

Filtering features of long acoustic-gravity waves in a windless atmosphere and its ionospheric manifestation

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¹ НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ
УНИВЕРСИТЕТ

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Introduction

In this report, we are going to discuss a possibility for filtering the features of long acoustic-gravity waves in a windless atmosphere and its ionospheric manifestation. We will introduce a theoretical analysis of linear AGW behavior under conditions where the horizontal phase wave velocity is equal to the local value of the sound velocity. In recent papers, we have shown that there is a range of wave phase velocities (or a frequency range with fixed horizontal dimensions of the source) in which the wave does not pass through a resonance domain up from the ground. We have found that local disturbances of the wave pressure component are formed at the narrow resonance level.

We suggest that the resonance mentioned above can be a reason for ionospheric irregularities with relatively small vertical scales in the D and E layers. It is possible that such irregularities, flattened vertically, contribute to the generation of sporadic E layers of the ionosphere with a large range of translucency.

1. Basic equations for acoustic-gravity waves in the nonisothermal atmosphere

The linearized system of equations of gas dynamics for the pressure disturbances p , the horizontal velocity u , and the vertical velocity w is well known.

$$\begin{aligned}\rho_0 \frac{\partial u}{\partial t} &= -\frac{\partial p}{\partial x}, \\ \rho_0 \frac{\partial w}{\partial t} &= -\frac{\partial p}{\partial z} - \rho g, \\ \frac{\partial \rho}{\partial t} + w \frac{d\rho_0}{dz} + \rho_0 \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) &= 0, \\ \frac{\partial p}{\partial t} + w \frac{dp_0}{dz} &= c_s^2 \left(\frac{\partial \rho}{\partial t} + w \frac{d\rho_0}{dz} \right).\end{aligned}$$

We select axis z in the vertical direction against the gravity acceleration \vec{g} and axis x in the horizontal direction; ρ_0 is the basic state density, c_s is a sound speed.

Really, the background temperature T depends on z such as the sound velocity. The regular pressure p_0 depends on altitude according to a thermodynamic equilibrium condition. The relation

$$p_0(z) = p_{00} \exp \left[-\int_0^z \frac{dz'}{H(z')} \right] \text{ describes the regular density } \rho_0(z) = \frac{p_0(z)}{gH(z)},$$

2. Wave fields near the resonance level

Let us discuss the model of atmospheric wave propagation from the ground sources to the ionosphere with a realistic high-altitude temperature profile. The linearized system of equations for the wave perturbation can be reduced to the following form:

$$\left[-\omega^2 + \omega_g^2(z) \right] W - i \frac{\omega}{\rho_E} \left[\frac{\partial}{\partial z} + \Gamma(z) \right] P = 0,$$

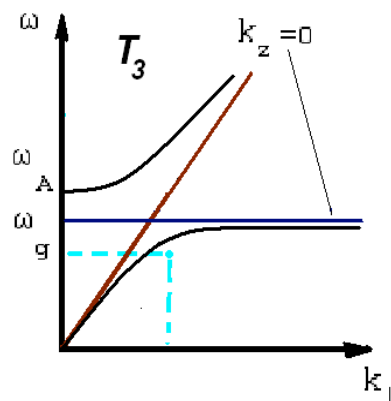
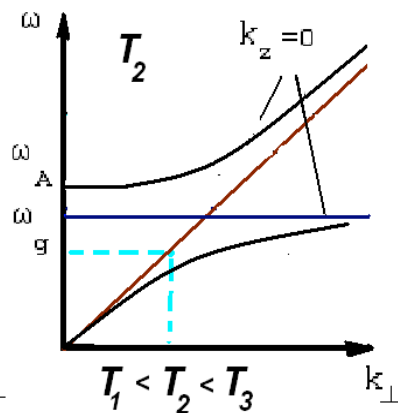
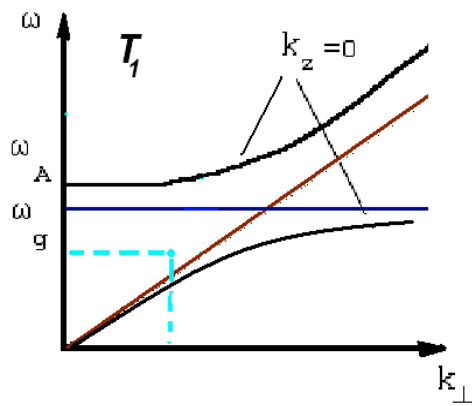
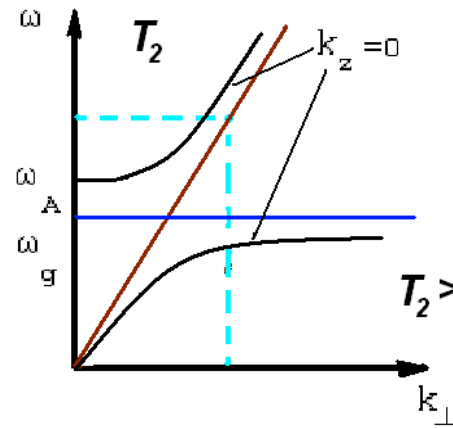
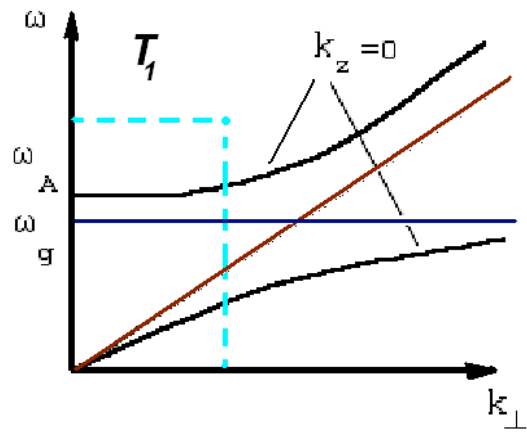
$$V = (k_{\perp} / \omega) P$$

$$\left[-\omega^2 + c_s^2(z) k_{\perp}^2 \right] P - i \omega \rho_E c_s^2(z) \left[\frac{\partial}{\partial z} - \Gamma(z) \right] W = 0.$$

Here,

$V = (\rho / \rho_E)^{1/2} v_{\sim}$, $W = (\rho / \rho_E)^{1/2} w_{\sim}$, and $P = (\rho_E / \rho)^{1/2} p_{\sim}$, where ρ and ρ_E are the basic state densities in the current layer and at the ground level, respectively. Field variables are proportional to $\exp(-i\omega t + i k_{\perp} x)$ for a monochromatic signal with frequency ω in plane atmospheric layers.

In this model, the horizontal wave number is altitude independent, and its main value is defined by the scale of the sources.

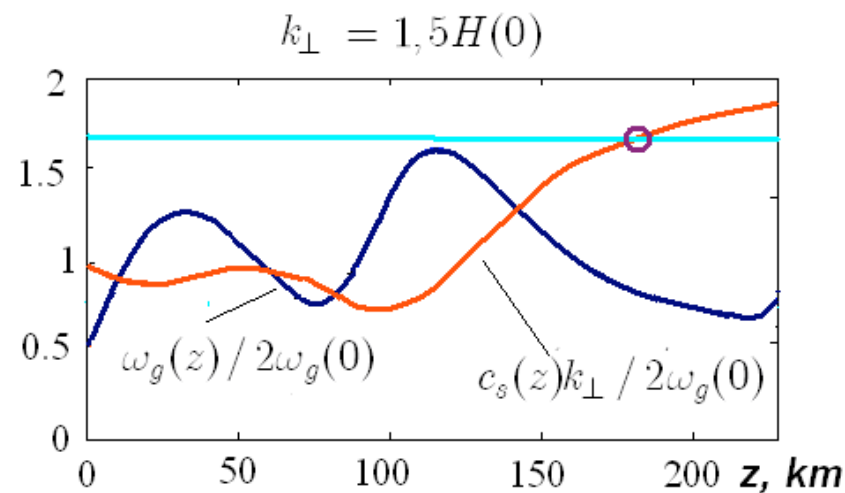
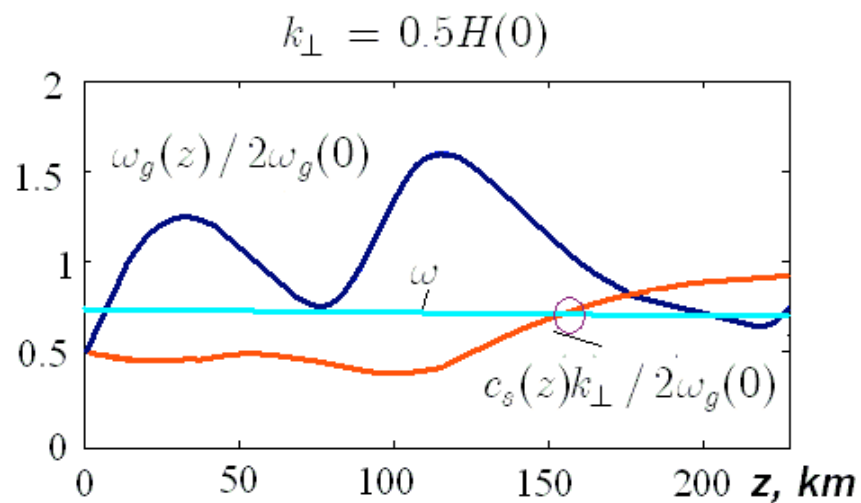
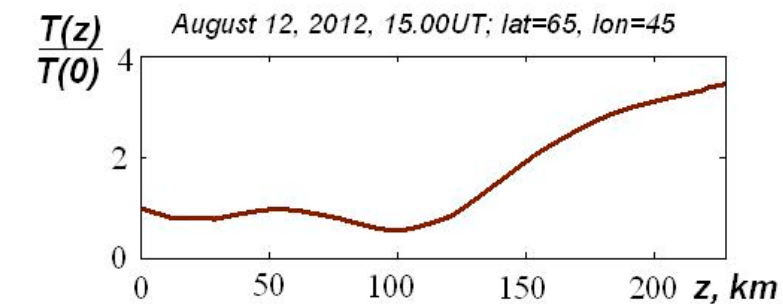


Assume that the source is ground-based. The simplest solution of this problem is founding the geometrical-optics approximation. In this pictures we can see transformation of the dispersion dependence between a wave frequency and a horizontal wave number when the atmospheric temperature increases with altitude. The up propagating wave reflects from higher-temperature domain, but some energy leaks up and the level where $\omega - c_s(z)k_{\perp} = 0$.

Thus the geometrical-optics approximation is not correct for this problem.

We have to analyze theoretically a general system of equations.


We note at once, that the resonance is possible at different heights for different ω and k_{\perp} .




It is seen that the frequency and the wave number can satisfy the relationship

$$\omega - c_s(z)k_{\perp} = 0$$

Let us return to the main system of equations


$$\begin{aligned} & \left[-\omega^2 + \omega_g^2(z) \right] W - i \frac{\omega}{\rho_E} \left[\frac{\partial}{\partial z} + \Gamma(z) \right] P = 0, \\ & \left[-\omega^2 + c_s^2(z) k_{\perp}^2 \right] P - i \omega \rho_E c_s^2(z) \left[\frac{\partial}{\partial z} - \Gamma(z) \right] W = 0. \end{aligned}$$

In some altitude domain near $z = z_*$, where $0 < [\omega - c_s(z) k_{\perp}]^2 / \omega^2 < \varepsilon$ ε is small in the mathematical meaning, and $W \approx 0$, the first equation of the system is reduced to the following form (it is clear that the temperature is almost constant in the domain in question, then the sound velocity and the Ekkard parameter are also constants).


$$\left[\frac{\partial}{\partial z} + \Gamma(z) \right] P = 0, \quad z \neq z_*, \quad W \approx 0$$

Let us consider in greater detail the processes near the resonance level $z = z_*$, which are described by the system of equations: Pressure disturbances both above and below the level $z = z_*$ are satisfied according to equation (*), but not according to equation (**), because $\omega \neq c_s k$, and W is infinitely small, but its derivative can be finite.

$$\begin{aligned} [-\omega^2 + \omega_g^2(z)]W - i\frac{\omega}{\rho_E}\left[\frac{\partial}{\partial z} + \Gamma(z)\right]P &= 0, \\ [-\omega^2 + c_s^2(z)k_\perp^2]P - i\omega\rho_E c_s^2(z)\left[\frac{\partial}{\partial z} - \Gamma(z)\right]W &= 0. (*) \end{aligned}$$

Pressure disturbances both above and below the level $z = z_*$ are satisfied according to equation (*), but not according to equation (**), because $\omega \neq c_s k$, and W is infinitely small, but its derivative can be finite.

$$[-\omega^2 + c_s^2(z)k_\perp^2]P = 0, \quad z = z_* \quad (**)$$

Exactly at the level $z = z_*$, the conditions $W = 0$ and $\frac{dW}{dz} = 0$ are fulfilled. Therefore, as follows from the equation, P can be arbitrary and equation is not defined by this equation at this point.

The absence of disturbances of both the vertical velocity and its derivative leads to the conclusion that above the level $z = z_*$ the solutions both for W and for P are identically equal to zero. Consequently, in order to counter balance the pressure jump at the level in question, the finite mass should be concentrated at the level $z = z_*$, which is taken into account in the solution using a delta function.

A formal solution of equation system near $z = z_*$ can be written as follows (Savina and Bespalov, 2014):

$$P = P_* E(z_* - z) \exp(\Gamma_*(z_* - z)) + \frac{P_* c_{s*}^2}{g} \delta(z - z_*),$$

$$W(z = z_*) = 0$$

Here $E(z - z_*)$ is the Heaviside step function and $\delta(z - z_*)$ is the Dirac delta function; by a subscript asterisk we mark the values of the variables at the layer $z = z_*$. Solution (3) depends on a single constant P_* , which is determined by the boundary condition on the Earth's surface. Thus, a local disturbance consists of a special feature in the altitude distribution of the pressure disturbance and characteristic structures $P(z)$ and $W(z)$ adjoining it from below. Actually, one or more resonance levels can exist in the real atmosphere for specified ω and k_\perp .

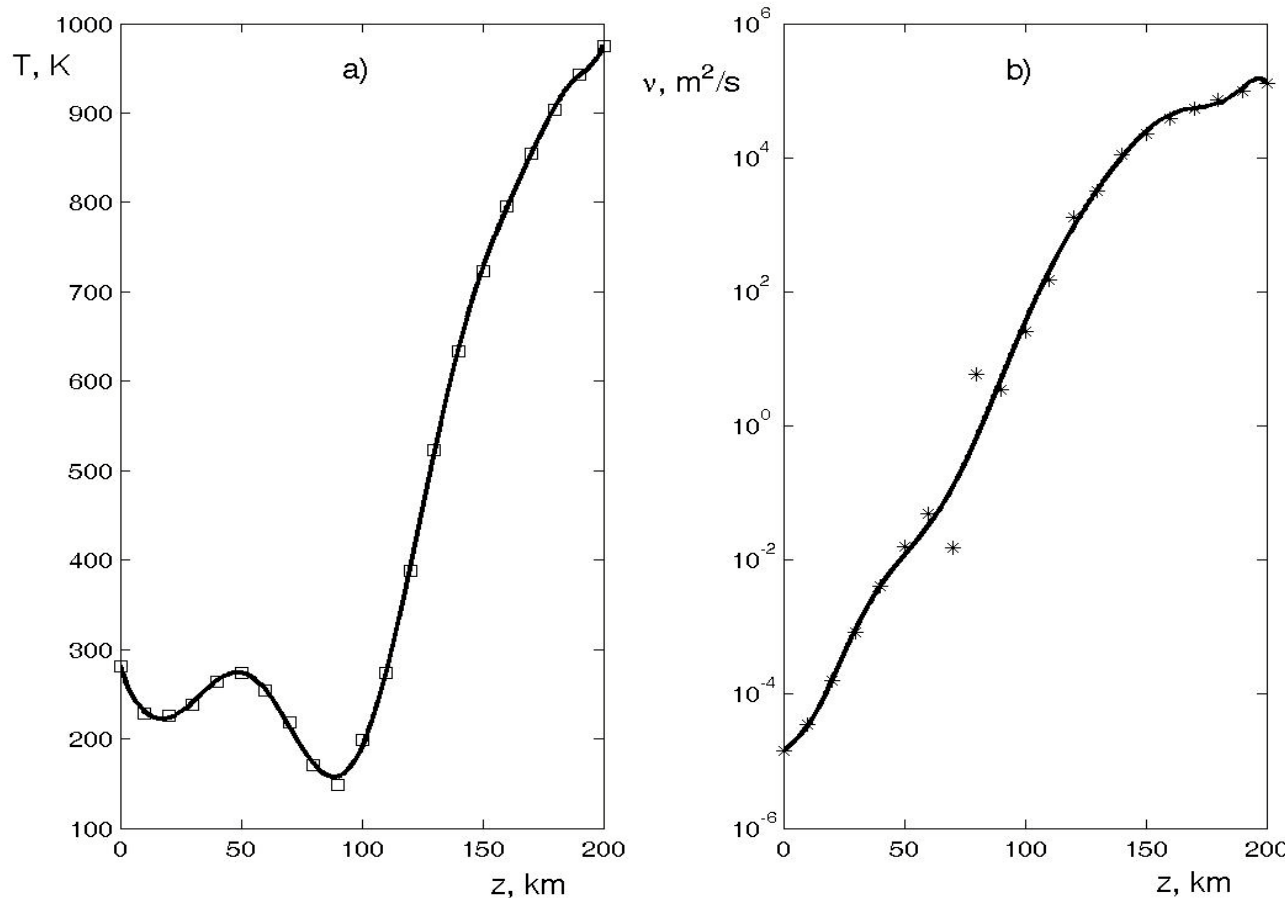
Perturbations of the velocity and the pressure are absent everywhere above the first resonance level.

Hence, if for the wave perturbation in the nonisothermal atmosphere at some level $z = z_*$ a condition $\omega = c_s(z_*)k_\perp$ is satisfied, then the averaged vertical energy flux is equal to zero. Above the first of such levels, wave perturbation are absent along the vertical propagation path. This effect is responsible for the formation of a waveguide channel between the Earth's surface and the resonance level for the waves whose horizontal phase velocity is equal to the local sound velocity.

Under the actual conditions, the resonance in the form of a delta function in the pressure disturbance, as well as in the horizontal velocity perturbation that is proportional to it ($V = P/c_s(z = z_*)\rho_E$), is smeared due to the molecular viscosity and the nonlinearity. In the numerical calculations given below the molecular viscosity was taken into account.

3. *Full-wave calculations for resonance atmospheric perturbation*

We selected the altitude temperature profile following the MSIS-E-90 model (Hedin, 1991). For example, we chose one temperature profile $T(z)$ for 10.08.2012 at 15.00 UT for geographic latitude 65° and longitude 45° . We determine the analytical function for this temperature profile (Fig. 1a) and kinematic viscosity profile (Fig 1b) (Kikoin, 1976) using the model values marked by boxes and stars in Fig.a and b, respectively. The temperature and viscosity profiles were approximated by polynomials of the tenth order.



Typical atmosphere temperature profile $T(z)$ as obtained from the MSIS-E-90 model marked by boxes and spline curve $T(z)$ (left panel); typical altitude profile of molecular kinematic viscosity $v(z)$ in the Earth's atmosphere (right panel).

4. The results of numerical simulation

The results of numerical simulation for an inhomogeneous plane wave with fixed frequency and the horizontal wave number. Therefore, simulations are one-dimensional, and task is two-dimensional. Numerical calculation is conducted in much the same way as in papers (Rapoport et al., 2004, Bespalov and Savina, 2014).

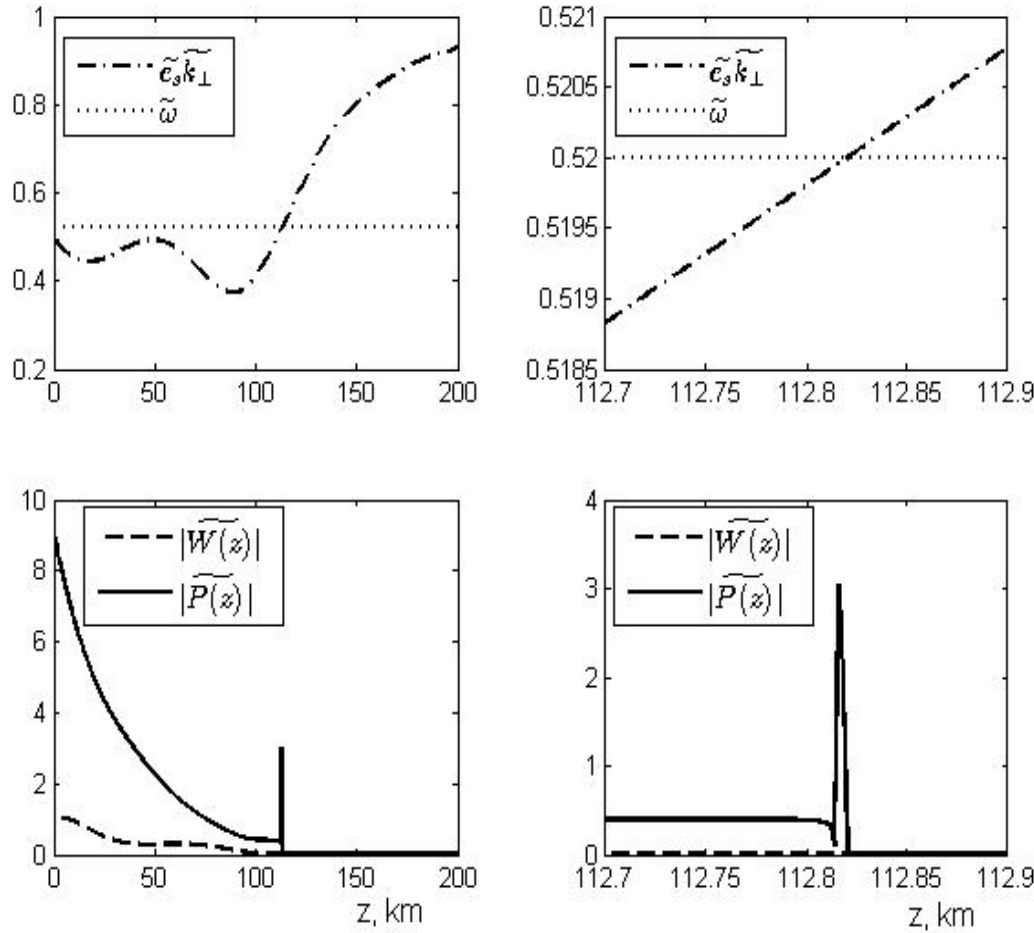
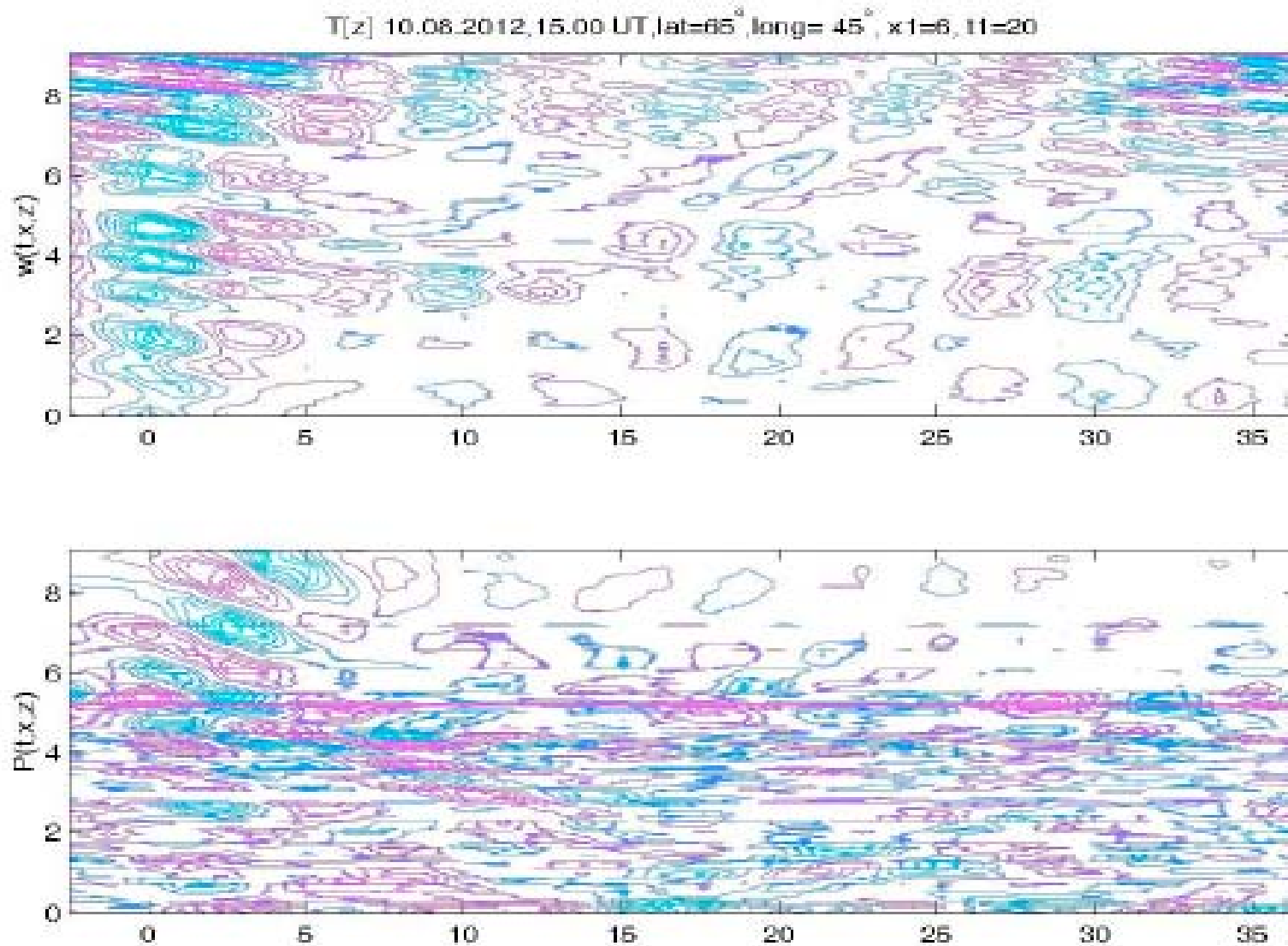


Figure shows the local disturbance at an altitude near 113 km. In the upper panels, the dashed dotted curves are the altitude dependence $\tilde{c}_s(z)\tilde{k}_\perp$ and the dotted lines are the frequency $\tilde{\omega}$. In the bottom panels, the solid curves are the altitude dependence $|\tilde{P}(z)|$ and the dashed curves are the altitude dependence $|\tilde{W}(z)|$. In the figure on the right bottom panel the dependencies $|\tilde{P}(z)|$ and $|\tilde{W}(z)|$ are given in greater detail for altitudes near 113 km, which correspond to the local disturbance. Calculations show that in this case the averaged vertical energy flux $\tilde{S} = 0$.

5. Instantaneous wave field disturbances

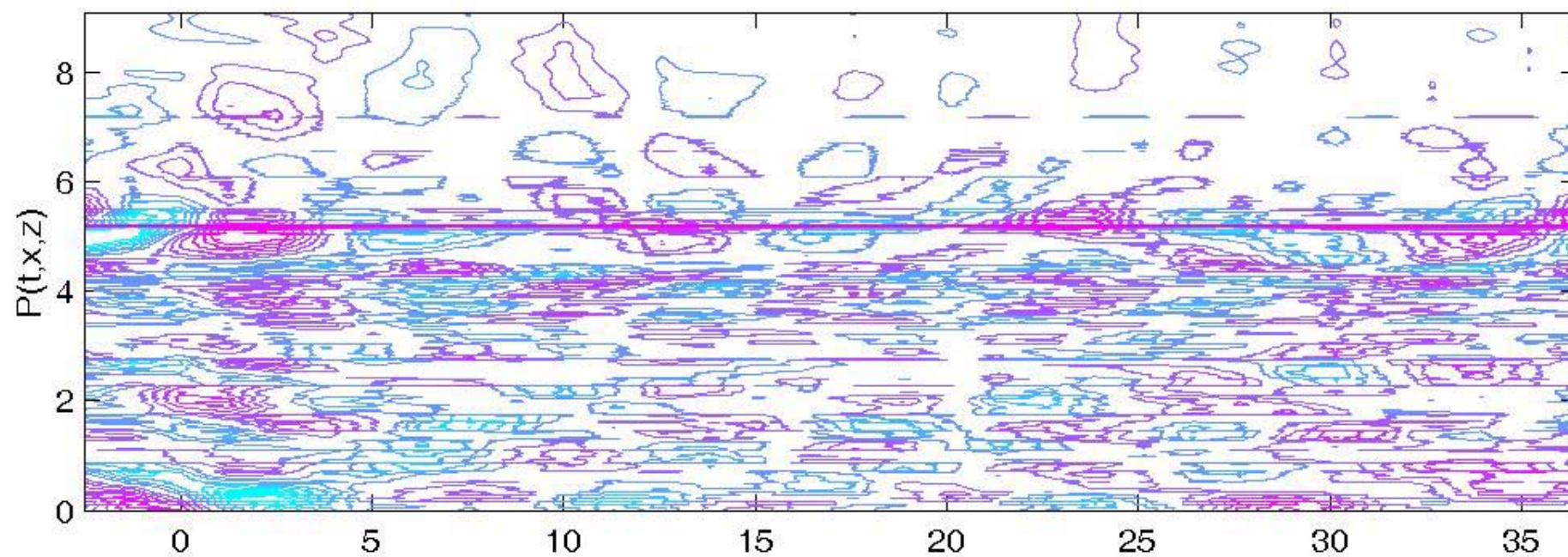
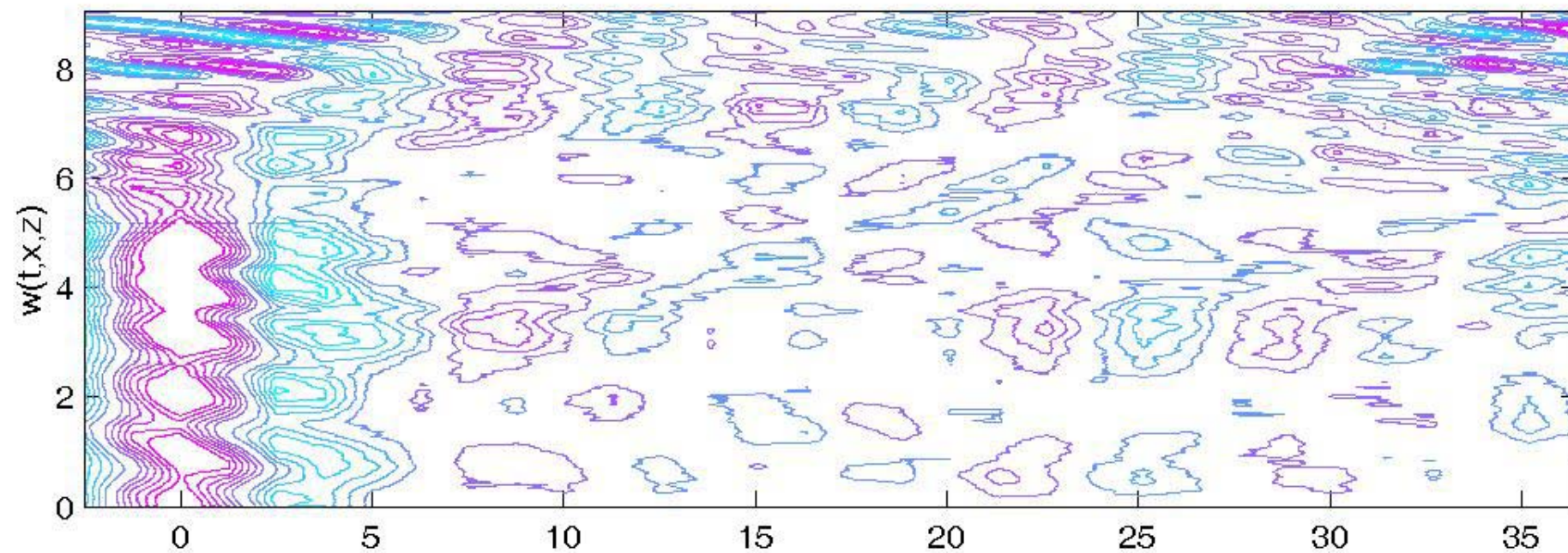
Instantaneous wave field disturbances of vertical velocity (upper figure) and pressure (lower figure) from the local pulse ground-based source (along the X-axis it is deposited on the standardized horizontal coordinate).



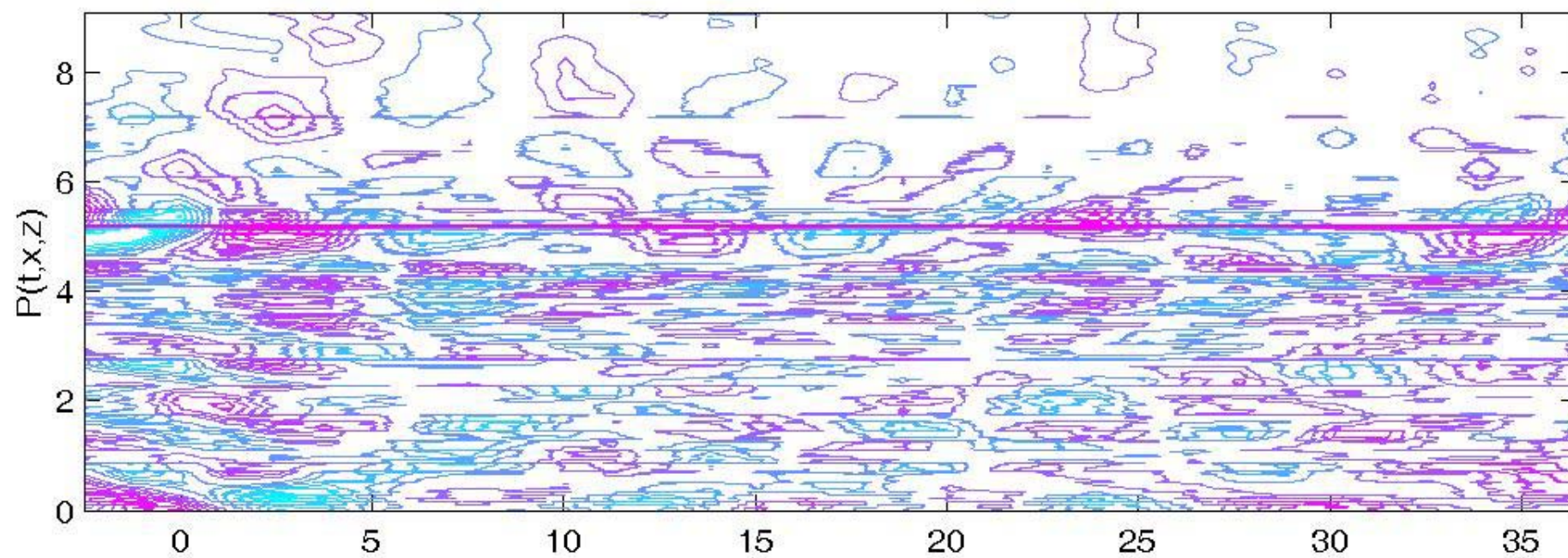
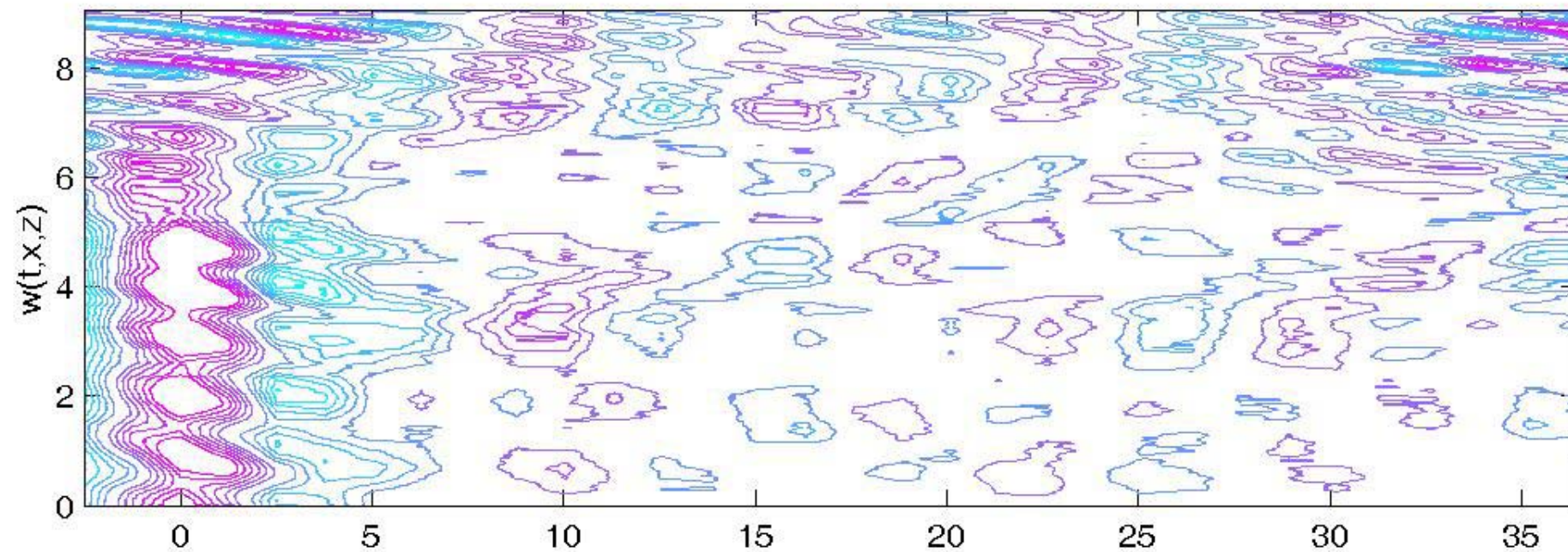
High-frequency component forms the disturbance, which departs to the upper levels of the atmosphere.

The disturbance above the resonance level does not pass.

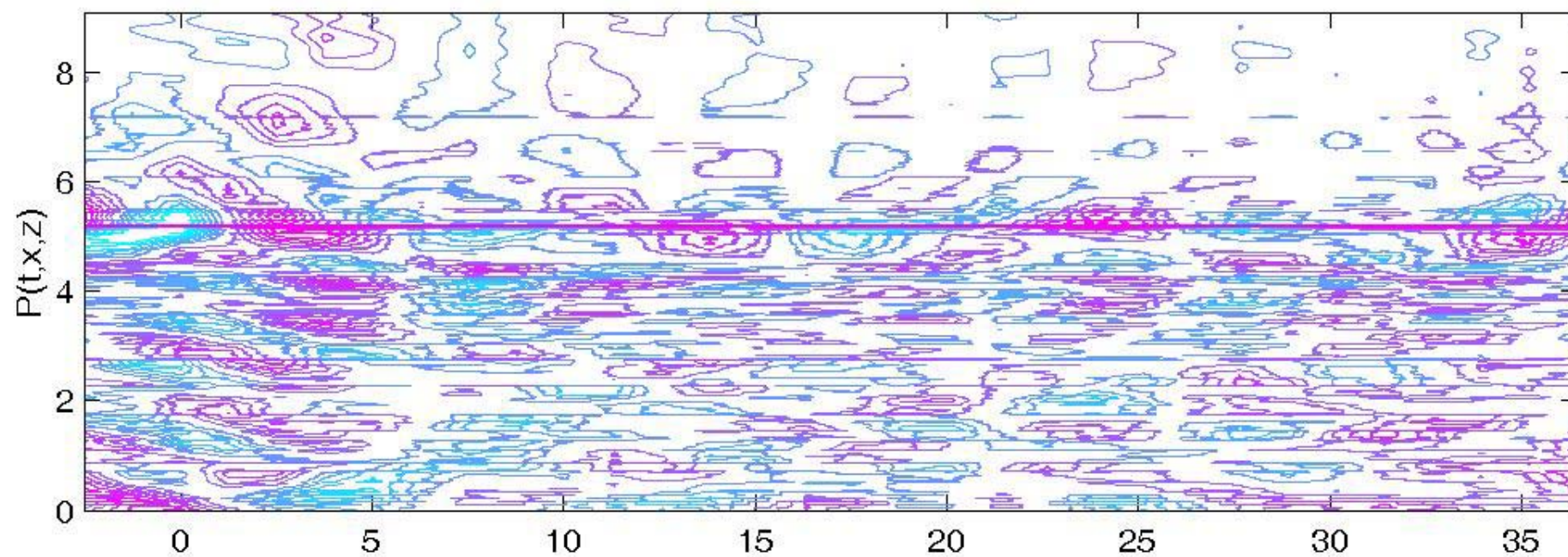
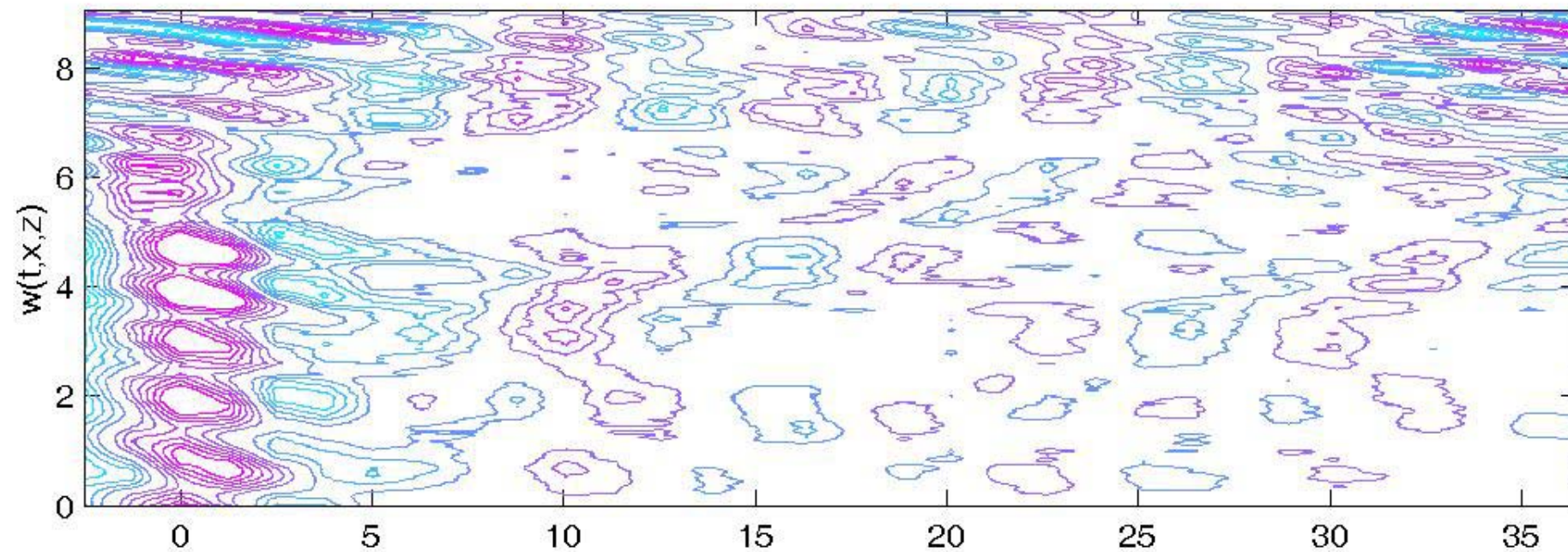
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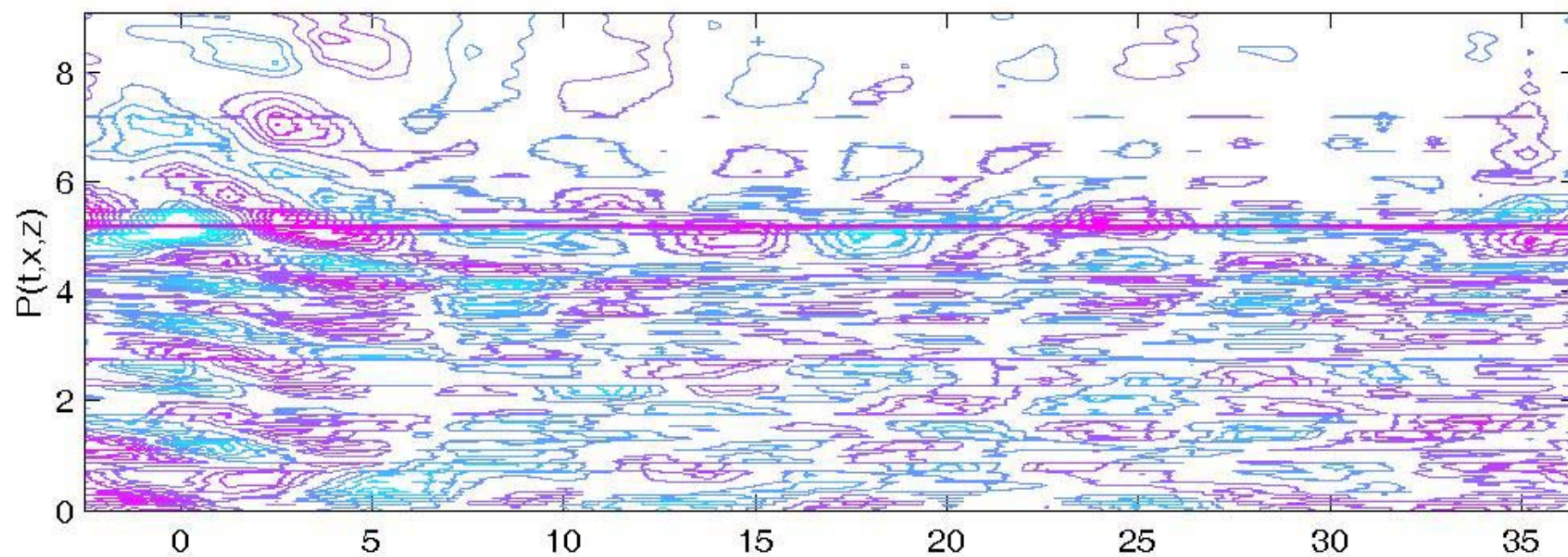
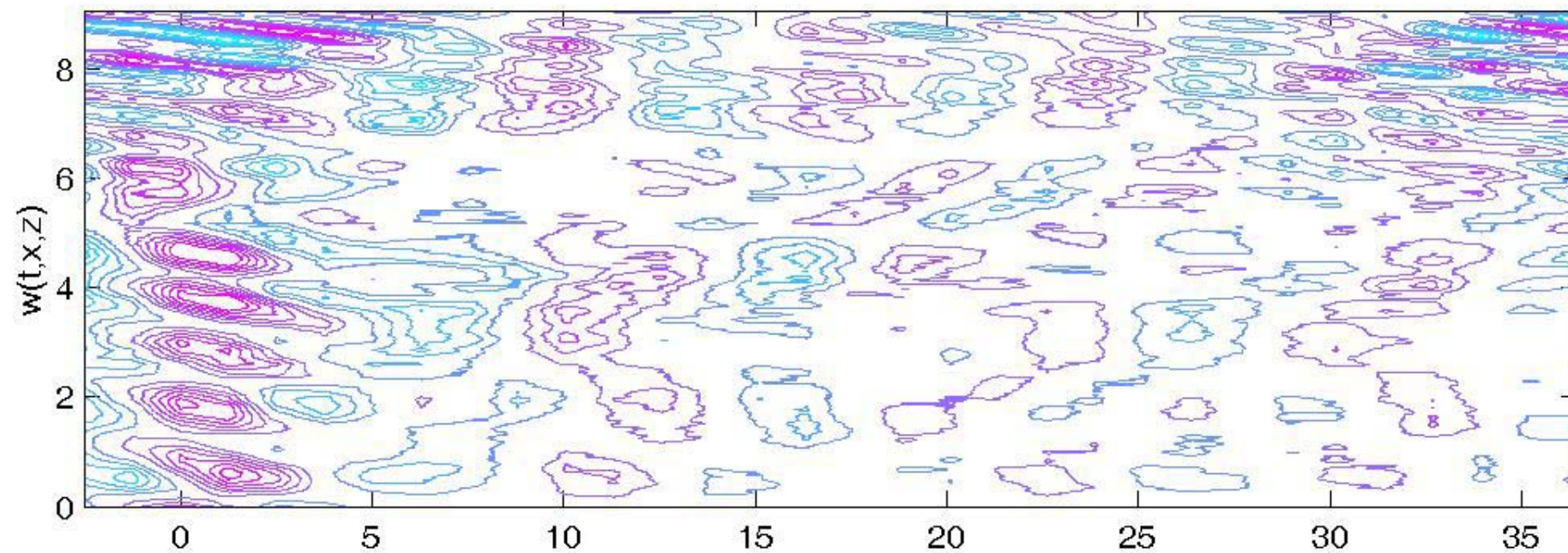
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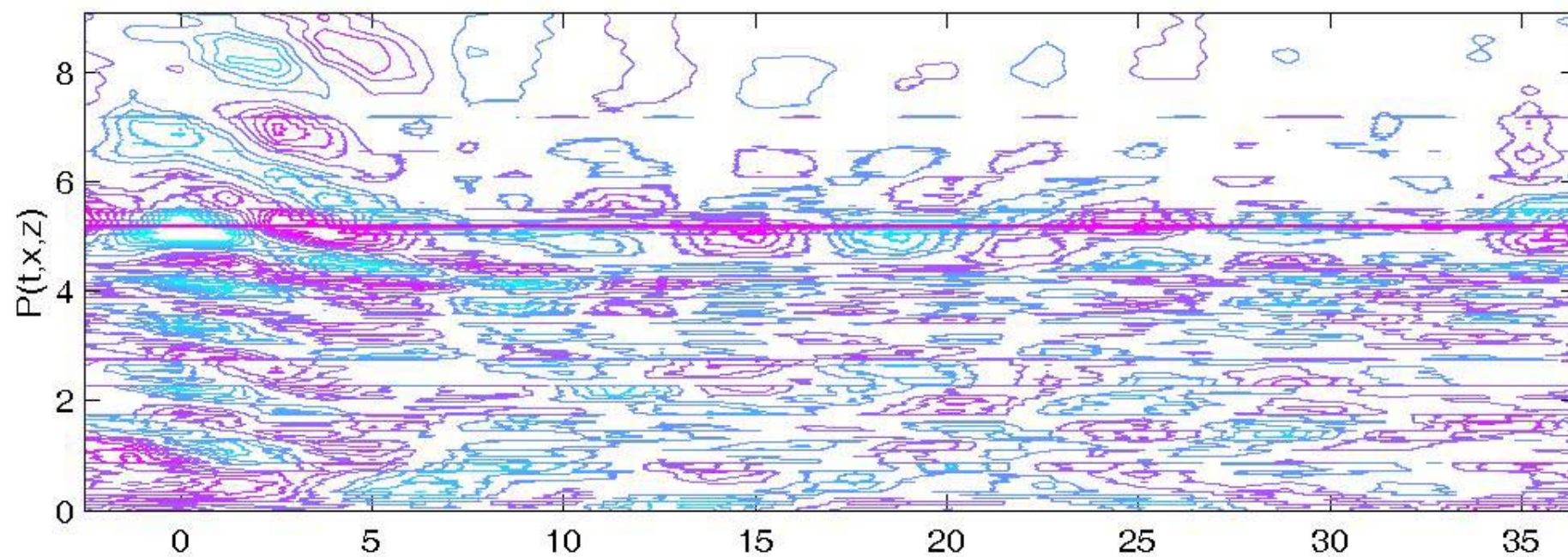
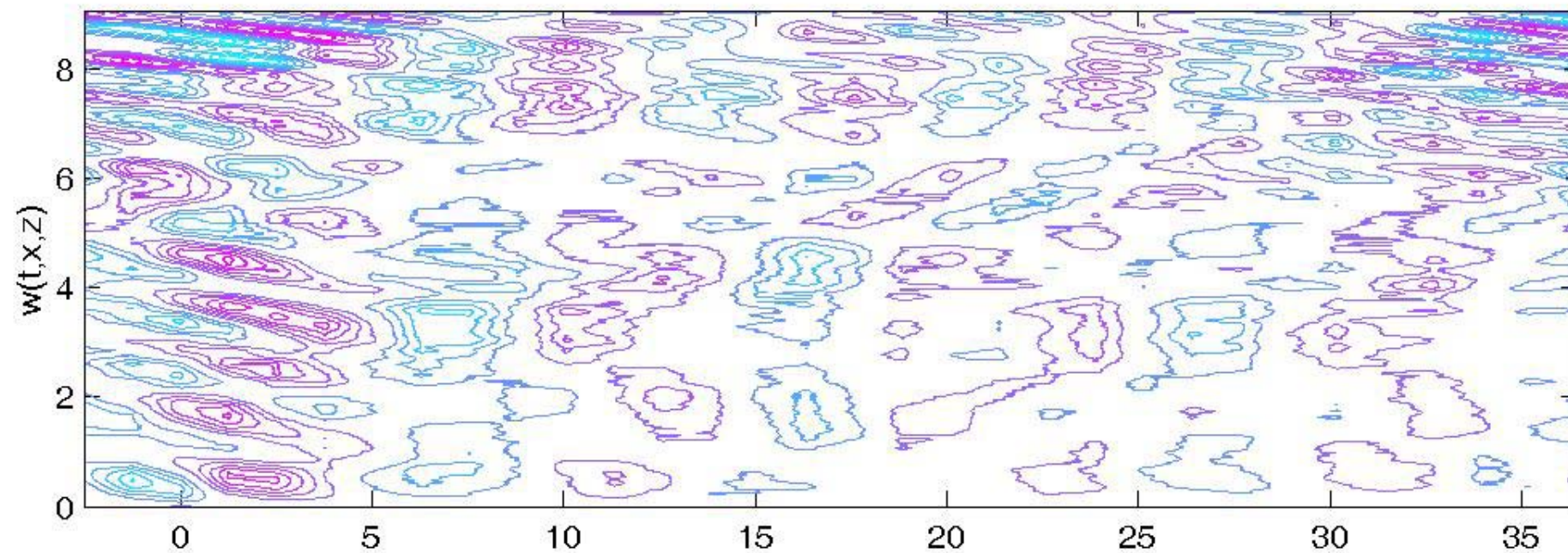
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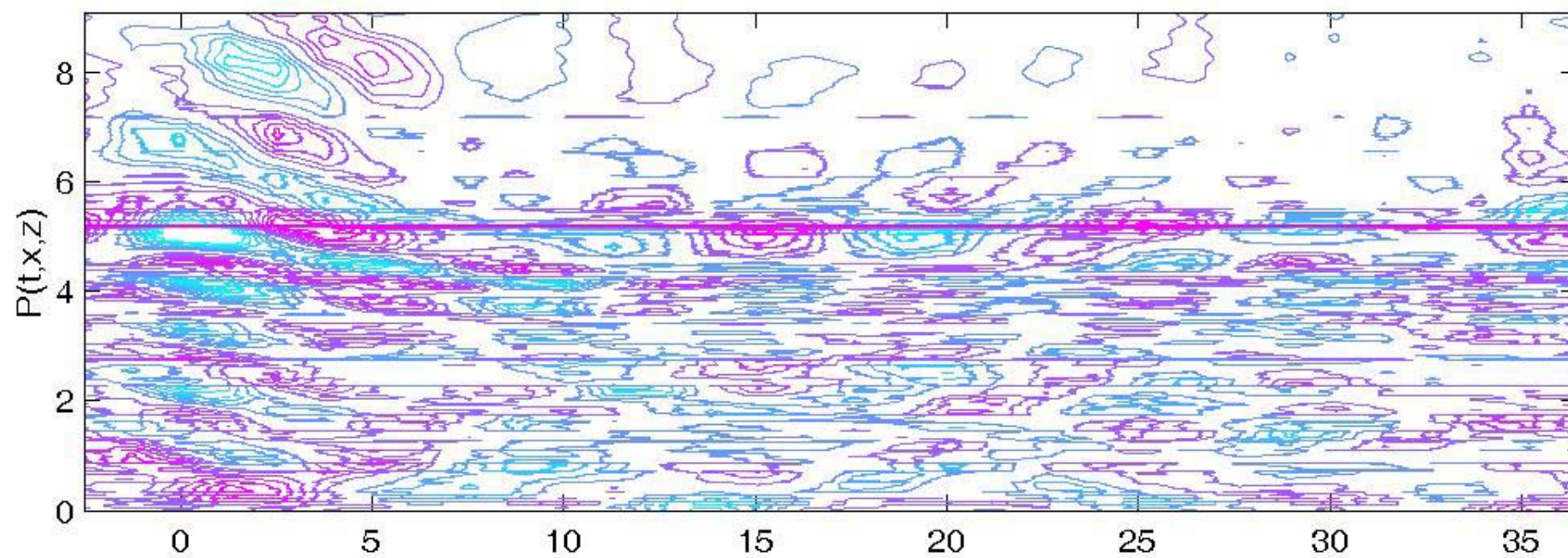
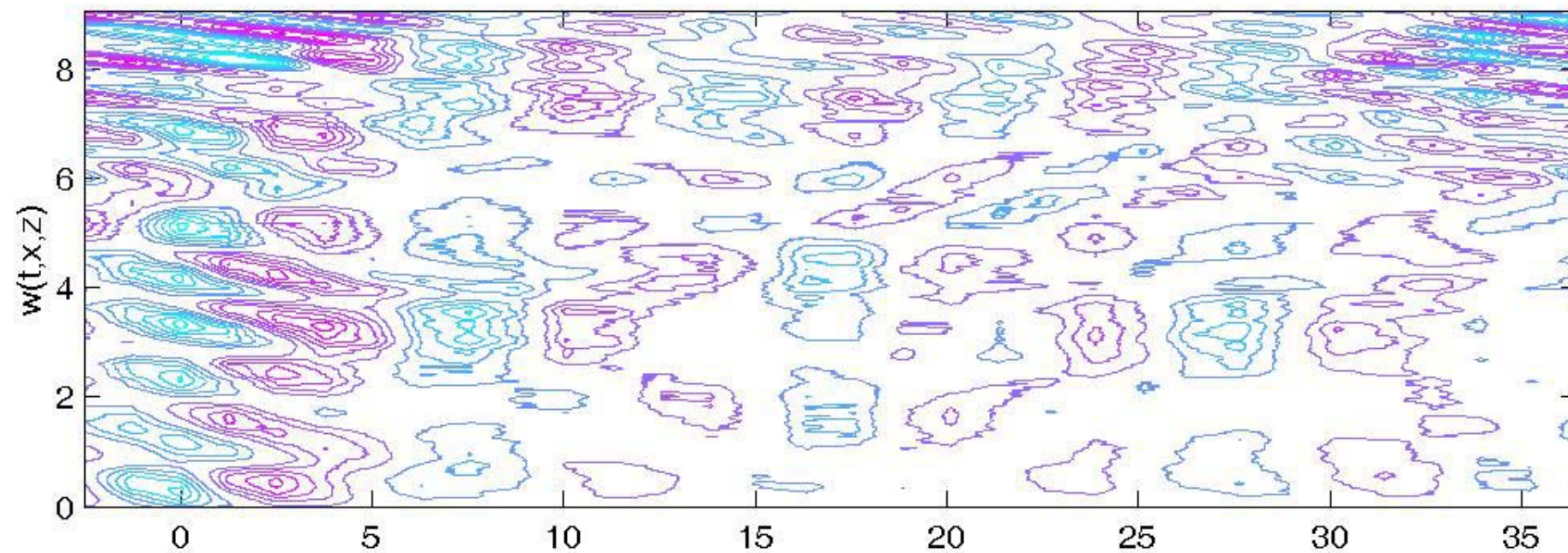
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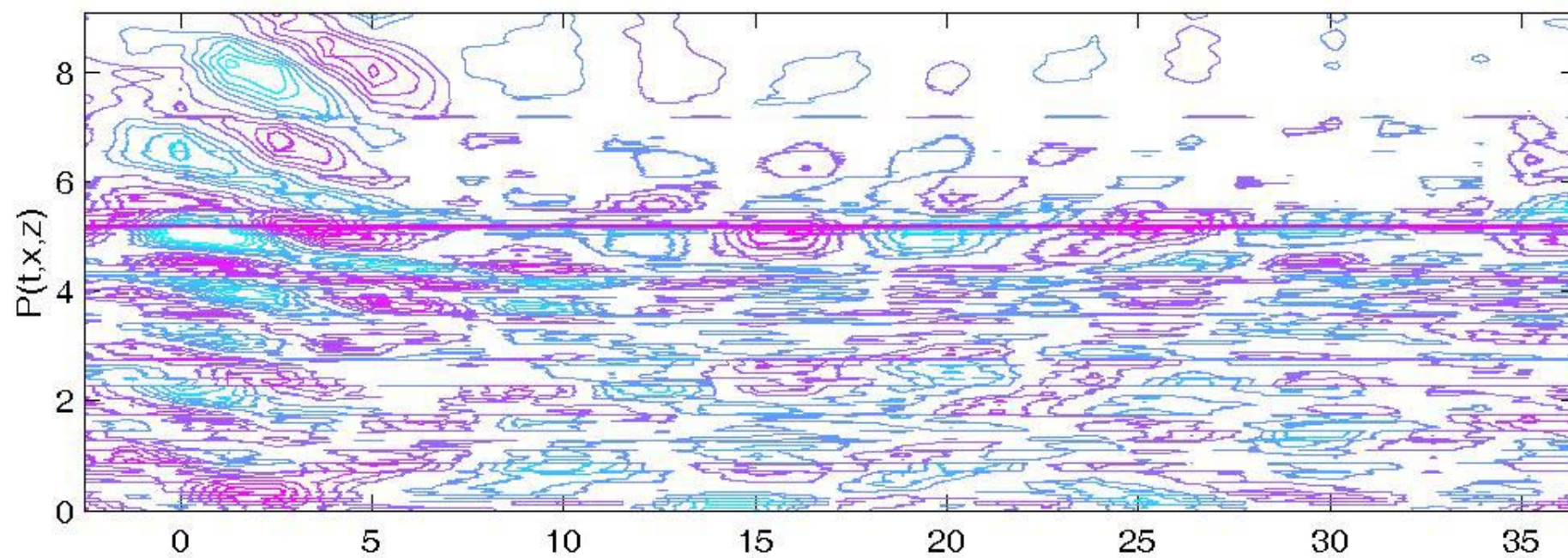
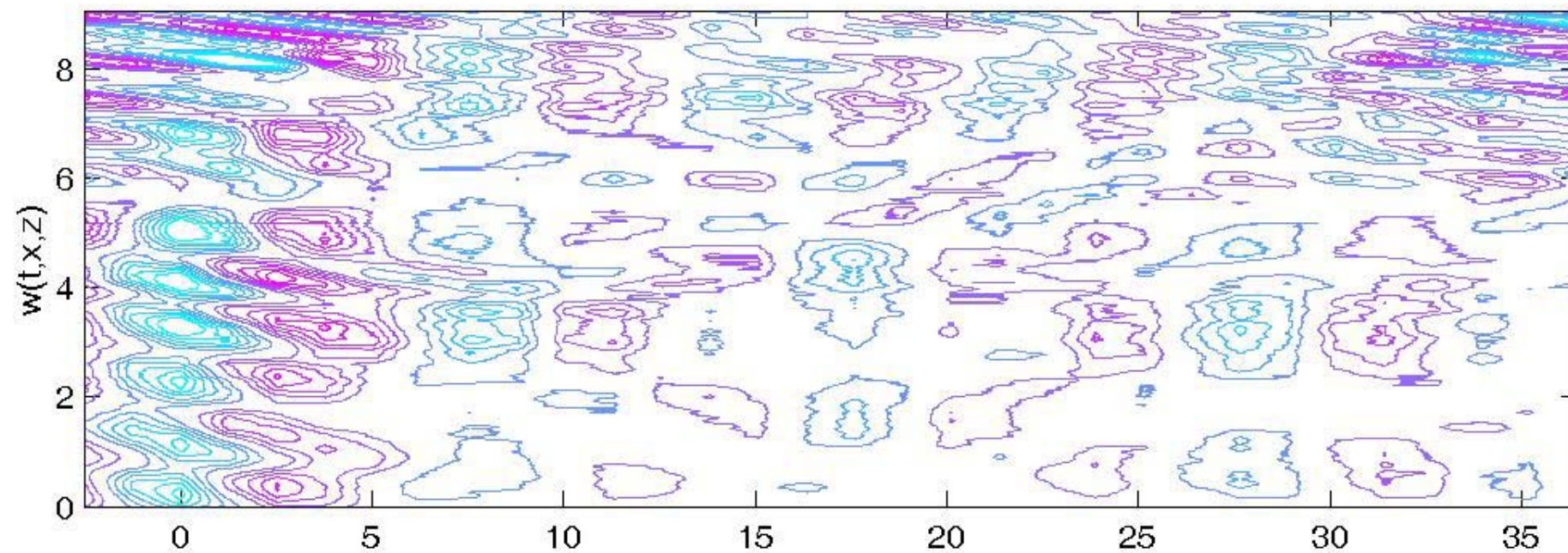
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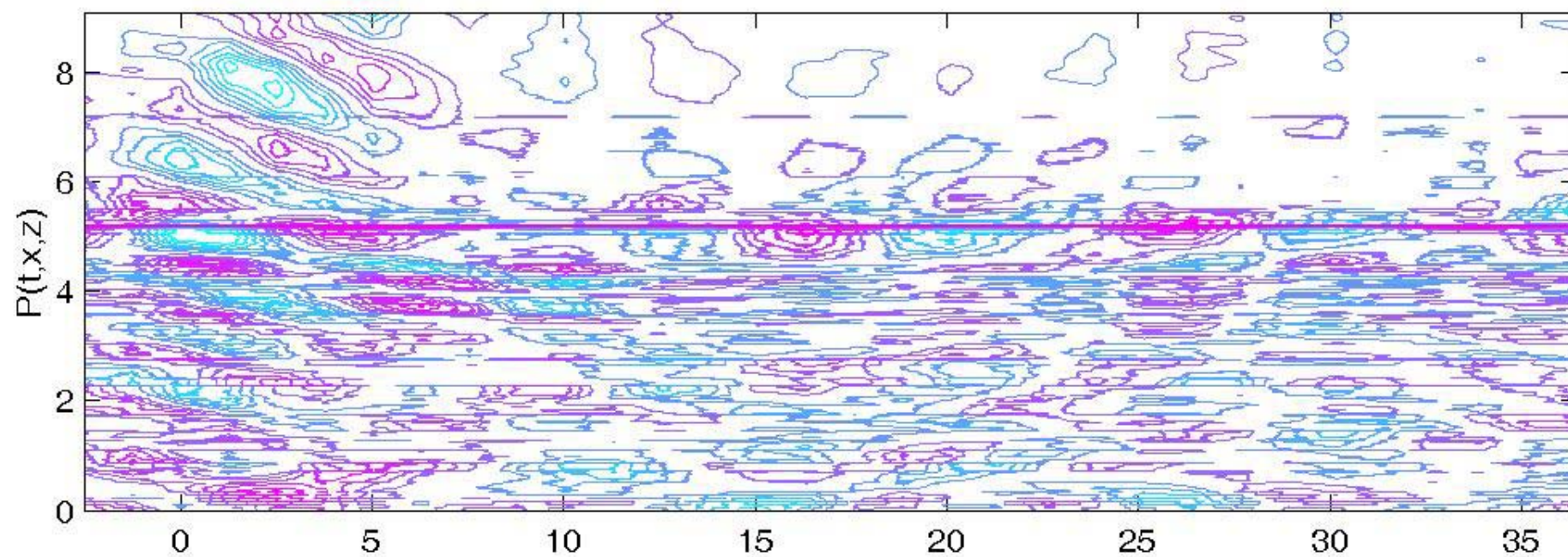
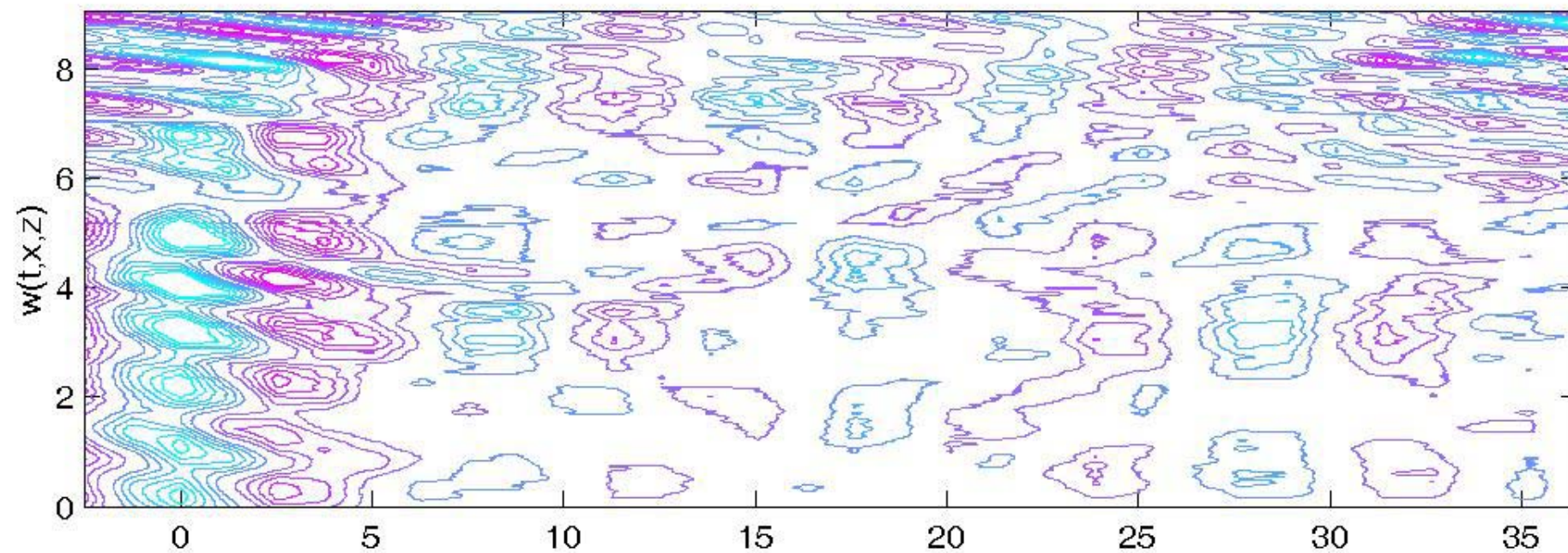
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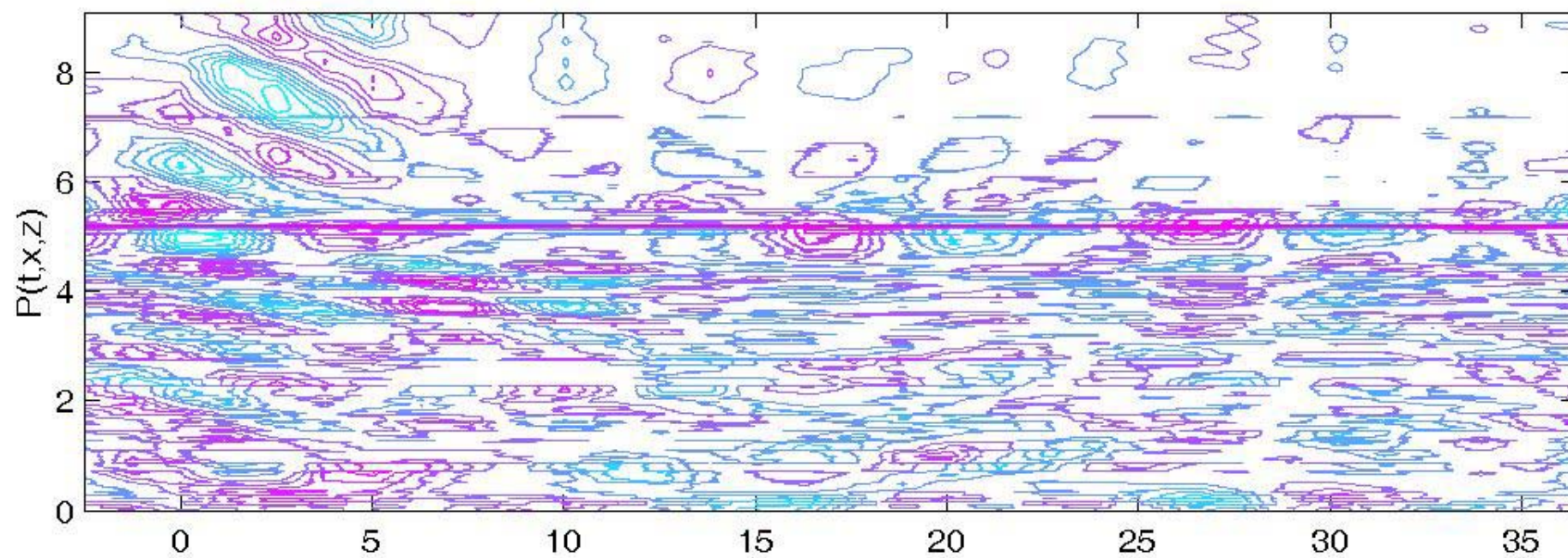
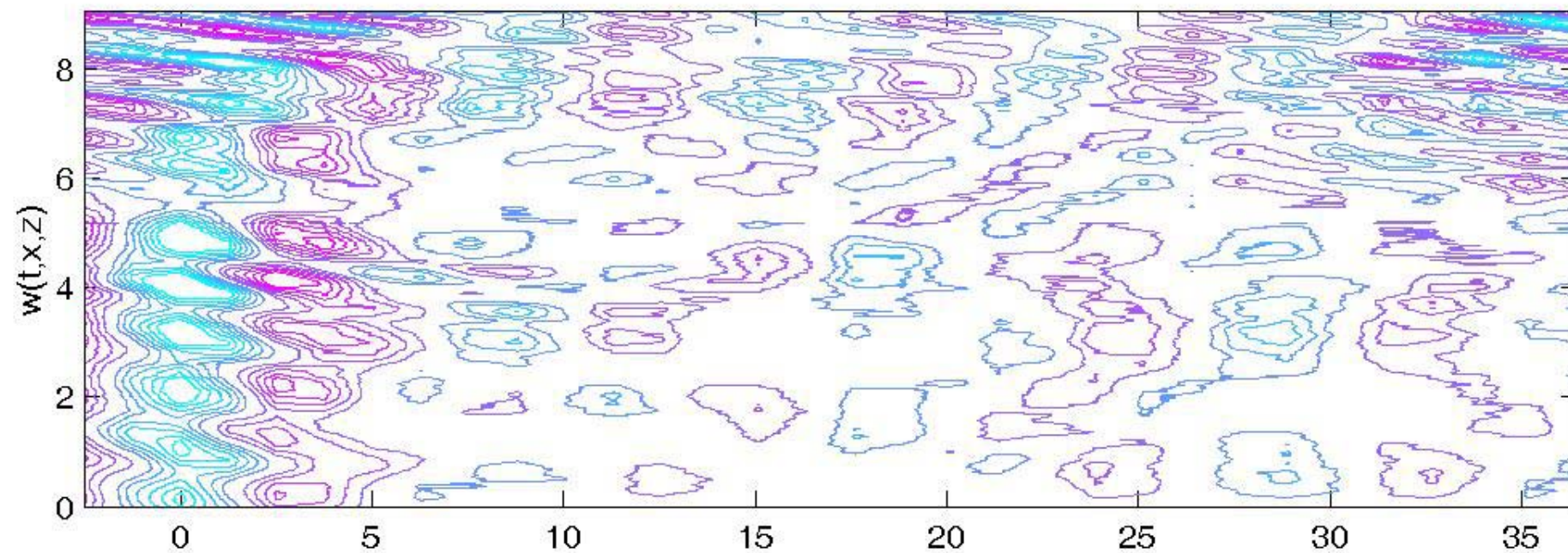
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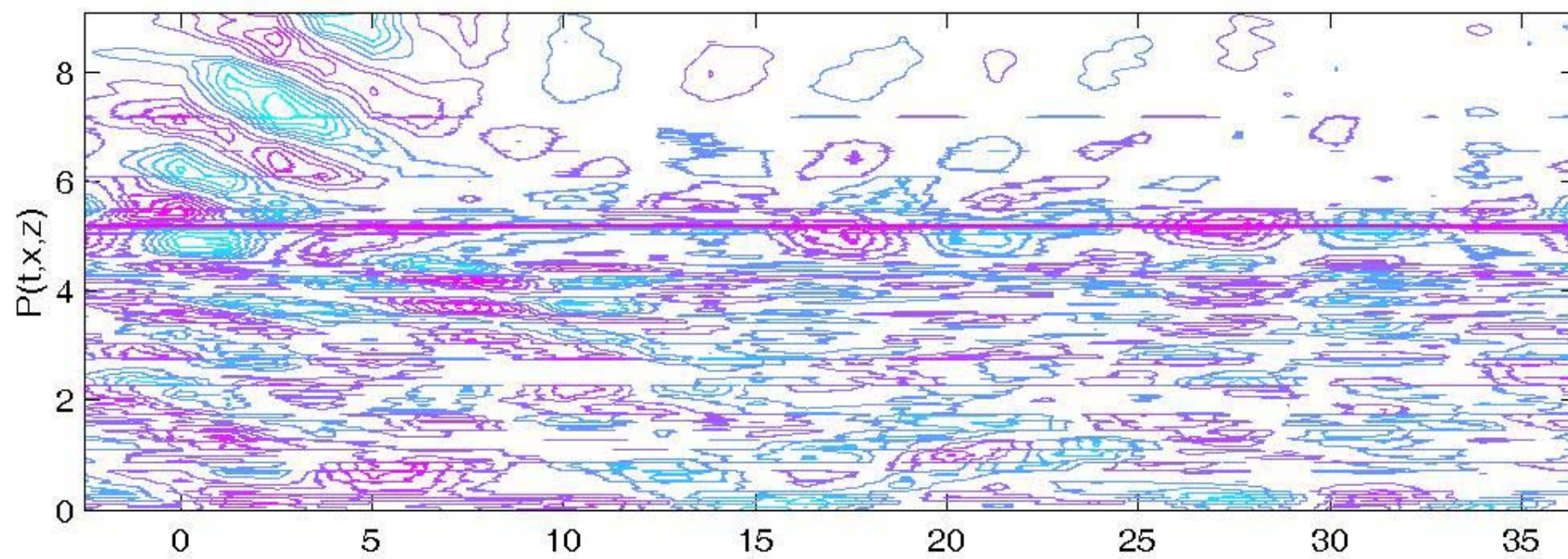
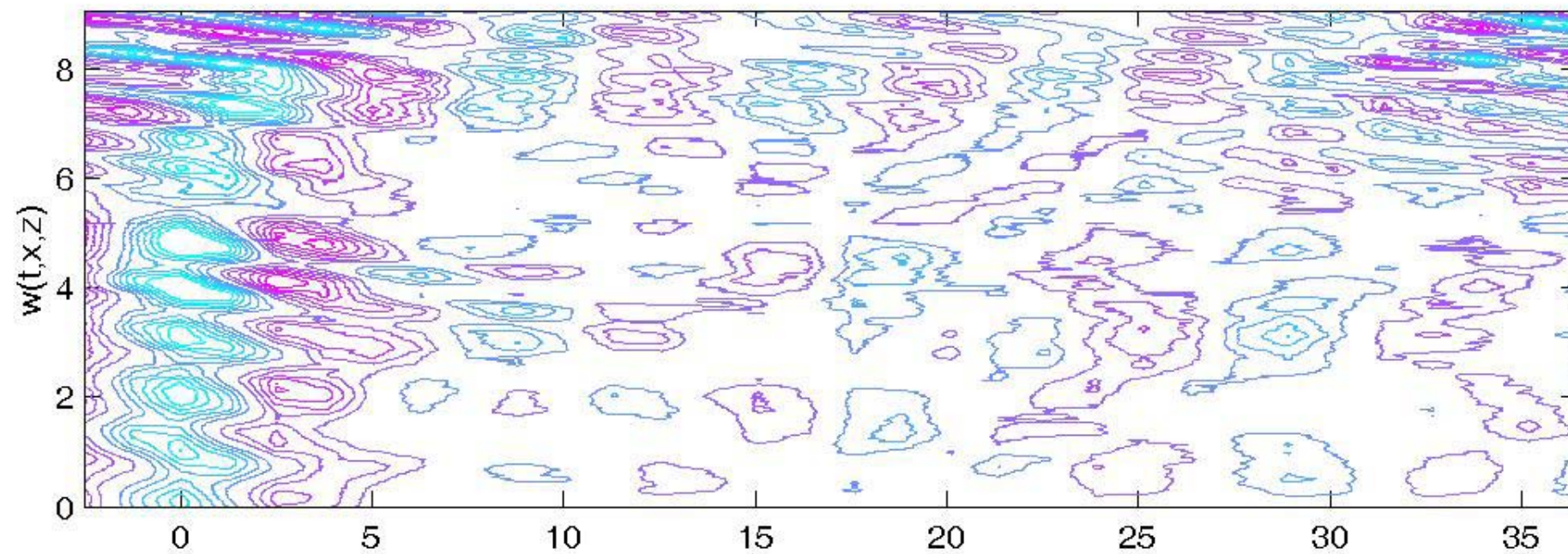
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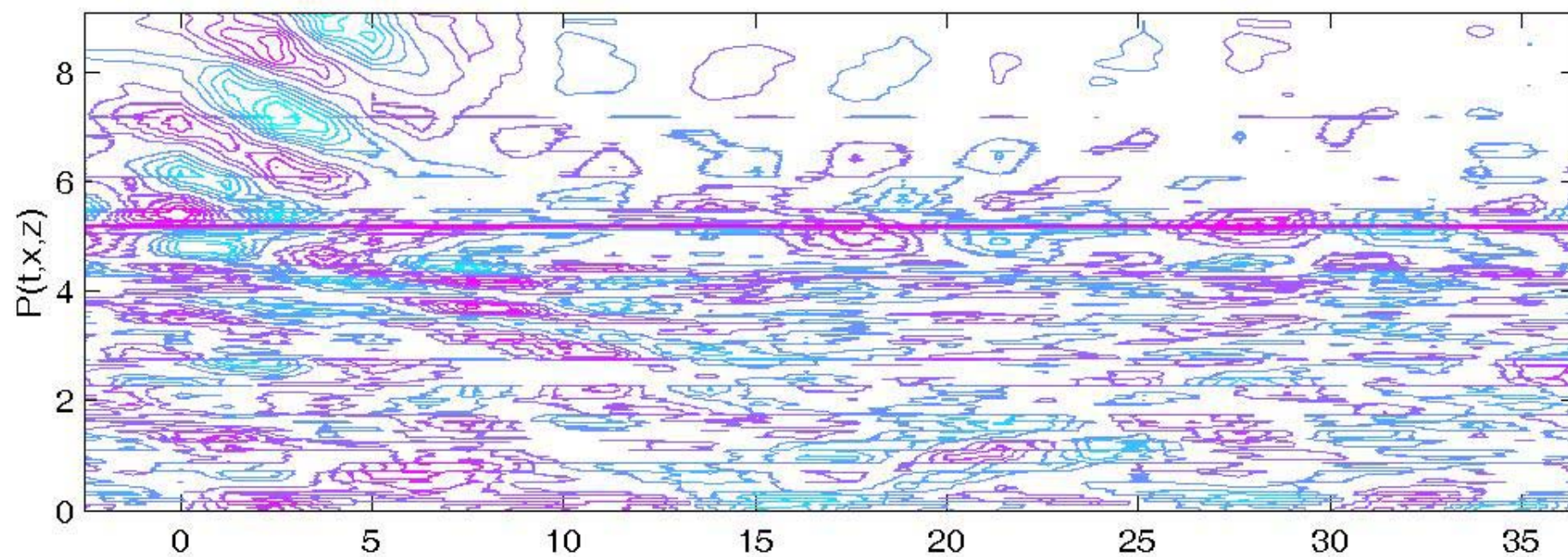
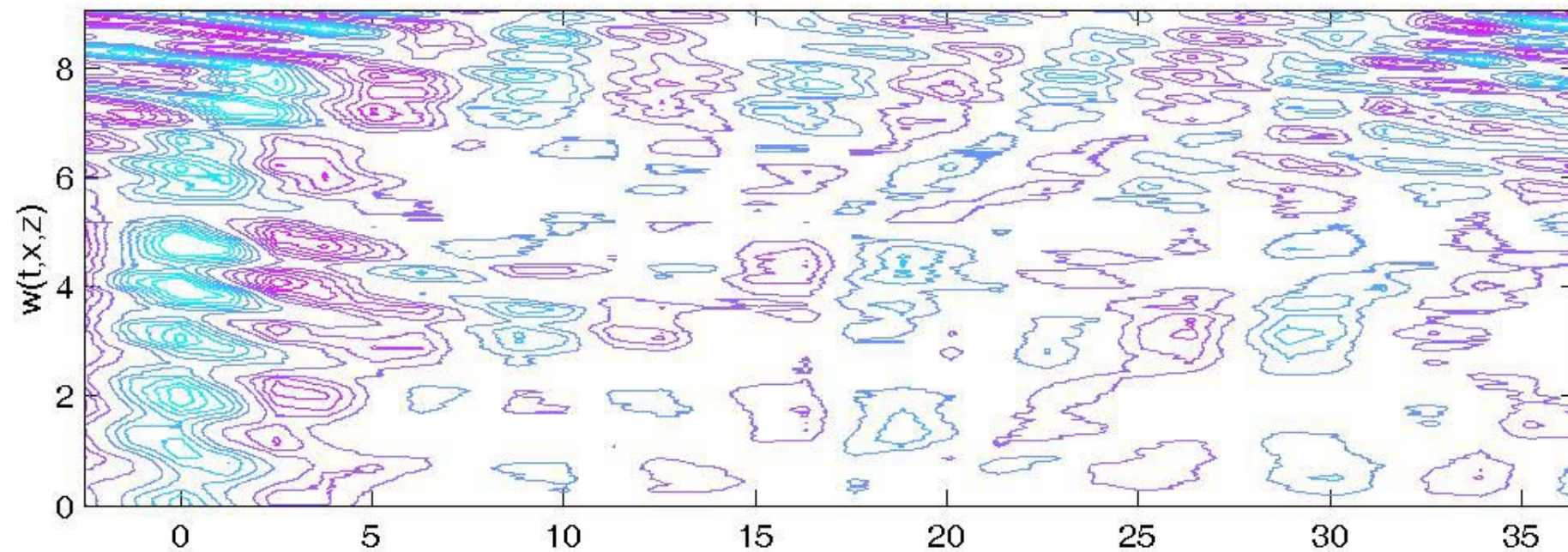
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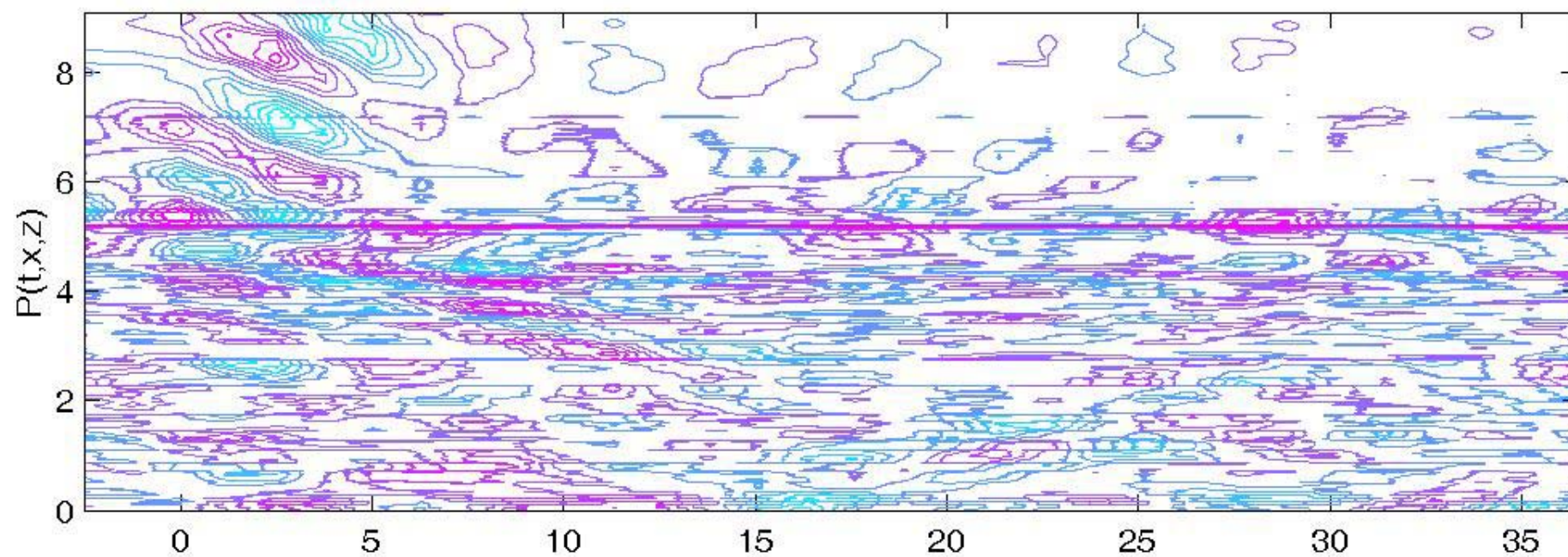
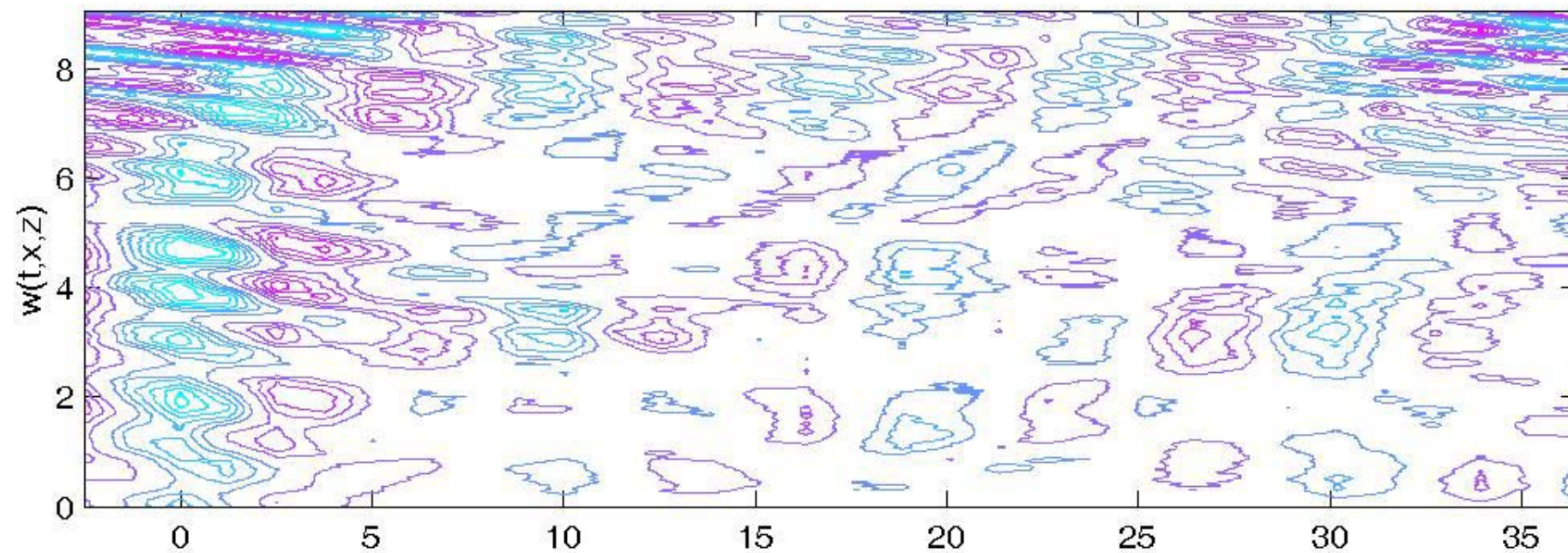
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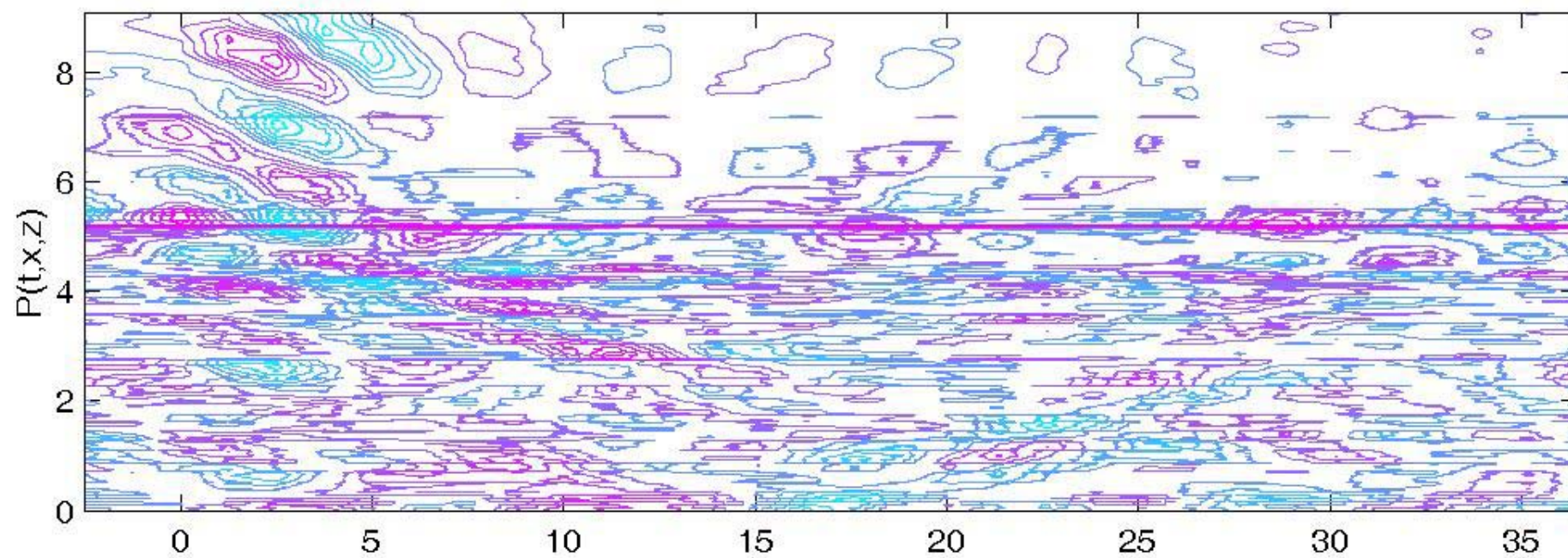
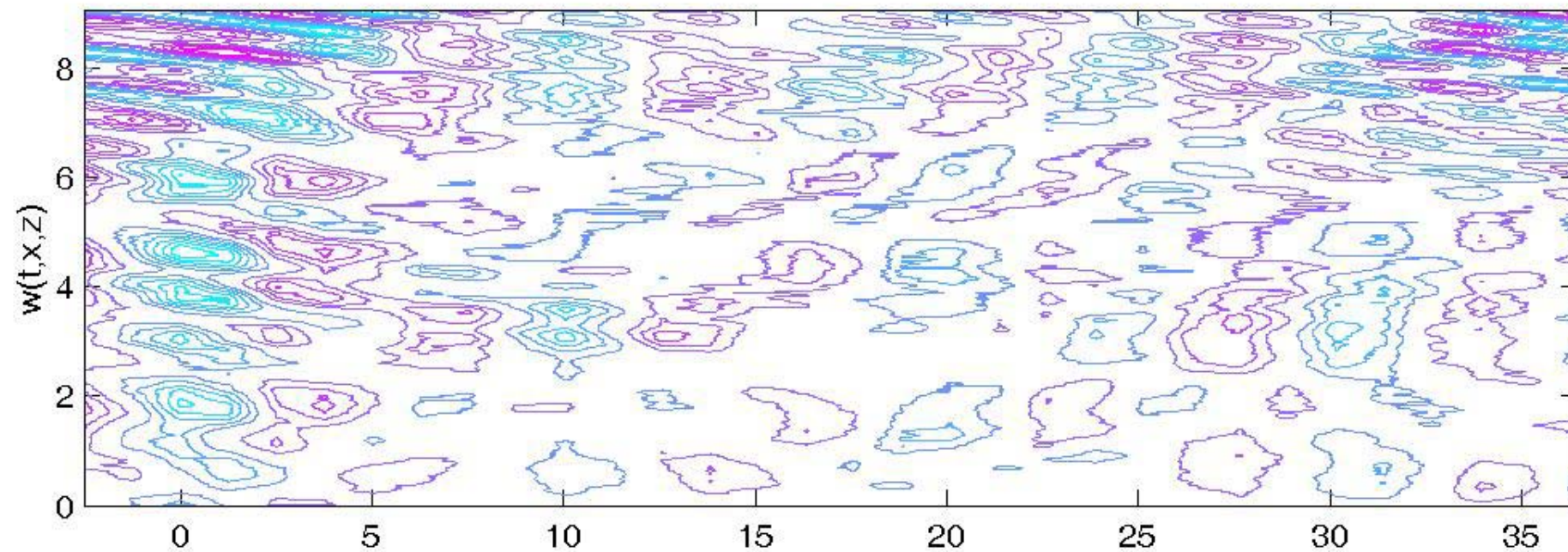
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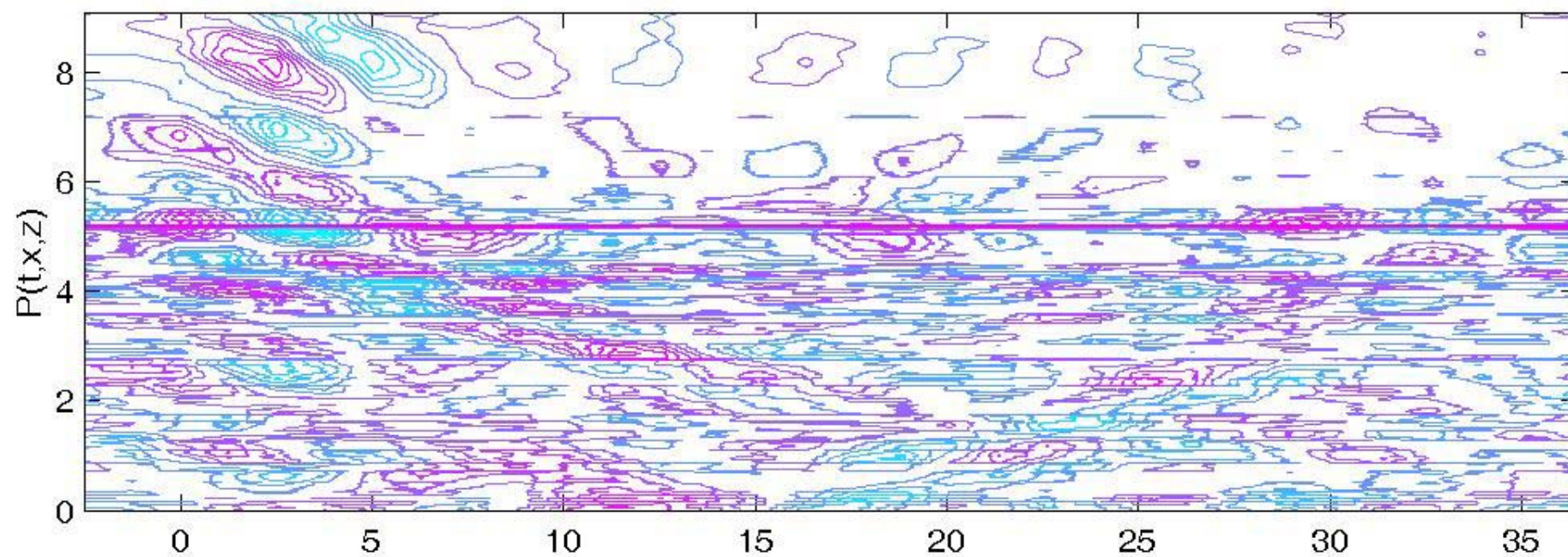
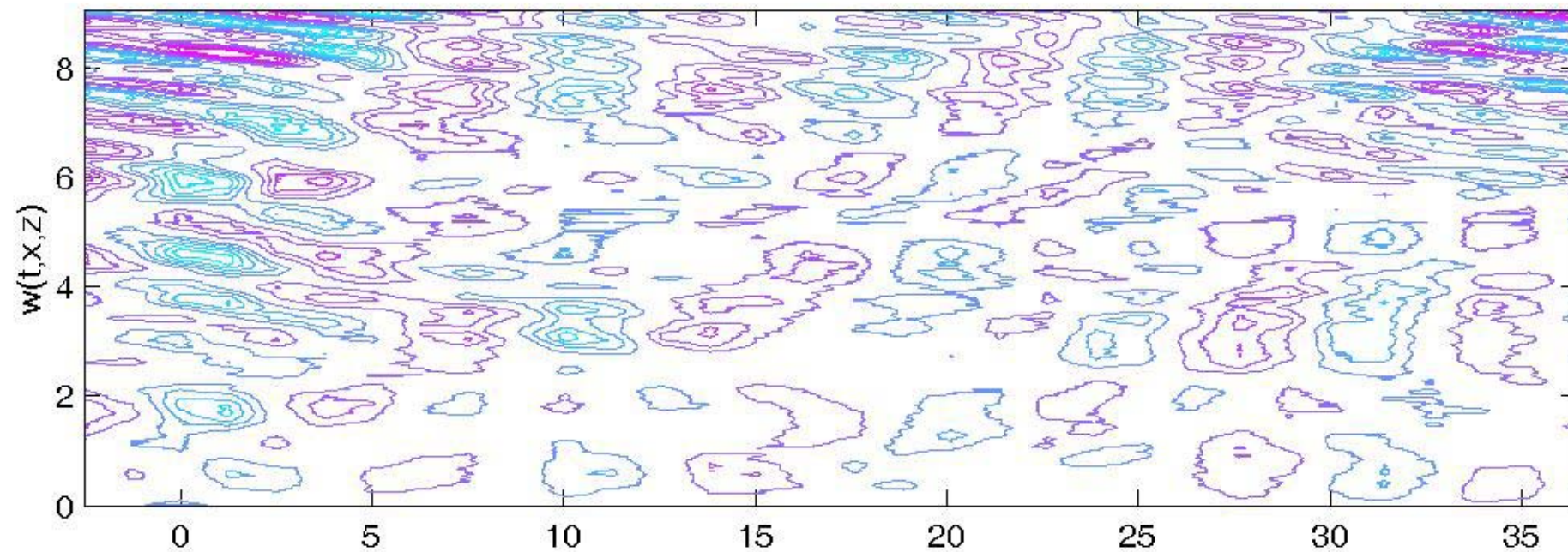
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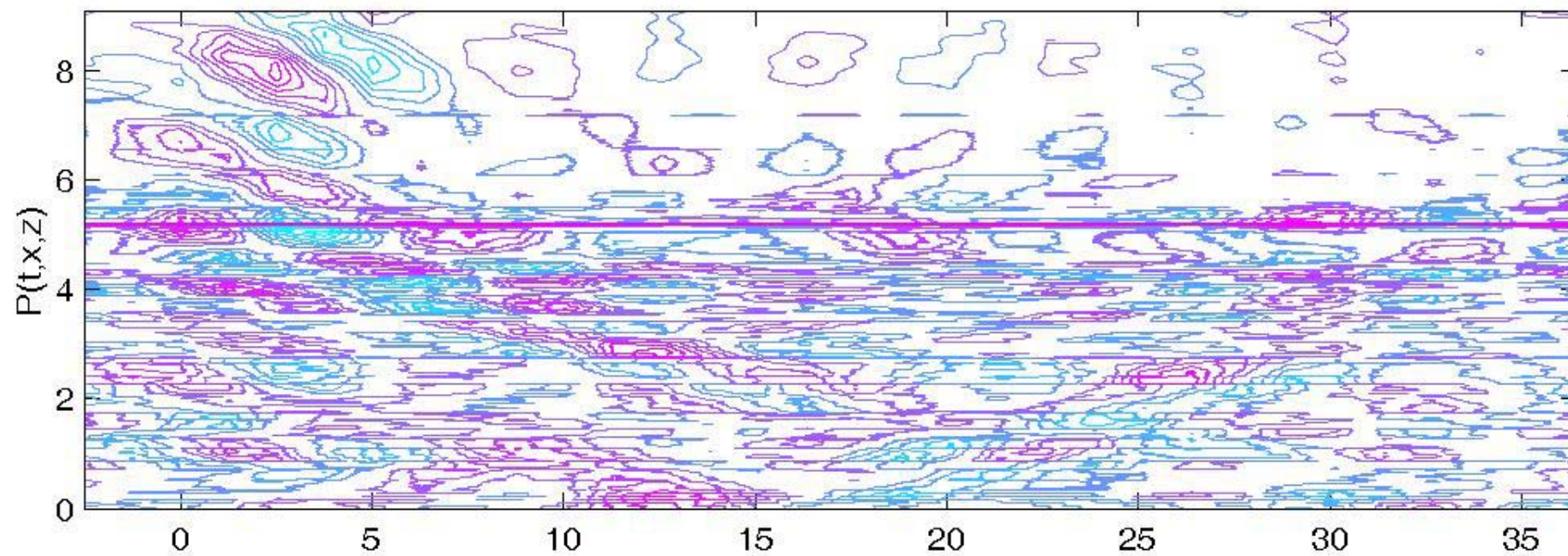
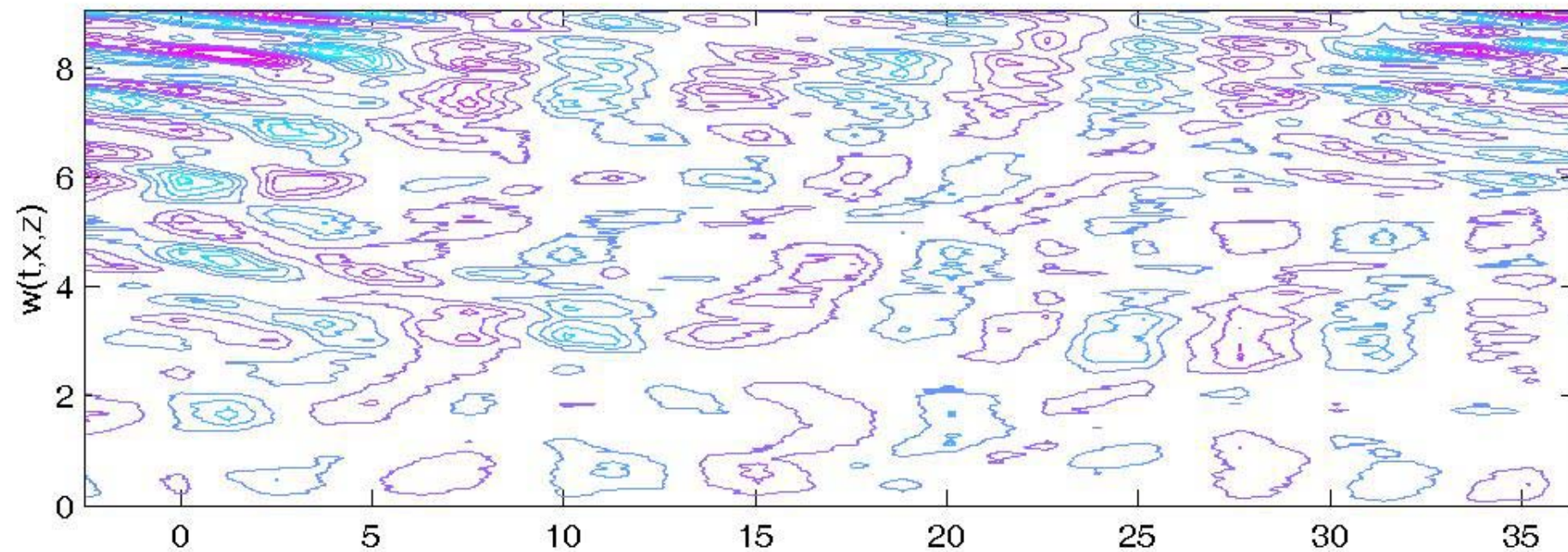
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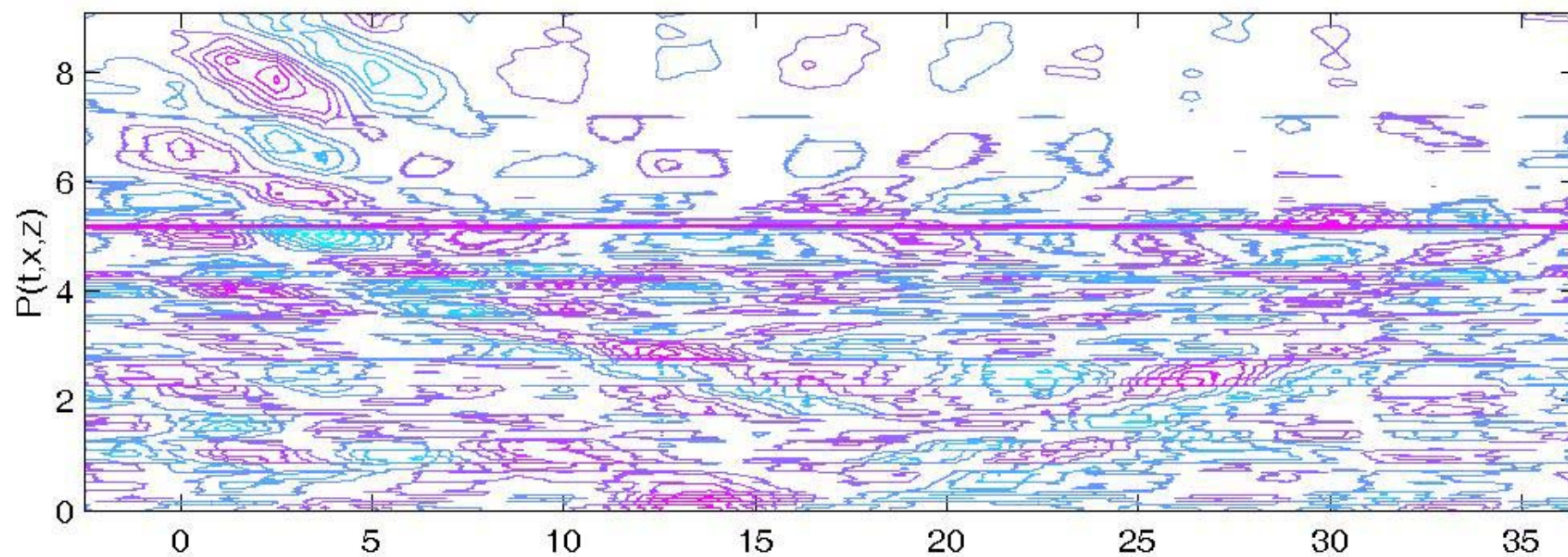
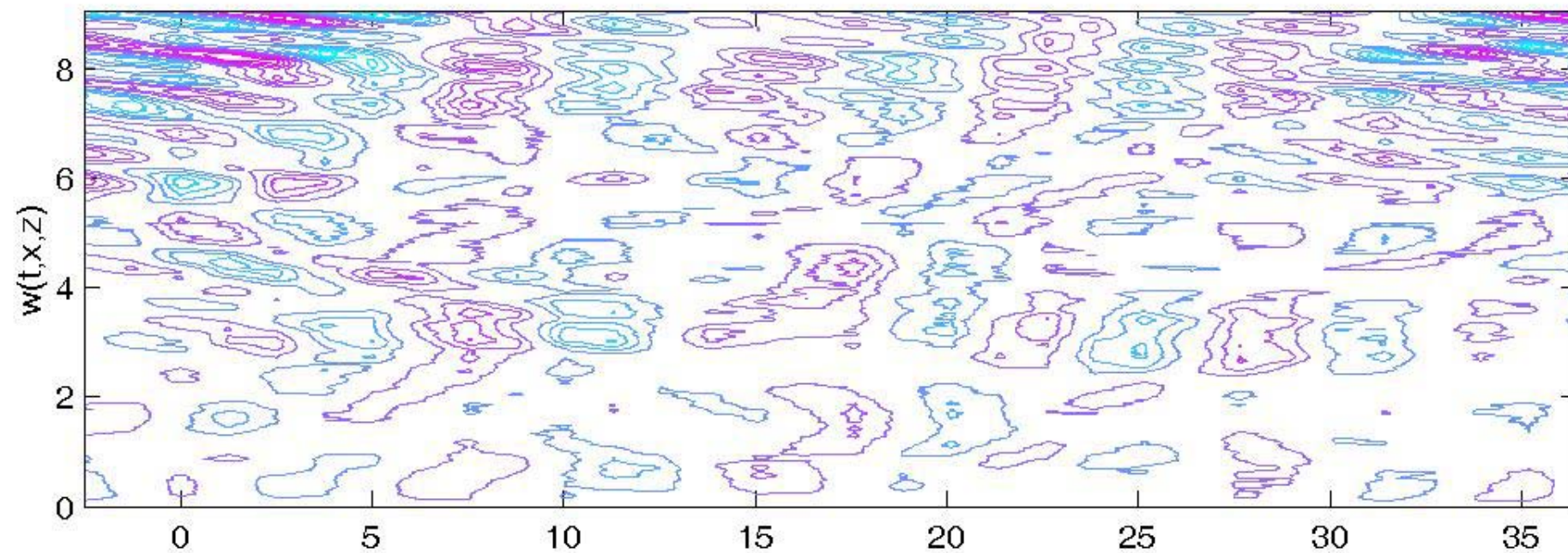
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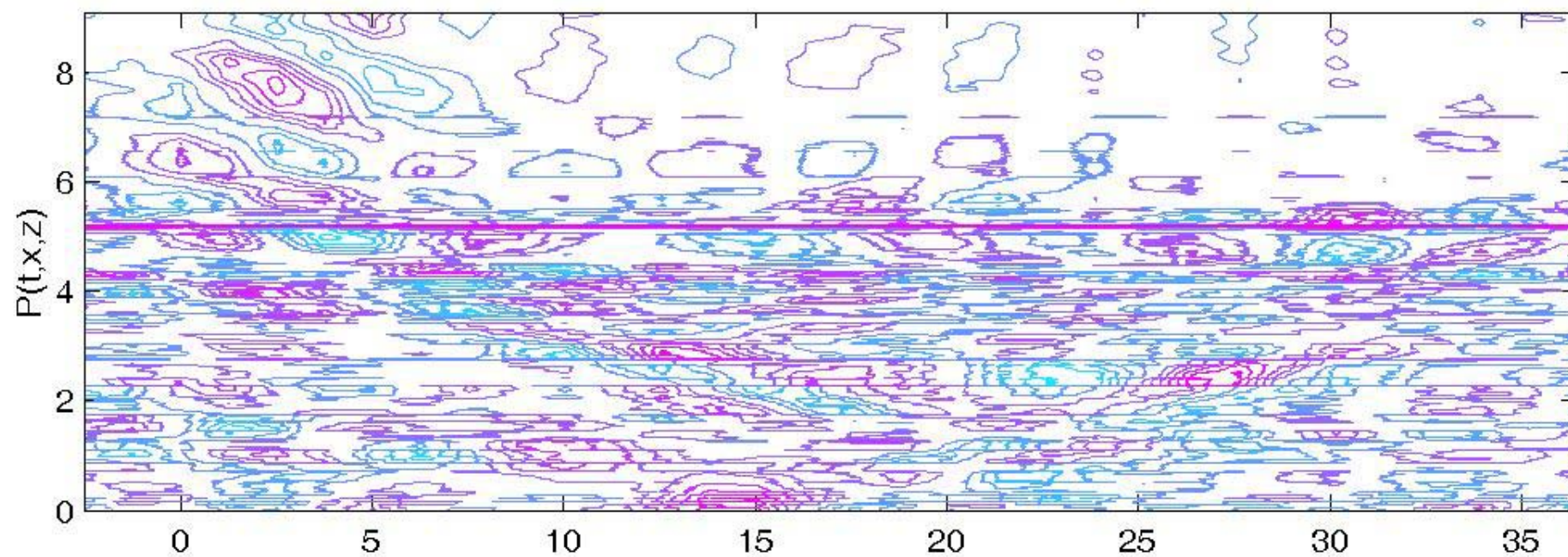
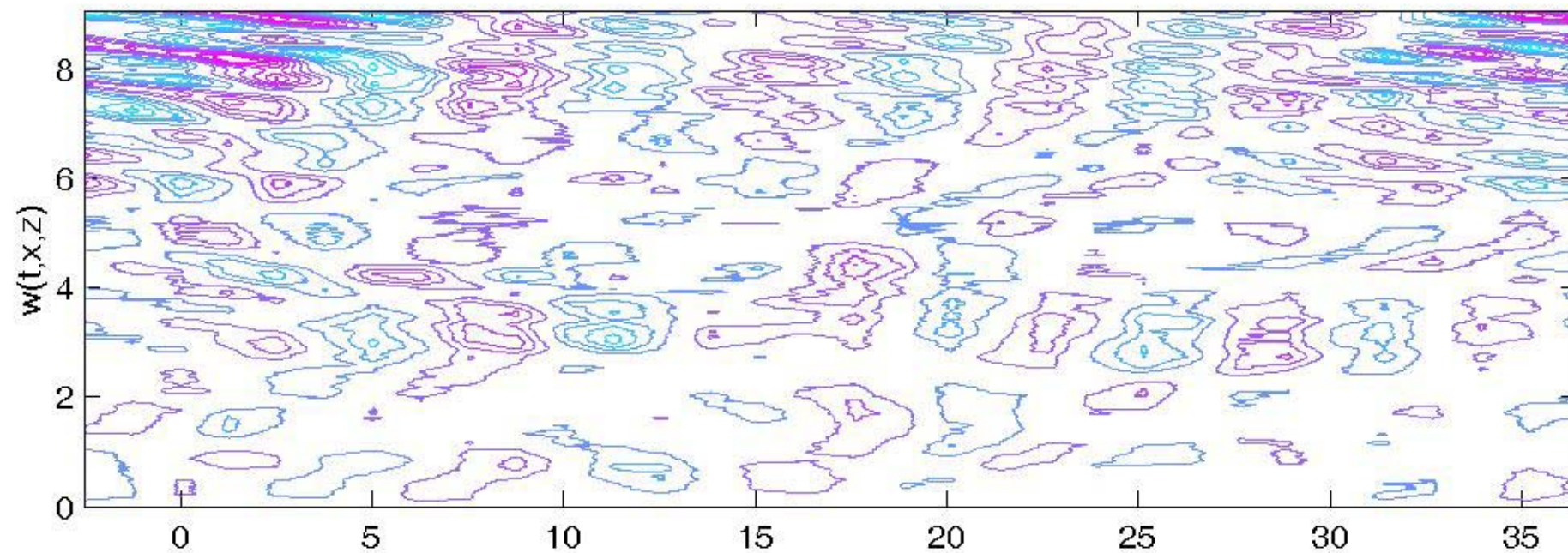
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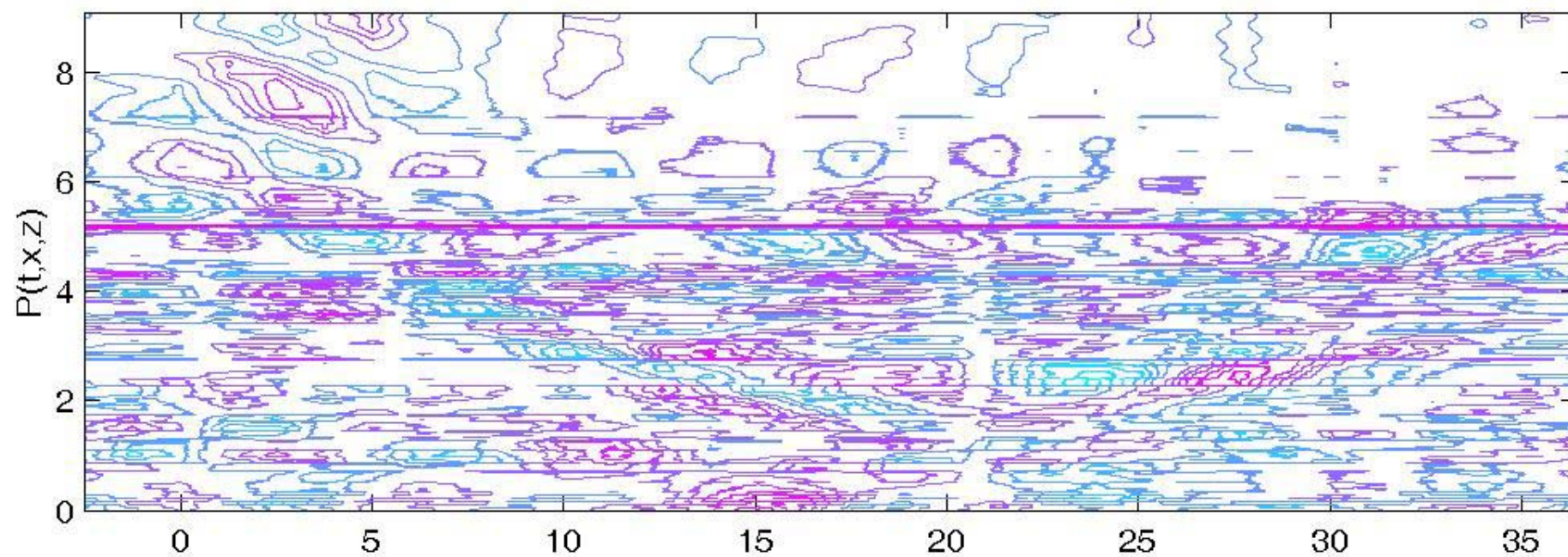
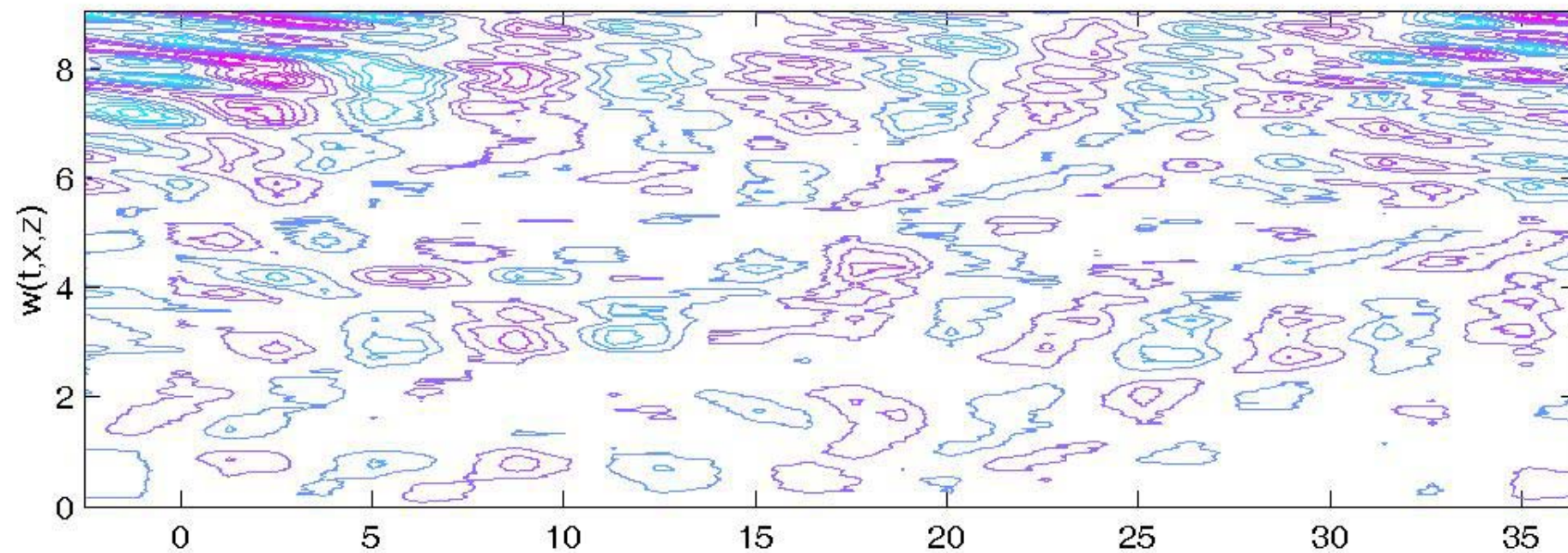
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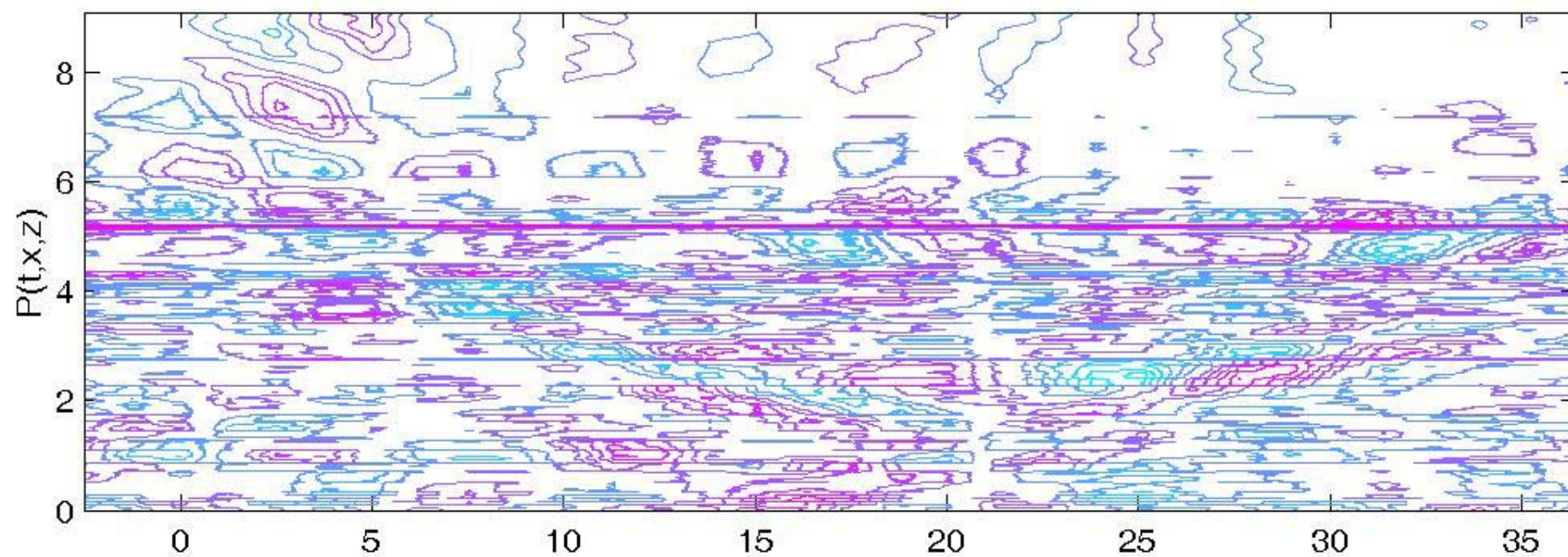
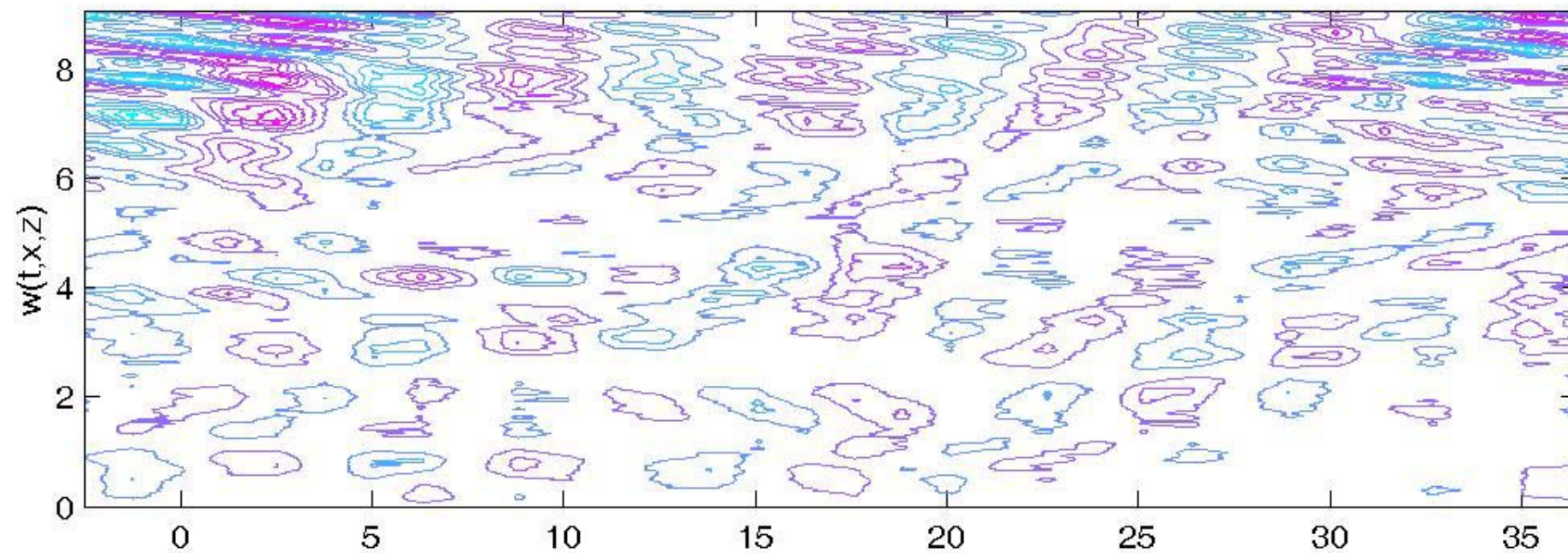
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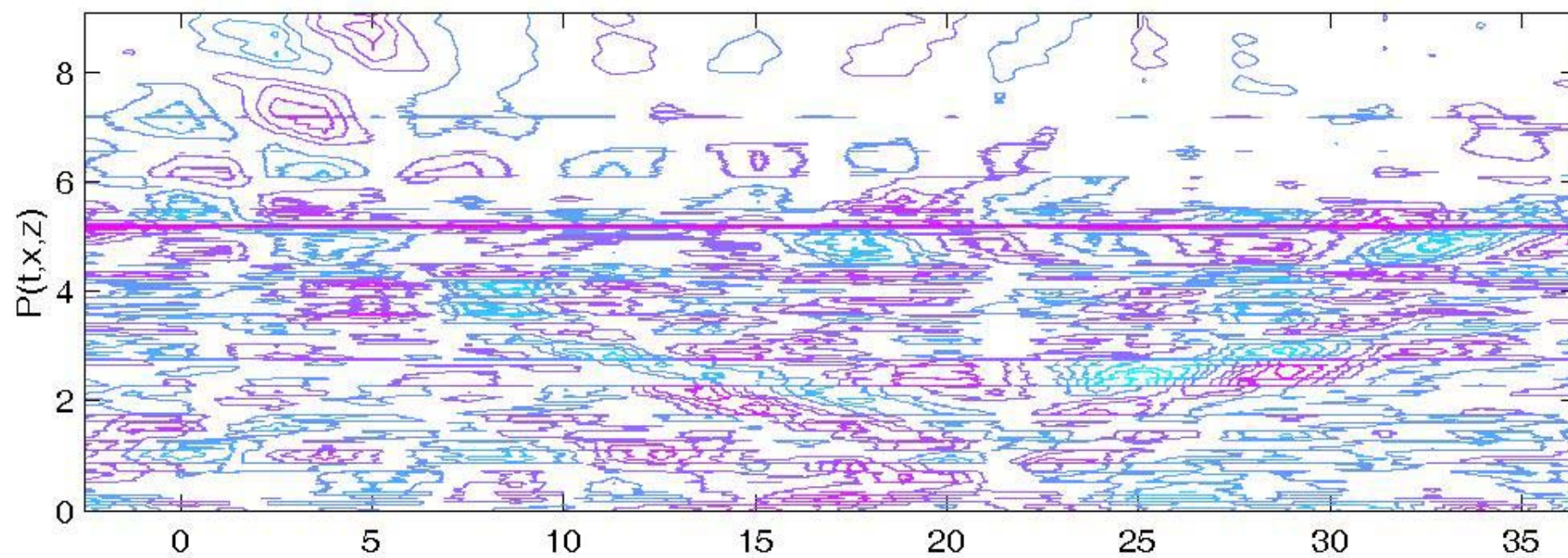
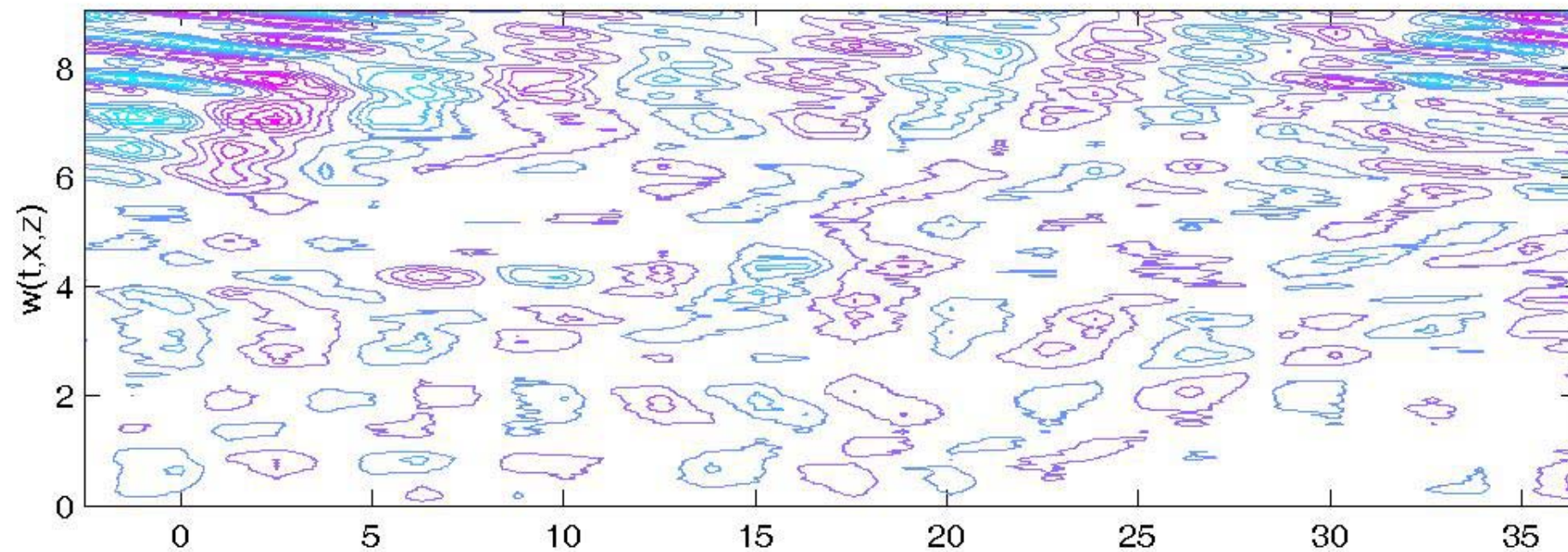
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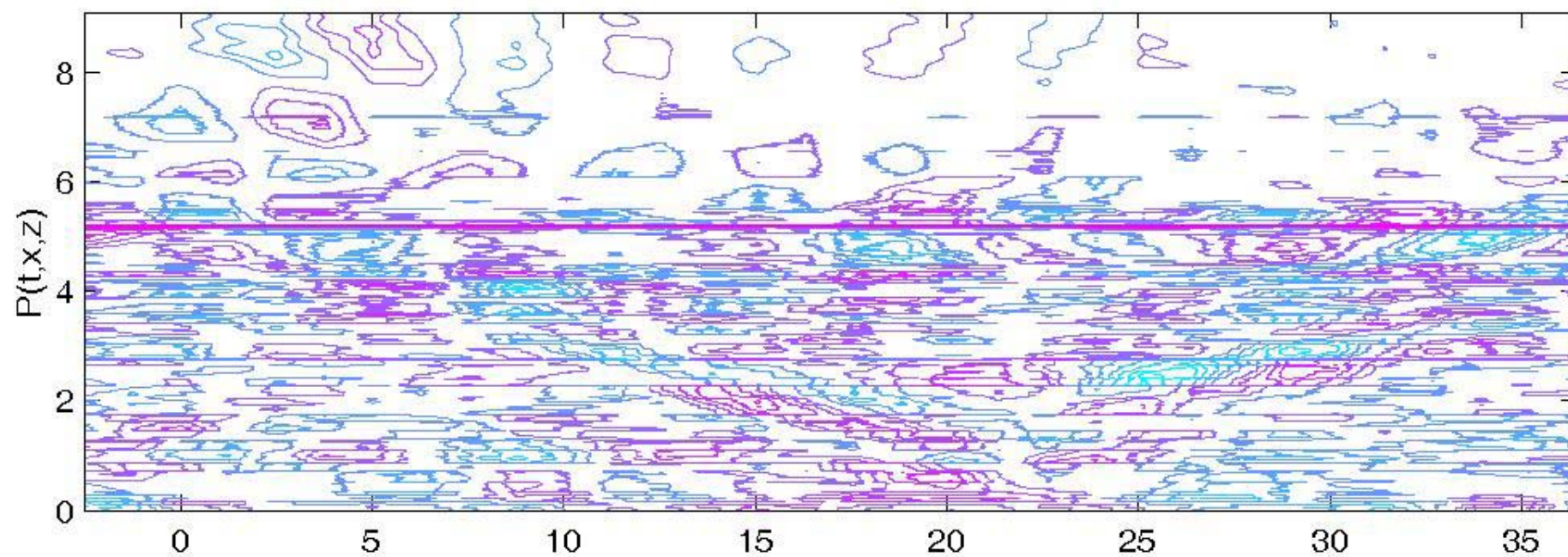
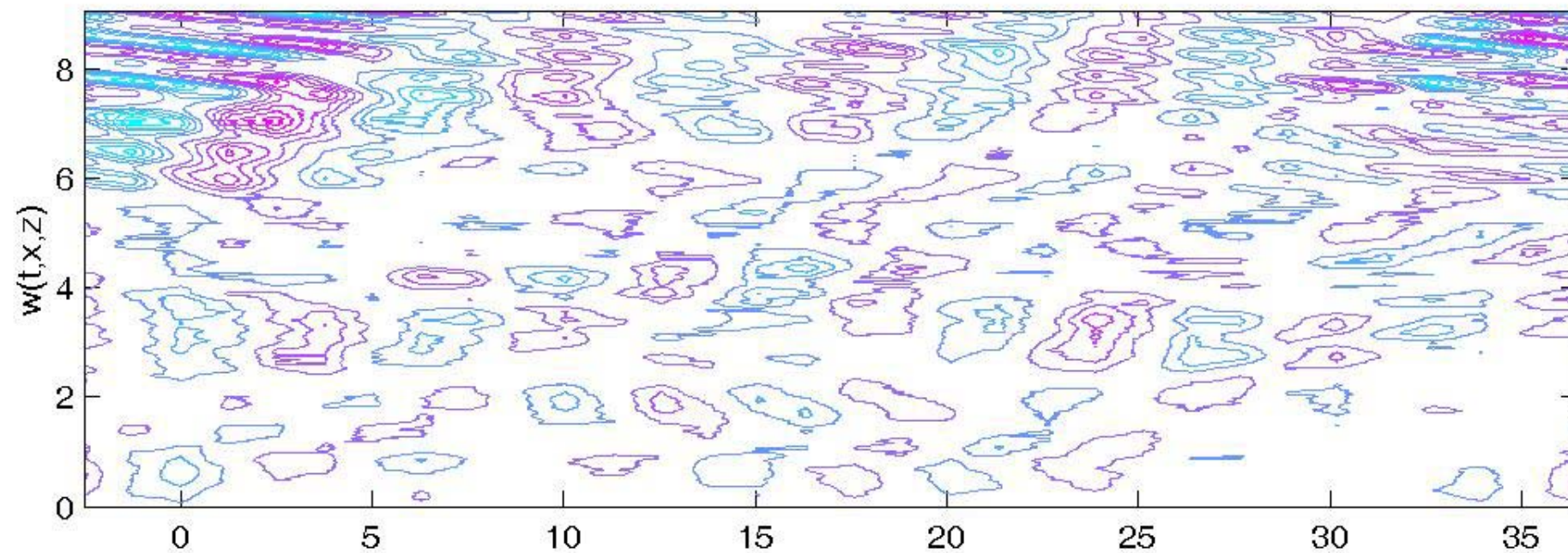
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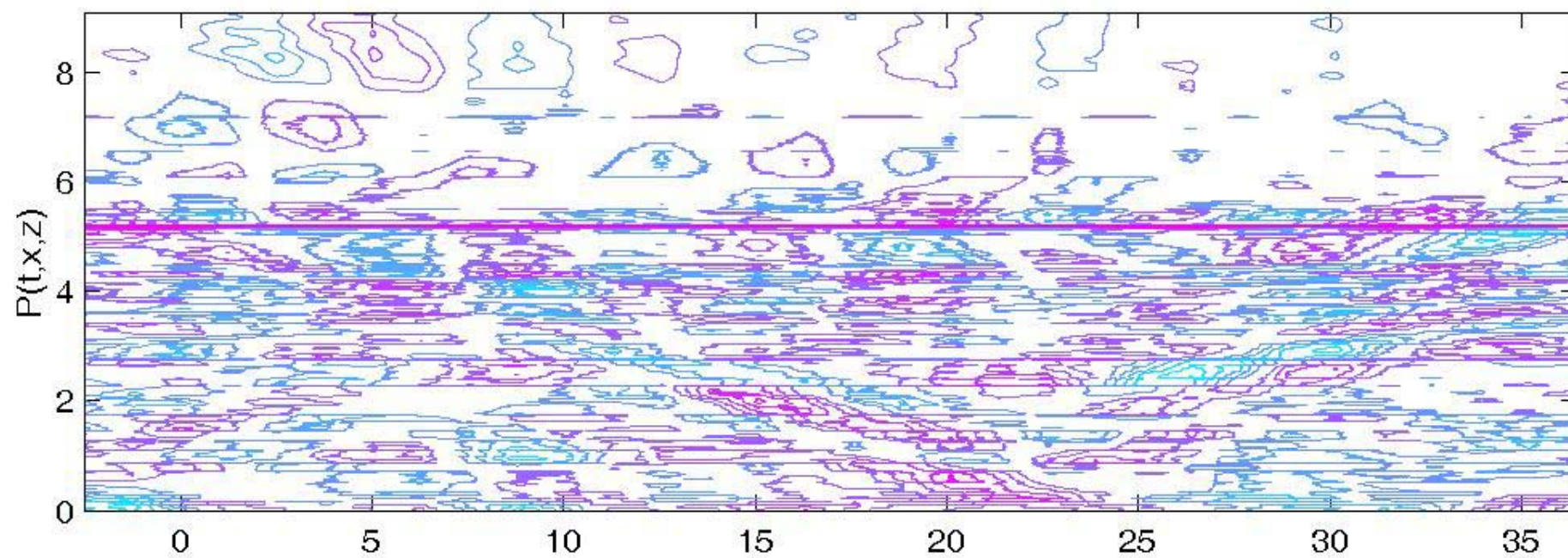
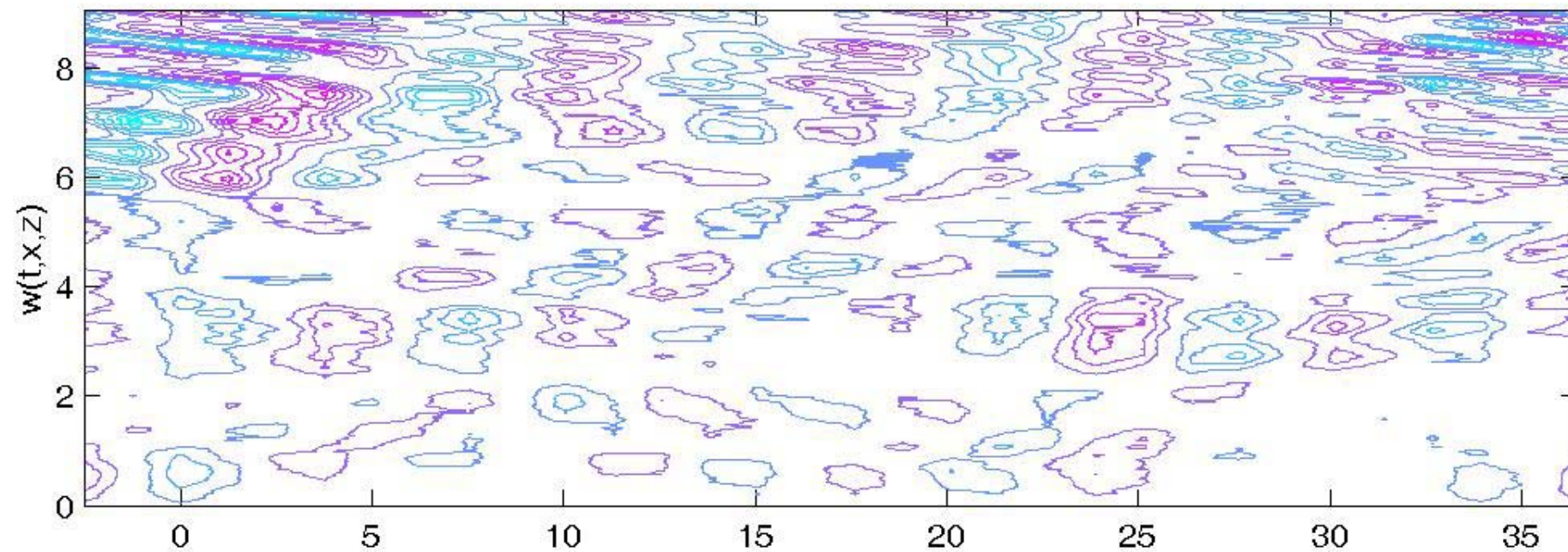
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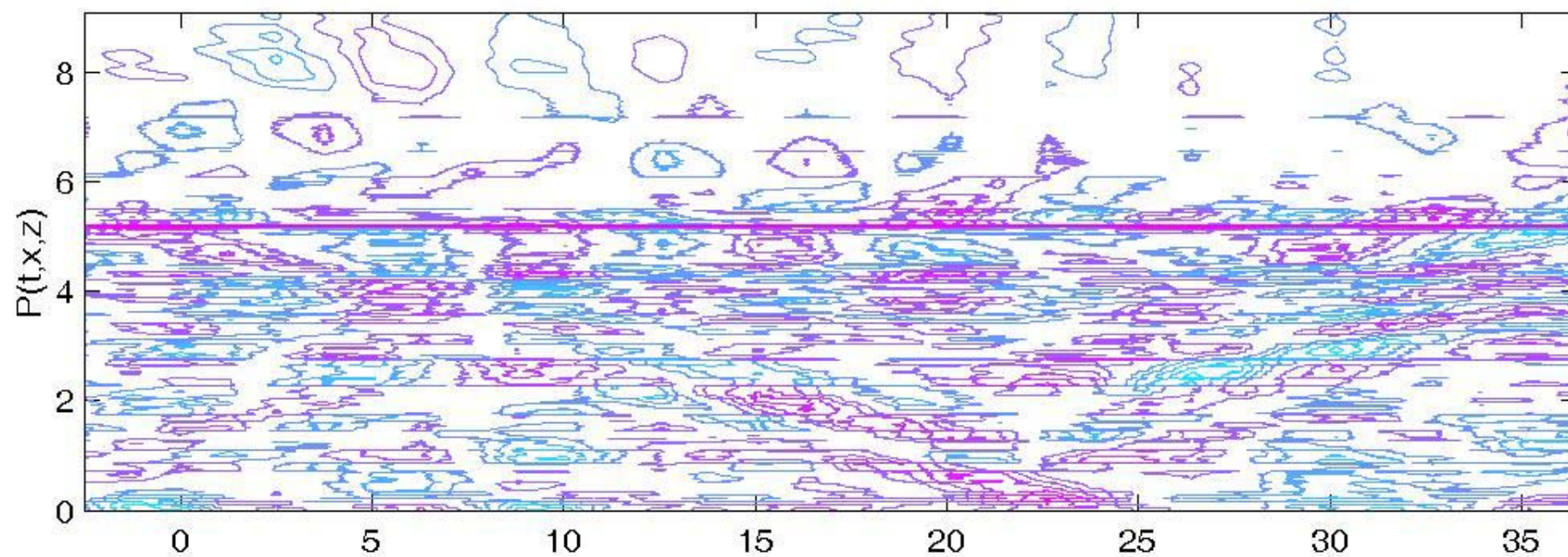
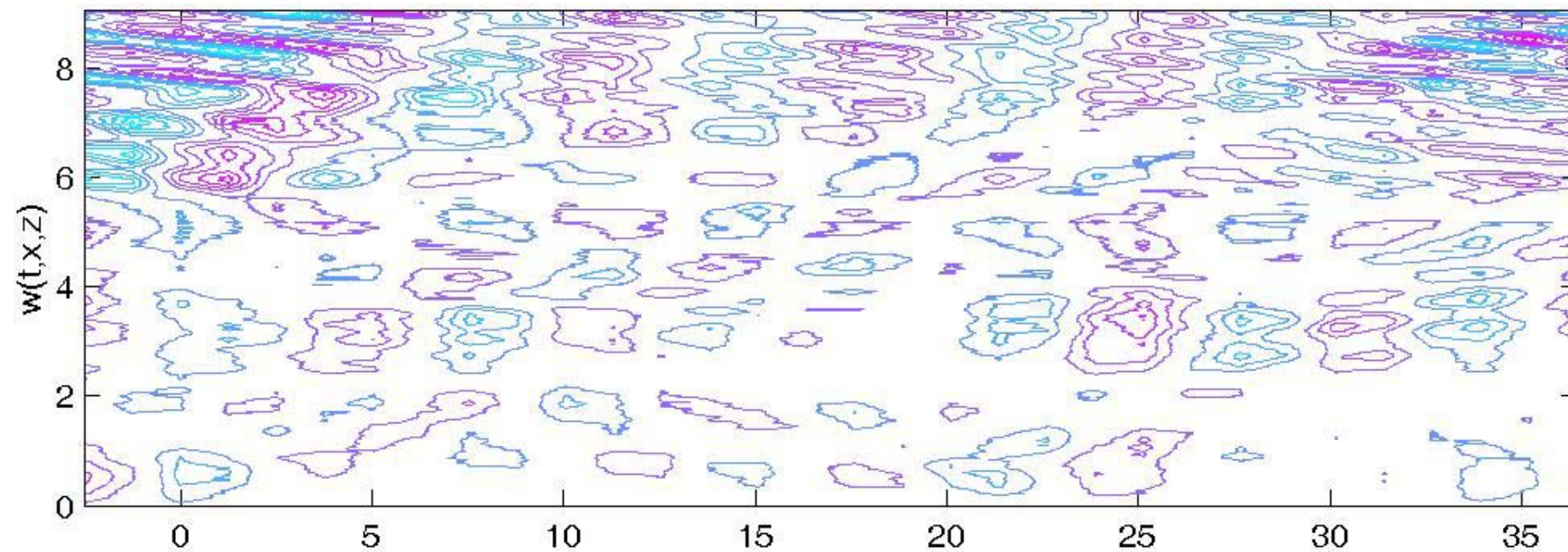
T[z] 10.08.2012, 15.00 UT, lat=65°, long=-45°, x1=6, t1=20



T[z] 10.08.2012, 15.00 UT, lat=65°, long=-45°, x1=6, t1=20



T[z] 10.08.2012, 15.00 UT, lat=65°, long=-45°, x1=6, t1=20



6. Ionospheric response to the atmospheric wave singularity

At the altitudes of the D and E layers, the frequency of the acoustic gravity wave is much lower than the frequencies ν_{in} and ν_{en} of the ion and electron collisions with neutrals ($\nu_{in} \approx 10^3 \text{ s}^{-1}$ and $\nu_{en} \approx 10^4 \text{ s}^{-1}$ at an altitude close to 110 km). Then, neglecting the collisions between charged particles, for the velocity of drag of the electrons and ions by neutrals in the absence of external electric fields and sharp density gradients one can write (Gershman, 1974)

$$\vec{u}_{e,i} = \frac{\nu_{en,in}^2}{\nu_{en,in}^2 + \omega_{He,i}^2} \left\{ \vec{u}_n \mp \frac{\omega_{He,i}}{\nu_{en,in}} \left[\vec{u}_n \vec{h}_0 \right] + \frac{\omega_{He,i}^2}{\nu_{en,in}^2} \vec{h}_0 \left(\vec{h}_0 \vec{u}_n \right) \right\},$$

where $\vec{u}_{e,i,n}$ are the velocities of electrons, ions, and neutrals, \vec{h}_0 is a unit vector along the geomagnetic field direction, and $\omega_{He,i}$ are the electron and ion gyrofrequencies, respectively. The minus (plus) sign is chosen for the electron (ion) velocity.

The disturbance of the densities $N \approx N_i \approx N_e$ of the ionospheric plasma, which we assume to be quasi-neutral, can be estimated from the continuity equation

$$\frac{\partial N}{\partial t} + \operatorname{div} (N \vec{u}_{i,e}) = 0.$$

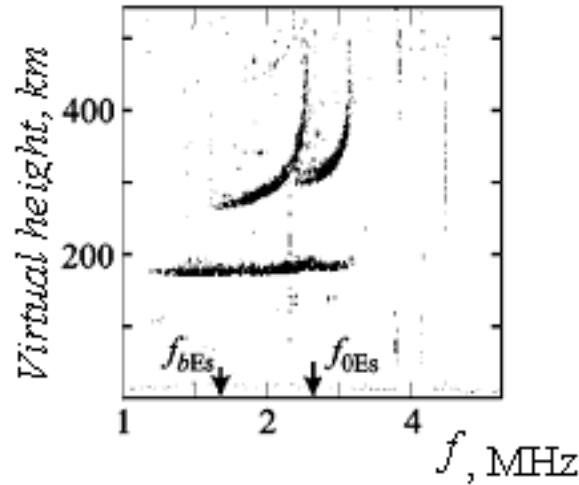
At the altitudes of the D layer and in the lower part of the E layer, where the conditions $\nu_{en,in} > \omega_{He,i}$ are fulfilled, it can be assumed that the ionospheric plasma is completely dragged by neutrals and $\vec{u}_i \approx \vec{u}_e \approx \vec{u}_n$. The time τ of onset of forced distributions of electrons and ions is less than the period of the considered atmospheric gas oscillations. This means that the disturbances of the ionospheric plasma density in our case have temporal and horizontal scales corresponding to the disturbance of neutral gas. Considering that in the linear approximation the disturbances of the electrons and ions densities are small and assuming that the velocities of charged particles are close to v_z , equation leads to the following formula:

$$N = \frac{k_{\perp} V}{\omega} \sqrt{\frac{\rho_E}{\rho}} N_0 = \frac{P}{c_s^2} \frac{N_0}{\sqrt{\rho \rho_E}},$$

where N_0 is the basic state of the ionospheric plasma density.

The disturbance of ionospheric plasma density is proportional to the pressure disturbance of the neutral gas, it should be expected that thin (several ten meters) and extended (of the order of the acoustic gravity wave horizontal scale determined by the horizontal scales of the source on the Earth's surface) ionospheric irregularities with a periodically varied plasma density will be generated in the ionospheric D layer with a weak gradient of the background density in the resonance level.

The impact of this resonance effect on the plasma at the altitudes of the ionospheric E layer, where $\nu_{in} \sim \omega_{Hi}$, $\nu_{en} \ll \omega_{He}$ (electrons are strongly magnetized) and there are conditions for the sporadic E layers generation, is more difficult for analysis. This is due to the fact that the electron density gradient should be taken into account and the Whitehead force, which leads to a still greater reduction of the domain thickness, should play an important role. In this case the formation of finite mass on the resonance level (Fig. 2) must lead to the formation of narrow (in vertical directive) horizontal ionospheric irregularities. Irregularities of such a type can be observed on the ionograms in the form of mildly sloping weakly diffuse sporadic layers with a large range of translucency (Fatkullin et al., 1985).



Ionospheric plasma at these altitudes and below behaves as a passive impurity, and therefore ionospheric irregularities with the same characteristic features as the neutral gas disturbances should be observed, including the case where the condition of the resonance feature generation is fulfilled. It is shown in paper (Erukhimov and Savina, 1980) that irregularities of such a structure are the main reason for the formation of weakly diffuse sporadic E layers with a large range of translucence. In the framework of the model in question, the time of existence, altitude, and space scales of such layers are fundamentally dependent on the parameters of the ground-based acoustic gravity waves sources and ionospheric conditions

Conclusion

- *Local acoustic-gravity disturbance in a nonisothermal atmosphere has been studied. According to the analytical results, the pressure wave amplitude has a local wave singularity near the layer at which the horizontal phase velocity is equal to the sound velocity and the vertical velocity in the disturbance becomes zero.*
- *In real conditions, many resonance layers (for different ω and k_{\perp}) can exist. The fact is that the atmospheric viscosity and nonlinearity limit the pressure wave singularity. The vertical dimensions of singularity domain will be of the order of the mean free path of the molecules at the resonance level. Estimates show that at altitudes of about one hundred kilometers the viscosity limits the scales to several hundreds of meters.*
- *Ionospheric irregularities with the same characteristic features as the neutral gas disturbances should be observed, including the case where the condition of the resonance feature generation is fulfilled.*