including into account some additional factors: real temperature profile and nonlinearity.