A branch and bound algorithm for the cell formation problem

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Overview

Formulation

	1	2	3	4	5	6	7
1	0	1	0	1	1	1	0
2	1	0	1	0	0	0	0
3	1	0	1	0	0	0	1
4	0	1	0	1	0	1	0
5	1	0	0	0	0	0	1

$$A = \begin{cases} 1 & \text{if machine } i \text{ processes part } j \\ 0 & \text{otherwise} \end{cases}$$

Goal is to find the optimal partitioning of machines and parts into groups (production cells, or shops), in order to minimize the inter-cell movement of parts from one cell to another and to maximize intra-cell processing operations

Restrictions:

1. Each part and each machine must be assigned to only one cell;
2. All parts and machines must be partitioned into groups.

	2	4	5	6	1	3	7
1	1	1	1	1	0	0	0
4	1	1	0	1	0	0	0
1	0	0	0	0	1	1	0
3	0	0	0	0	1	1	1
5	0	0	0	0	1	0	1

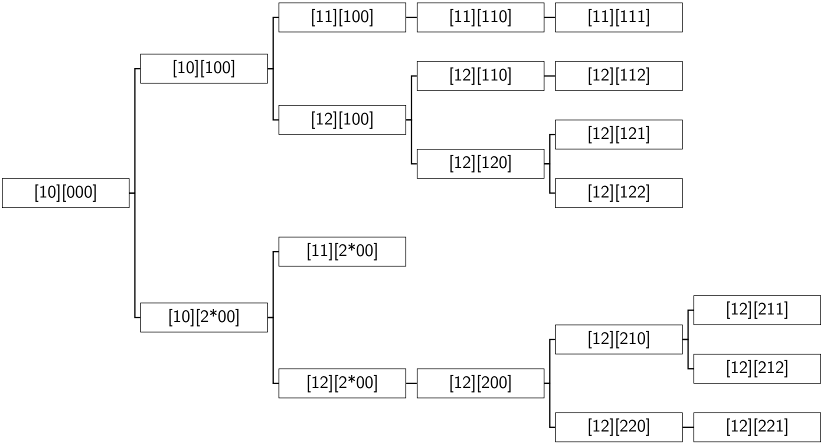
Definitions

- $A(m \times p)$ - an input matrix;
- $M(1 \times m)$ - a vector, which contains the assignment of machines to cells;
- $P(1 \times p)$ - a vector, which contains the assignment of parts to cells;
- f^* - is the optimal value of function f;

$$f = \frac{n_1^{in}}{n_1 + n_0^{in}}$$

n_1 - the number of ones in the input matrix,
n_0 - the number of zeroes in the input matrix,
n_1^{in} - the number of ones inside cells,
n_1^{out} - the number of ones outside cells,
n_0^{in} - the number of zeroes inside cells,
n_0^{out} - the number of zeroes outside cells.

Branching


[10][000]
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[12][220]
[12][211]
[12][212]
[12][221]

	1	3	5	6	4	2	7	8
2	1	1	1	1	1	0	0	1
4	1	1	0	1	0	0	0	1
1	0	0	1	0	1	1	1	0
3	1	0	1	1	1	0	1	0
5	0	0	0	0	0	0	1	1

	1	3	5	6	4	2	7	8
2	1	1	1	1	1	0	0	1
4	1	1	0	1	0	0	0	1
1	0	0	1	0	1	1	1	0
3	1	0	1	1	1	0	1	0
5	0	0	0	0	0	0	1	1

	1	3	5	6	4	2	7	8
2	1	1	1	1	1	0	0	1
4	1	1	0	1	0	0	0	1
1	0	0	1	0	1	1	1	0
3	1	0	1	1	1	0	1	0
5	0	0	0	0	0	0	1	1

	1	3	5	6	4	2	7	8
2	1	1	1	1	1	0	0	1
4	1	1	0	1	0	0	0	1
1	0	0	1	0	1	1	1	0
3	1	0	1	1	1	0	1	0
5	0	0	0	0	0	0	1	1

a_1 - the number of "1" inside cells for first alternative
b_1 - the number of "0" inside cells for first alternative
a_2 - the number of "1" inside cells for second alternative
b_2 - the number of "0" inside cells for second alternative

a_1 - the number of "1" inside cells for first alternative
b_1 - the number of "0" inside cells for first alternative
a_2 - the number of "1" inside cells for second alternative
b_2 - the number of "0" inside cells for second alternative

$$\frac{a'}{b'} = \frac{16}{21} \approx 0.76 > \frac{a}{b} = \frac{18}{24} = 0.75$$

a_1 - the number of "1" inside cells for first alternative
b_1 - the number of "0" inside cells for first alternative
a_2 - the number of "1" inside cells for second alternative
b_2 - the number of "0" inside cells for second alternative

$$\frac{a'}{b'} = \frac{16}{21} \approx 0.76 > \frac{a}{b} = \frac{18}{24} = 0.75$$

$$\frac{a' + a_1}{b' + b_1} = \frac{16 + 1}{21 + 31} \approx 0.326 < \frac{a' + a_2}{b' + b_2} = \frac{16 + 2}{21 + 34} \approx 0.327$$

a_1 - the number of "1" inside cells for first alternative
b_1 - the number of "0" inside cells for first alternative
a_2 - the number of "1" inside cells for second alternative
b_2 - the number of "0" inside cells for second alternative

$$\frac{a'}{b'} = \frac{16}{21} \approx 0.76 > \frac{a}{b} = \frac{18}{24} = 0.75$$

$$\frac{a' + a_1}{b' + b_1} = \frac{16 + 1}{21 + 31} \approx 0.326 < \frac{a' + a_2}{b' + b_2} = \frac{16 + 2}{21 + 34} \approx 0.327$$

$$\frac{a + a_1}{b + b_1} = \frac{18 + 1}{24 + 31} \approx 0.345 > \frac{a + a_2}{b + b_2} = \frac{18 + 2}{24 + 34} \approx 0.344$$

Theorem (Theorem 1)

For positive numbers a, b, a', b' *and non-negative* a_0, b_0 *if we have:*

$$\frac{a'}{b'} > \frac{a}{b}, \tag{1}$$

$$\frac{a' + a_0}{b' + b_0} > \frac{a'}{b'} \tag{2}$$

then the following inequality is true:

$$\frac{a' + a_0}{b' + b_0} > \frac{a + a_0}{b + b_0} \tag{3}$$

Theorem (Theorem 2)

If the unknown maximum value of the objective function $\frac{a}{b}$ for the relaxed CFP problem without assignment of machine i (considering all its ones and zeroes to be outside cells) can be estimated as $\frac{a}{b} \in [l, u]$, $b \in [b_l, b_u]$, then alternative (a_1, b_1) for machine i is better than alternative (a_2, b_2) if:

$$b_l \left(l - \frac{\Delta a}{\Delta b} \right) \geq b_1 \frac{\Delta a}{\Delta b} - a_1 \tag{4}$$

and is worse than (a_2, b_2) if:

$$b_u \left(u - \frac{\Delta a}{\Delta b} \right) \leq b_1 \frac{\Delta a}{\Delta b} - a_1 \tag{5}$$

Here $\Delta a = a_2 - a_1$, $\Delta b = b_2 - b_1 > 0$ (if $\Delta b < 0$ we can always swap the alternatives).

$$l = \frac{a_c}{b_c}$$

$$u = \frac{n_1 - \bar{n}_1^{out} - n_1^i}{b_c}$$

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	0	0	0	0	1
3	0	0	0	0	0	1	1	0	0
4	0	1	1	0	0	0	0	1	1
5	0	0	0	0	1	1	0	0	1

$n_1^{in} = 11, n_0^{in} = 1, \bar{n}_1^{out} = 0, \bar{n}_0^{out} = 9, n_1 = 19, n_0 = 26, a_c = 11, b_c = 20.$

	Alt 1	Alt 2	Alt 3
4			
5			

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	0	0	0	0	1
3	0	0	0	0	0	1	1	0	0
4	0	1	1	0	0	0	0	1	1
5	0	0	0	0	1	1	0	0	1

$n_1^{in} = 11, n_0^{in} = 1, \bar{n}_1^{out} = 0, \bar{n}_0^{out} = 9, n_1 = 19, n_0 = 26, a_c = 11, b_c = 20.$

	Alt 1	Alt 2	Alt 3
4	(4,3)		
5			

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	0	0	0	0	1
3	0	0	0	0	0	1	1	0	0
4	0	1	1	0	0	0	0	1	1
5	0	0	0	0	1	1	0	0	1

$n_1^{in} = 11, n_0^{in} = 1, \bar{n}_1^{out} = 0, \bar{n}_0^{out} = 9, n_1 = 19, n_0 = 26, a_c = 11, b_c = 20.$

	Alt 1	Alt 2	Alt 3
4	(4,3)	(2,2)	
5			

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	0	0	0	0	1
3	0	0	0	0	0	1	1	0	0
4	0	1	1	0	0	0	0	1	1
5	0	0	0	0	1	1	0	0	1

$n_1^{in} = 11, n_0^{in} = 1, \bar{n}_1^{out} = 0, \bar{n}_0^{out} = 9, n_1 = 19, n_0 = 26, a_c = 11, b_c = 20.$

	Alt 1	Alt 2	Alt 3
4	(4,3)	(2,2)	(2,0)
5			

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	0	0	0	0	1
3	0	0	0	0	0	1	1	0	0
4	0	1	1	0	0	0	0	1	1
5	0	0	0	0	1	1	0	0	1

$n_1^{in} = 11, n_0^{in} = 1, \bar{n}_1^{out} = 0, \bar{n}_0^{out} = 9, n_1 = 19, n_0 = 26, a_c = 11, b_c = 20.$

	Alt 1	Alt 2	Alt 3
4	(4,3)	(2,2)	(2,0)
5	(2,4)		

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	0	0	0	0	1
3	0	0	0	0	0	1	1	0	0
4	0	1	1	0	0	0	0	1	1
5	0	0	0	0	1	1	0	0	1

$n_1^{in} = 11, n_0^{in} = 1, \bar{n}_1^{out} = 0, \bar{n}_0^{out} = 9, n_1 = 19, n_0 = 26, a_c = 11, b_c = 20.$

	Alt 1	Alt 2	Alt 3
4	(4,3)	(2,2)	(2,0)
5	(2,4)	(2,1)	

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	0	0	0	0	1
3	0	0	0	0	0	1	1	0	0
4	0	1	1	0	0	0	0	1	1
5	0	0	0	0	1	1	0	0	1

$n_1^{in} = 11, n_0^{in} = 1, \bar{n}_1^{out} = 0, \bar{n}_0^{out} = 9, n_1 = 19, n_0 = 26, a_c = 11, b_c = 20.$

	Alt 1	Alt 2	Alt 3
4	(4,3)	(2,2)	(2,0)
5	(2,4)	(2,1)	(1,0)

So we need to compare only two alternatives $(a_1, b_1) = (2, 0)$ and $(a_2, b_2) = (4, 3)$. We have:
$n_1^i = 4, n_0^i = 5, \Delta a = 2, \Delta b = 3, l = a_c/b_c = 11/20, u = (n_1 - \bar{n}_1^{out} - n_1^i)/b_c = 15/20, b_l = 20, b_u = n_1 + n_0 - \bar{n}_0^{out} - n_0^i = 31$. And the values we need to apply theorem 2 are:

$$b_1 \frac{\Delta a}{\Delta b} - a_1 = -2, \quad b_l \left(l - \frac{\Delta a}{\Delta b} \right) = -\frac{7}{3}, \quad b_u \left(u - \frac{\Delta a}{\Delta b} \right) = \frac{31}{12}$$

So neither of the conditions in theorem 2 is satisfied and we cannot determine which alternative is better. In this case we build an alternative $(\max(a_1, a_2), \min(b_1, b_2))$, which is better than both incomparable alternatives, and use it to obtain an upper bound on the solution of the relaxed CFP problem. In our example it is alternative $(4, 0)$.

It is clear that alternative $(2,4)$ is worse than $(2,1)$. For $(a_1,b_1)=(1,0)$ and $(a_2,b_2)=(2,1)$ we have:

$$b_1\frac{\Delta a}{\Delta b}-a_1=-1, \quad b_l\left(l-\frac{\Delta a}{\Delta b}\right)=-9, \quad b_u\left(u-\frac{\Delta a}{\Delta b}\right)=-6$$

So $b_u\left(u-\frac{\Delta a}{\Delta b}\right)\leq b_1\frac{\Delta a}{\Delta b}-a_1$ and by theorem 2 alternative $(a_2,b_2)=(2,1)$ is better. Thus $(2,1)$ is the best alternative for machine 5.

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	0	0	0	0	1
3	0	0	0	0	0	1	1	0	0
4	0	1	1	0	0	0	0	1	1
5	0	0	0	0	1	1	0	0	1

$n_1^{in} = 11, n_0^{in} = 1, \bar{n}_1^{out} = 0, \bar{n}_0^{out} = 9, n_1 = 19, n_0 = 26, a_c = 11, b_c = 20.$

	Alt 1	Alt 2	Alt 3
8			
9			

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	0	0	0	0	1
3	0	0	0	0	0	1	1	0	0
4	0	1	1	0	0	0	0	1	1
5	0	0	0	0	1	1	0	0	1

$n_1^{in} = 11, n_0^{in} = 1, \bar{n}_1^{out} = 0, \bar{n}_0^{out} = 9, n_1 = 19, n_0 = 26, a_c = 11, b_c = 20.$

	Alt 1	Alt 2	Alt 3
8	(0,2)		
9			

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	0	0	0	0	1
3	0	0	0	0	0	1	1	0	0
4	0	1	1	0	0	0	0	1	1
5	0	0	0	0	1	1	0	0	1

$n_1^{in} = 11, n_0^{in} = 1, \bar{n}_1^{out} = 0, \bar{n}_0^{out} = 9, n_1 = 19, n_0 = 26, a_c = 11, b_c = 20.$

	Alt 1	Alt 2	Alt 3
8	(0,2)	(0,1)	
9			

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	0	0	0	0	1
3	0	0	0	0	0	1	1	0	0
4	0	1	1	0	0	0	0	1	1
5	0	0	0	0	1	1	0	0	1

$n_1^{in} = 11, n_0^{in} = 1, \bar{n}_1^{out} = 0, \bar{n}_0^{out} = 9, n_1 = 19, n_0 = 26, a_c = 11, b_c = 20.$

	Alt 1	Alt 2	Alt 3
8	(0,2)	(0,1)	(0,0)
9			

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	0	0	0	0	1
3	0	0	0	0	0	1	1	0	0
4	0	1	1	0	0	0	0	1	1
5	0	0	0	0	1	1	0	0	1

$n_1^{in} = 11$, $n_0^{in} = 1$, $\bar{n}_1^{out} = 0$, $\bar{n}_0^{out} = 9$, $n_1 = 19$, $n_0 = 26$, $a_c = 11$, $b_c = 20$.

	Alt 1	Alt 2	Alt 3
8	(0,2)	(0,1)	(0,0)
9	(1,1)		

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	0	0	0	0	1
3	0	0	0	0	0	1	1	0	0
4	0	1	1	0	0	0	0	1	1
5	0	0	0	0	1	1	0	0	1

$n_1^{in} = 11, n_0^{in} = 1, \bar{n}_1^{out} = 0, \bar{n}_0^{out} = 9, n_1 = 19, n_0 = 26, a_c = 11, b_c = 20.$

	Alt 1	Alt 2	Alt 3
8	(0,2)	(0,1)	(0,0)
9	(1,1)	(0,1)	

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	0	0	0	0	1
3	0	0	0	0	0	1	1	0	0
4	0	1	1	0	0	0	0	1	1
5	0	0	0	0	1	1	0	0	1

$n_1^{in} = 11, n_0^{in} = 1, \bar{n}_1^{out} = 0, \bar{n}_0^{out} = 9, n_1 = 19, n_0 = 26, a_c = 11, b_c = 20.$

	Alt 1	Alt 2	Alt 3
8	(0,2)	(0,1)	(0,0)
9	(1,1)	(0,1)	(0,0)

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	0	0	0	0	1
3	0	0	0	0	0	1	1	0	0
4	0	1	1	0	0	0	0	1	1
5	0	0	0	0	1	1	0	0	1

$$UB = \frac{11+4+2+1}{20+0+1+1} = \frac{18}{22} \approx 0.82$$

#	Size	Best-known solution	f	Time, s	Bychkov, Time, s
1	5×7	0.8235	0.8235	0.00	0.63
2	5×7	0.6957	0.6957	0.00	2.29
3	5×18	0.7959	0.7959	0.00	5.69
4	6×8	0.7692	0.7692	0.00	1.86
5	7×11	0.6087	0.6087	0.00	9.14
6	7×11	0.7083	0.7083	0.00	5.15
7	8×12	0.6944	0.6944	0.00	13.37
8	8×20	0.8525	0.8525	0.00	18.33
9	10×10	0.5872	0.5872	0.19	208.36
10	10×15	0.7500	0.7500	0.00	6.25
11	10×15	0.9200	0.9200	0.00	2.93
12	14×24	0.7206	0.7206	2.89	259.19
13	14×24	0.7183	0.7183	5.51	259.19
14	16×24	0.5326	0.5326	97117.43	[b]20829.38
15	16×30	[c]0.6899	0.6899	837.93	[b]13719.99
16	16×43	0.5753	0.5753	7045.64	[b]24930.93
17	18×24	0.5773	0.5773	5668.25	[b]13250.01
18	20×20	0.4345	[a]0.4211	100800.00	[b]43531.77
19	20×23	0.5081	[a]0.4697	100800.00	[b]33020.13
20	20×35	0.7791	0.7791	88.62	[b]11626.98
21	20×35	0.5798	[a]0.5615	100800.00	[b]33322.08
22	24×40	1.000	1.000	0.00	0.00
23	24×40	0.8511	0.8511	33.70	[b]6916.24
24	24×40	0.7351	0.7351	86007.93	[b]14408.88
25	24×40	0.5329	[a]0.5185	100800.00	[b]34524.47

[a] The problem was not solved to optimality within the time limit of 28 hours

[b] Bychkov did not solve these problems.

[c] In some papers there was mistaken value of best-known solution.

#	Size	Best-known solution	f	Time, s	Bychkov, Time, s
26	24 × 40	0.4895	[a]0.4648	100800.00	[b]41140.94
27	24 × 40	0.4726	[a]0.4468	100800.00	[b]44126.76
28	27 × 27	0.5482	[a]0.5017	100800.00	[b]22627.28
29	28 × 46	0.4706	[a]0.4569	100800.00	[b]71671.08
30	30 × 41	0.6331	[a]0.5942	100800.00	[b]22594.20
31	30 × 50	0.6012	[a]0.5789	100800.00	[b]31080.82
32	30 × 50	0.5083	[a]0.4860	100800.00	[b]48977.01
33	30 × 90	0.4775	[a]0.4684	100800.00	[b]99435.64
34	37 × 53	0.6064	[a]0.5680	100800.00	[b]47744.04
35	40 × 100	0.8403	[a]0.8403	100800.00	[b]24167.76

[a] The problem was not solved to optimality within the time limit of 28 hours

[b] Bychkov did not solve these problems.

[c] In some papers there was mistaken value of best-known solution.

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