Integer Programming Approaches for Critical Elements Detection in Graphs

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Abstract. Graph-theoretical models arise in a variety of application areas due to their elegance and inherent ability to logically represent (as edges) important relationships, e.g., communication and transportation links, between structural elements (i.e., nodes) of complex systems. An interesting research question arising in this context is the problem of identifying the most "significant" nodes and edges of a given graph G, specifically, those whose removal (subject to a budgetary constraint) maximally degrades the connectivity of G, or equivalently, maximally fragments G, according to some pre-defined metric. These nodes (edges) and the corresponding optimization problem are often referred to as critical nodes (edges) and the Critical Elements Detection problem, respectively. Most of the attention in the literature has been focused on the node version of the problem, also referred to as the Critical Node Detection (CNP) problem.

The concept of critical nodes and edges allows for the characterization of vulnerability and robustness properties of a given networked system with respect to node and edge removals, which may be random failures or errors due to operating conditions, or natural disasters. In recent years, this stream of research has received a significant attention in the literature. In addition to apparent interpretations in telecommunication and transportation areas, the Critical Elements Detection problem has natural applications in a number of other important domains, e.g., social network analysis. For example, in social networks each node corresponds to a person, edges represent some type of relationships (or interactions) between the individuals (e.g., friendship, collaboration), and critical nodes are often referred to as the "key players" of the network (e.g., informal leaders of the organization or community).

In our lectures we review recent results on the topic of Critical Elements Detection in the Operations Research literature. Our focus will be on applications of integer programming approaches. In particular, we discuss theoretical computational complexity issues, derive mathematical programming models and describe related algorithmic approaches.