Network effects in economics and finance

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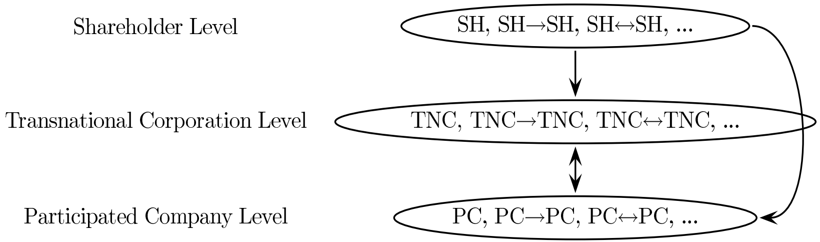
Outline

Global corporate network

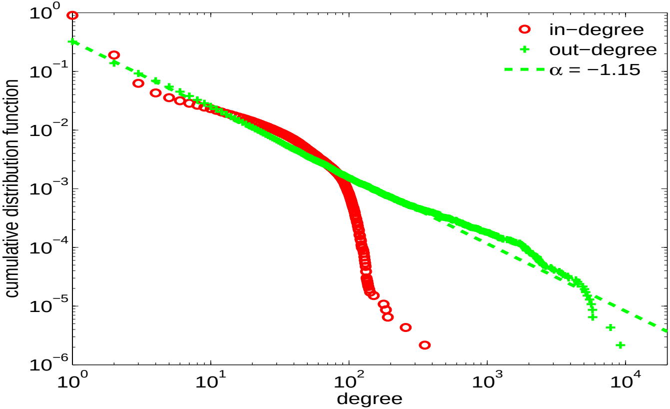
S. Vitali et al., (2011,2014)

Network characteristics:

- 43060 TCNs, 600508 nodes, 1006987 links
- 77456 SHs, 43060 TNCs, 479992 PCs

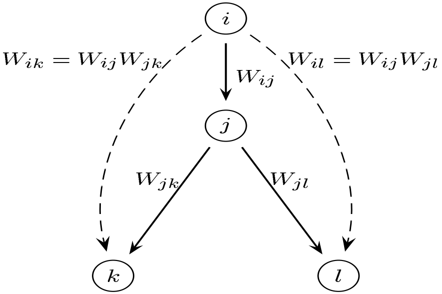


- Degree distribution



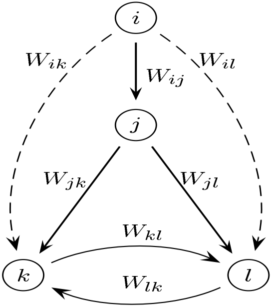
Global corporate network

- Let W be an ownership matrix such that W_{ij} is the percentage of ownership that an owner i holds in firm j. Ownership can be direct and indirect:



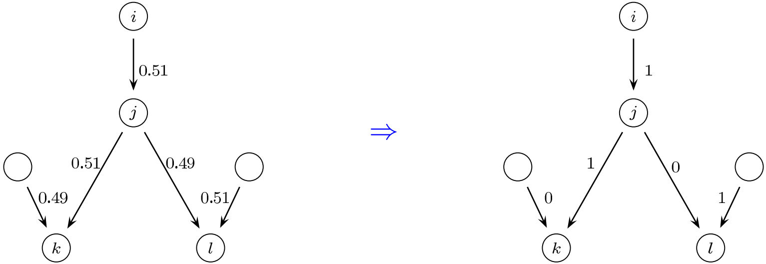
Global corporate network

- Complication: cross-ownership (loops), important for the analysis of control structure



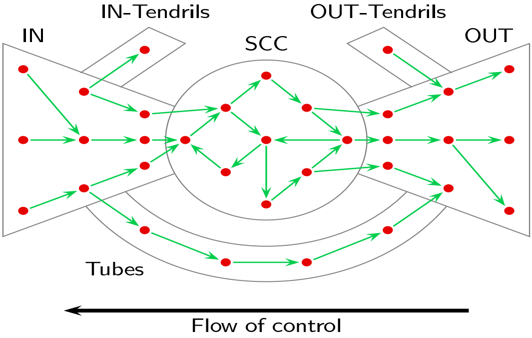
Global corporate network

- Control structure defined as a matrix $C_{ij} = \theta\left(W_{ij} - 0.5\right)$



Global corporate network

- Bow-tie structure of the network: In-section (IN), out-section (OUT), strongly connected component (SCC), tubes and tendrils (T & T)



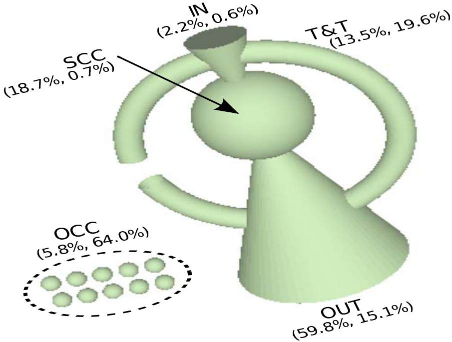
- Bow-tie structure of the corporate network:

Table: **Bow-tie statistics.** Percentage of total TNC operating revenue (OR) and number (#) of nodes in the sections of the bow-tie. Economic actors types are: shareholders (SH), participated companies (PC).

	TNC (#)	SH (#)	PC (#)	OR (%)
LCC	15491	47819	399696	94.17
IN	282	5205	129	2.18
SCC	295	0	1023	18.68
OUT	6488	0	318073	59.85
T&T	8426	42614	80471	13.46
OCC	27569	29637	80296	5.83

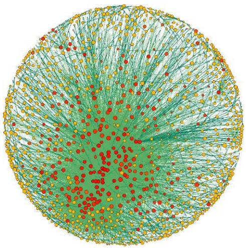
Global corporate network

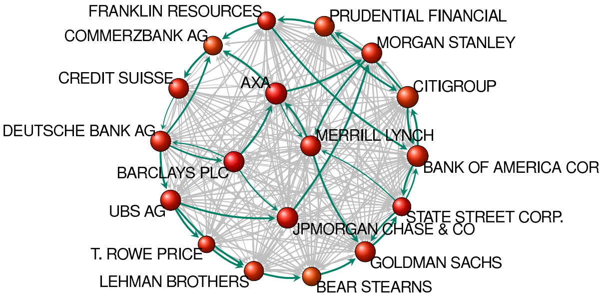
- Bow-tie structure of the largest connected component (LCC) and other connected components (OCC)



Global corporate network

- Left: strongly connected cluster (SCC) layout (1318 nodes, 12191 links). Right: Zoom on some major TNCs in the financial sector





Global corporate network

- Let v_i be the value of the firm i
- Direct control: $C_{ij} v_j$
- Total controlled amount:

$$c_i^{\mathrm{net}} = \sum_j C_{ij} v_j + \sum_j C_{ij} c_j^{\mathrm{net}}$$

- Total controlled amount:

$$c^{\mathrm{net}} = (I - C)^{-1} C v \equiv \tilde{C} v$$

- $\tilde{C}$ is C- related centrality

Global corporate network

- The above-defined procedure heavily overestimates network - related effects in the presence of loops (e.g. in the SCC).
- In the original scheme the matrix $\tilde{C}$ satisfies the following equation:

$$\tilde{C} = C + \tilde{C} \cdot C \quad \Rightarrow \quad \tilde{C} = (I - C)^{-1} \cdot C$$

- Corrected procedure:

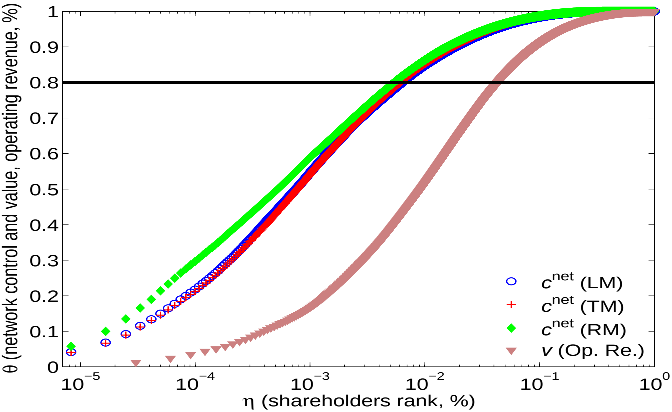
$$\hat{C}_{ij} = C_{ij} + \sum_{k \neq i} \hat{C}_{ik} C_{kj},$$

where

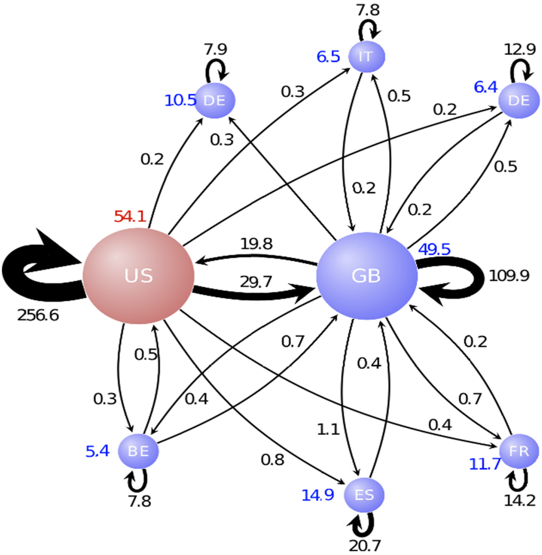
$$\hat{C} = \mathrm{diag}\left((I - C)^{-1}\right)^{-1} \cdot \tilde{C}$$

Global corporate network

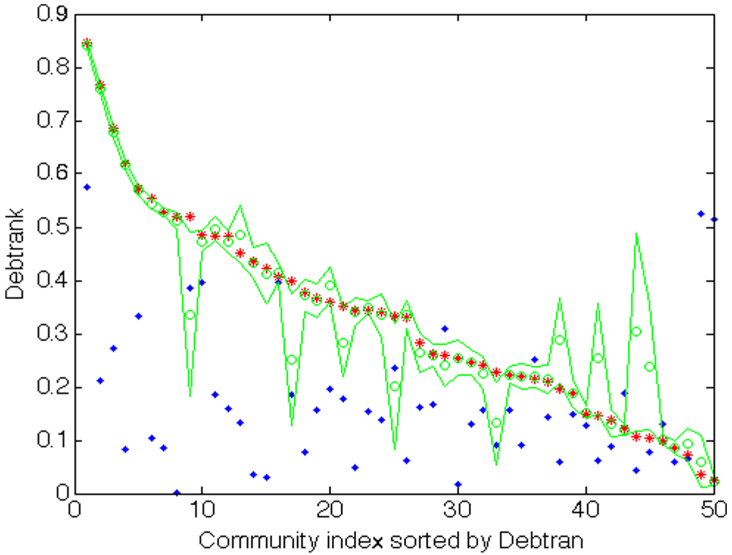
- Concentration of network control and operating revenue. 737 top holders have 80 % control over the cumulative value of TCN!



Global corporate network: community structure

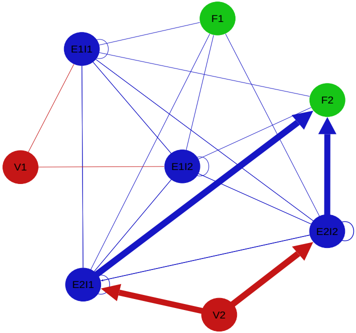


Global corporate network: DebtRank



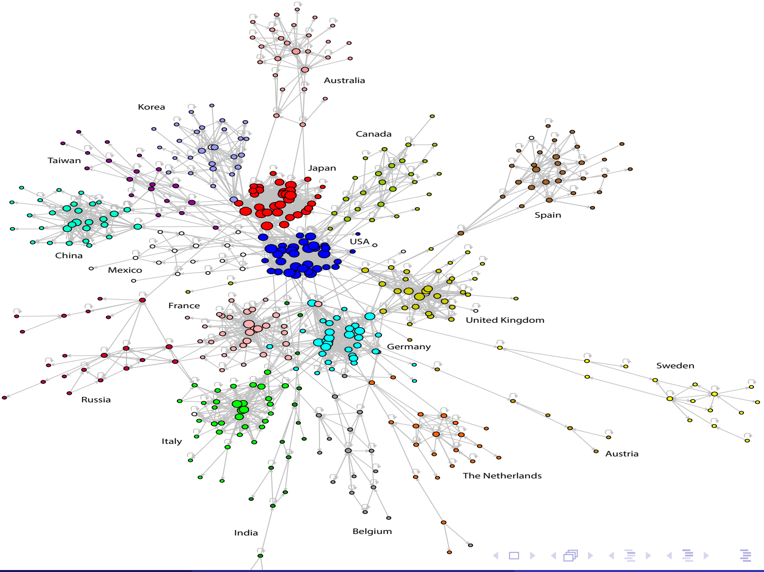
Blue points: financial sector removed

World input-output network: composition

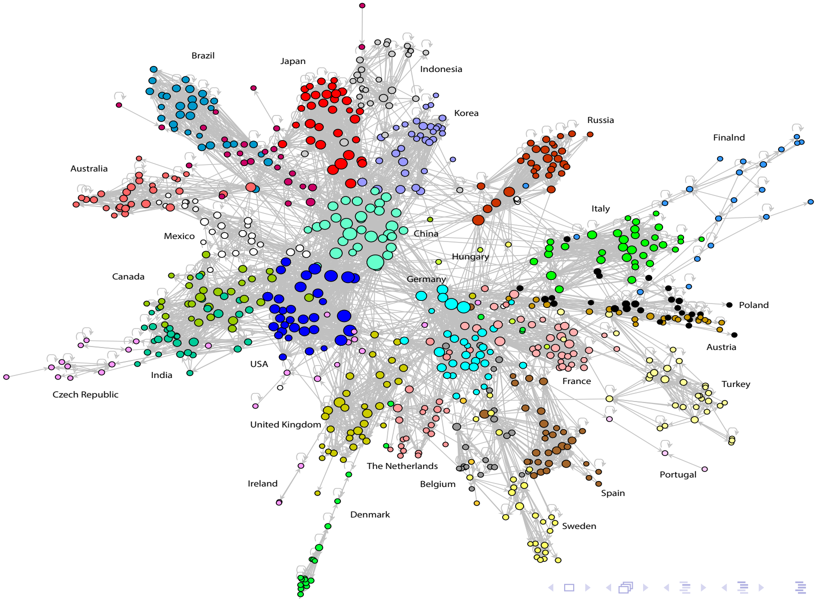


Blue: industries (sectors); red: value added; green: final demand

World input-output intersector network: 1995



World input-output intersector network: 2011



Input-output network: Leontieff inverse, etc

- Production balance:

$$x = Z \cdot 1 + f \equiv A \cdot x + f$$

where x: total output, Z: inter industrial transaction matrix; f: final demand; A: matrix of technical coefficients (matrix Z with columns normalized by x)

- Leontieff inverse

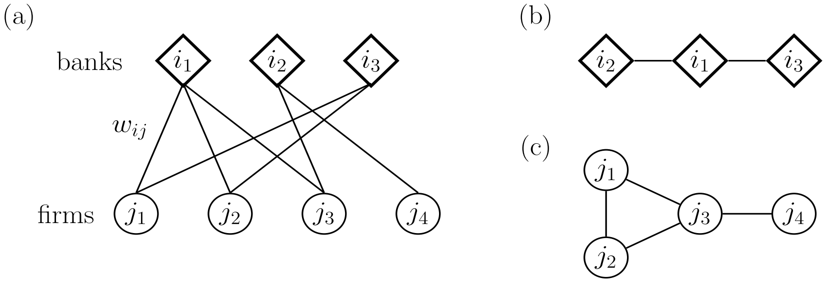
$$x = (I - A)^{-1} f \equiv L \cdot f$$

- Leontieff inverse is clearly just centrality measuring backward linkage effects for final demand driven direct and indirect effects
- Other centrality measures (Page rank, community coreness, ...) are also of interest. Different centrality measures give different results - especially important for developing dynamical picture of shock propagation through macroeconomic networks.

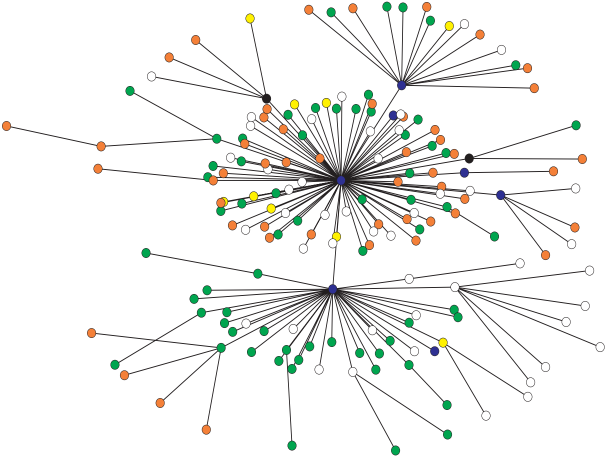
Banks and firms: bipartite network

J. De Masi et al. (2010)

- Consider a bipartite credit network of banks and firms, where banks credit firms, but there is no interbank borrowing



Banks and firms: MST cofinancing network

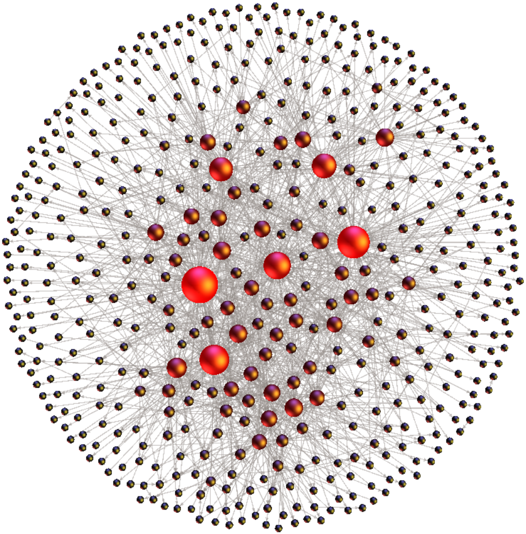


Russian interbank network: data description

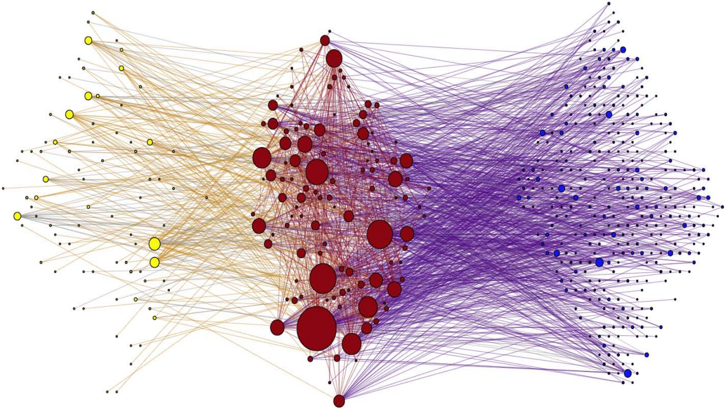
A.L., E. Rumyantsev (2011,2013,2014)

- Uncollaterized interbank rouble deposits of all maturities in the period from January 11, 2011 till December 30, 2013 are considered.
- Interbank network for N banks is fully characterized by an oriented weighted graph $G^W = (N, W)$, where $W = \{w_{ij}\}$ is an $N \times N$ matrix of $w_{ij} > 0$ of liabilities of the bank i with respect to the bank j.
- By definition the outgoing links correspond to liabilities, the incoming ones - to claims.
- The interbank network graph is scale-free in both in- and out- degrees and is characterized by significant clustering.

Russian interbank market



Russian interbank market: bow-tie structure



- Mathematical modeling of default propagation uses as its basic input the probability of having at least one incoming link capable of transmitting contagion from adjacent nodes to the node under consideration.
- The probability of default propagation depends on both on the weighted network topology around the node under consideration and characteristics of its balance sheet.
- The choice of mathematical formalism is crucially determined by the characteristic topology of default clusters. Our study shows that, despite of significant clustering of the original network, it is predominantly treelike, so one can use the formalism of generating functions generalized to take explicit account of the bow-tie topology of the interbank network.

Definition of vulnerability

- Solvency coefficient $H1$ as defined by CBRF:

$$H1 = \frac{K}{\sum_i A_i Kp_i + PP + OP + others}$$

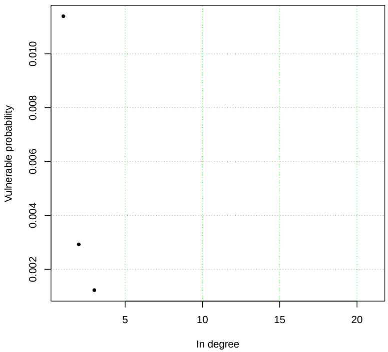
- Here K is capital, Kp_i - risk coefficients, PP - market risk, OP - operational risk, others - other contributions
- A default condition is

$$H1 = \frac{K}{\sum_i A_i Kp_i + PP + OP + others} < H1^*$$

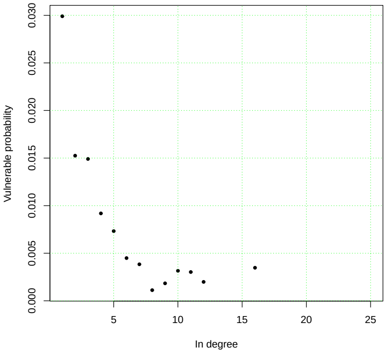
Main input: empirical default distributions

- Probability that at least one incoming link is vulnerable:

Out $\to$ In-Out

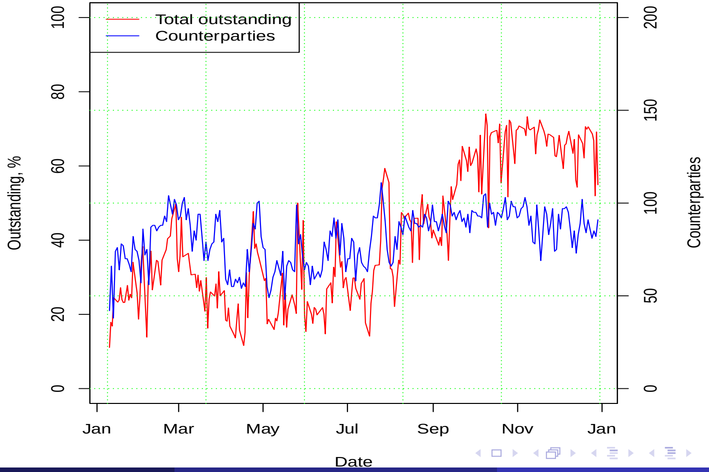


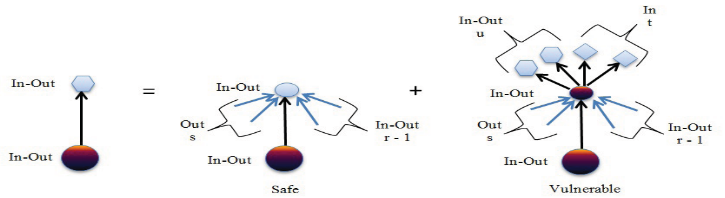
In-Out $\to$ In-Out



Strongly connected component

- There exists a strongly connected component
- The weight of this component did significantly increase





$$M_{k,l}(x, N(y)) = \sum_{u,t,s,r}^{\infty} P_{In-Out/In-Out}(u,t,s,r|k,l)$$

$$* \left[(1 - v_r^{In-Out/In-Out}) + x v_r^{In-Out/In-Out} M^u_{u,t}(x, N(y)) N^t_{u,t}(y) \right]$$

Contagion clusters In-Out → In-Out & In

- Let $F(x, y)$ be the generation function for the probability for a bank from In-Out being linked with In-Out and In components:

$$F(x, y) = \sum_{i,j=0}^{\infty} p_{ij}^{InOut} x^i y^j$$

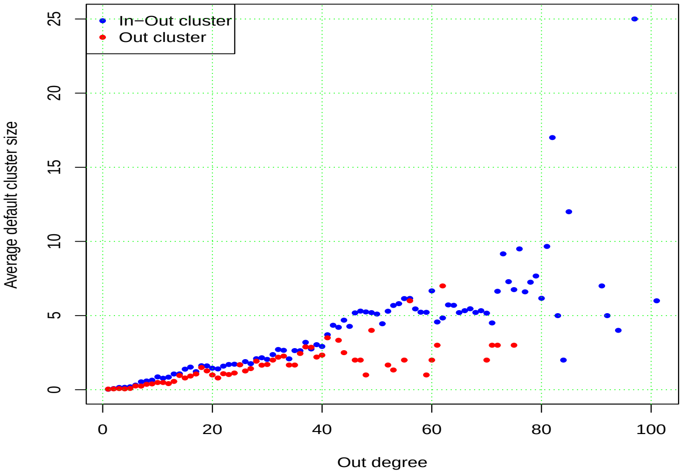
- The generation function for default cluster originating in In-Out then reads:

$$\mathcal{G}_{InOut}(x, y) = F(M(x, N(y)), N(y))$$

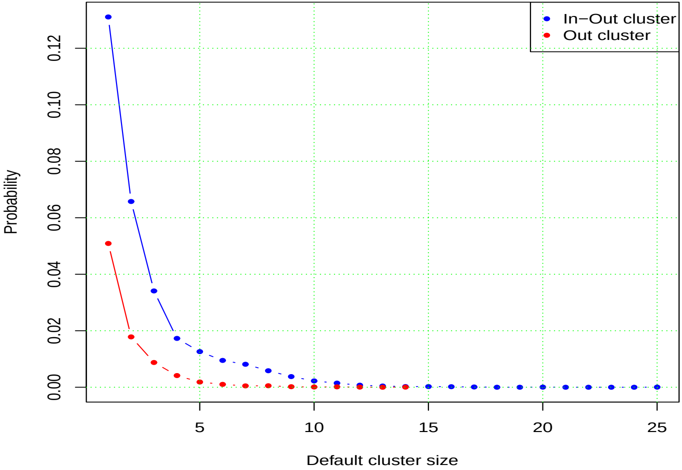
- The average size of default clusters is given by

$$\left. \frac{d\mathcal{G}_{InOut}(x, x)}{dx} \right|_{x=1} = 1$$

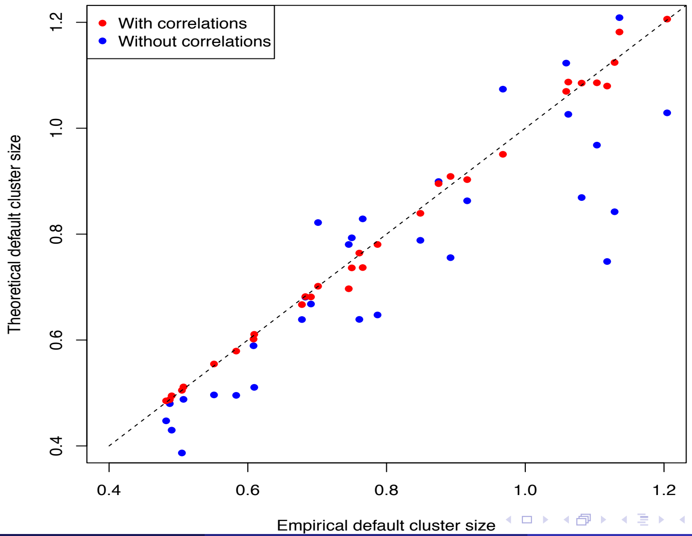
Simulation: dependence upon out-degree



Simulation: default cluster size distribution



Theory: comparison with simulations



Systemic risk: no giant cluster

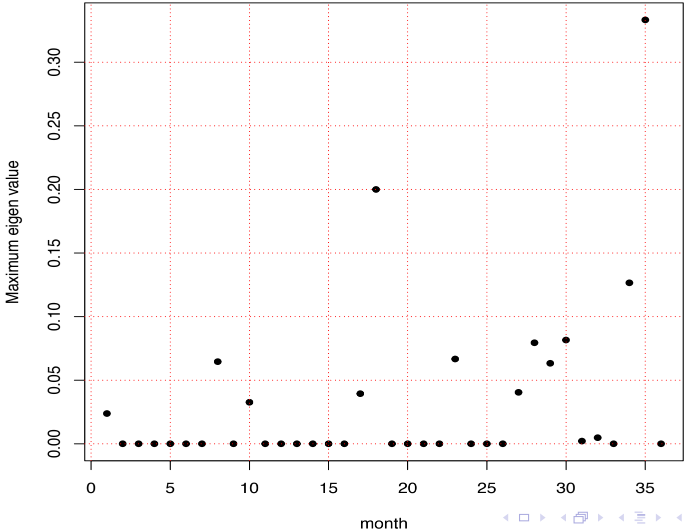
- For the (tree-like) graph of vulnerable nodes existence/non-existence of giant cluster can be studied using the matrix

$$A_{(k,l)(u,t)} = \sum_{r}^{\infty} u P^{\mathrm{IO}\to\mathrm{IO}}(u,t,r|k,l) v^{\mathrm{IO}\to\mathrm{IO}}(u,t,r|k,l)$$

- In the simplest form giant cluster appears if

$$\lambda_{\max}(A) > 1$$

Systemic risk: no giant cluster



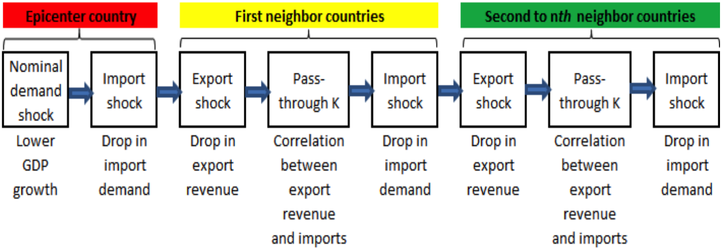
Conclusions

- Taking into account the bow-tie structure of the interbank network is very essential.
- Despite of the complicated topology of the original graph, the default clusters are (almost) always tree-like.
- This allows to describe default clusters in terms of generating functions taking into account the bow-tie structure of the original interbank network graph.
- The realistic contagion in the RF interbank market is a relatively small effect, nothing dramatic. Giant cluster is not formed.

Spillovers in international trade

A.L., A. Kireev (2015)

- International trade network is defined by the matrix W where w_{ij} is a volume of export from country i to country j.
- Spillover is an epidemic-type spreading of fluctuation of import/export in a given country over the network to first neighbors and beyond.



Spillover in international trade: cascade step

A cascade step is a process of transforming initial import shock into a secondary one. Schematically each step of the cascade is a composition of two steps:

1. Initial import shock $(\Delta M_1, \cdots, \Delta M_N)$ is (proportionally) distributed among exporters and, by definition, creates a vector of export shocks $(\Delta X_1, \cdots, \Delta X_N)$.

$$(\Delta M_1, \cdots, \Delta M_N) \quad \rightarrow \quad (\Delta X_1, \cdots, \Delta X_N)$$

2. Export shocks create secondary import shocks $(\Delta \tilde{M}_1, \cdots, \Delta \tilde{M}_N)$

$$(\Delta X_1, \cdots, \Delta X_N) \quad \rightarrow \quad (\Delta \tilde{M}_1, \cdots, \Delta \tilde{M}_N)$$

Spillover in international trade: cascade step

1. First step: $\Delta\vec{M} \to \Delta\vec{X}$:

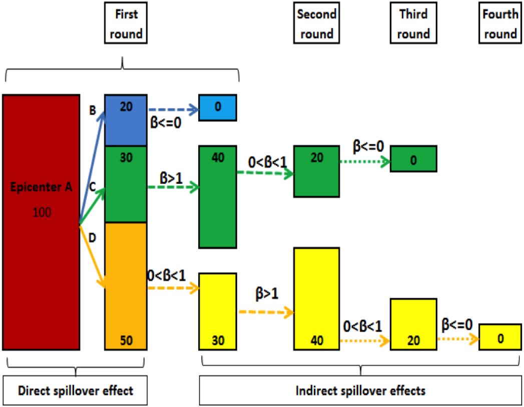
$$\Delta\vec{X} = \Omega \cdot \Delta\vec{M}, \qquad \Omega_{ij} = \frac{w_{ij}}{\sum_j w_{ij}}, \qquad W \to W - \Delta W$$

2. Second step: $\Delta\vec{X} \to \Delta\vec{\tilde{M}}$:

$$\log\left(\frac{M_i - \Delta\tilde{M}_i}{M_i}\right) = \alpha_i + \beta_i \log\left(\frac{X_i - \Delta X_i}{X_i}\right) + \varepsilon_i$$

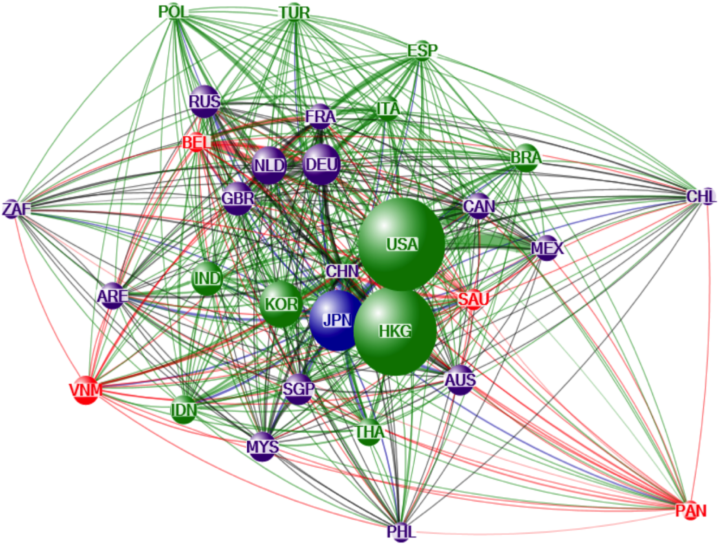
$$\Delta\tilde{M}_i = M_i\left(1 - e^{\alpha_i}\cdot\left(1 - \frac{\Delta X_i}{X_i}\right)^{\beta_i}\right)$$

Spillover in international trade: cascade step



Total spillover		
Direct	Indirect	Total
20	0	20
30	60	90
50	90	140

Spillover in international trade: example



Spillover in international trade: example