Network Bipartivity

$$Bipartivity \Leftrightarrow No\ odd\ \ cycles$$

$$No\ odd\ \ cycles \Leftrightarrow No\ odd\ \ closed\ \ walks$$

$$No\ odd\ \ closed\ \ walks \Leftrightarrow \left[\sinh\left(A\right)\right]_{ii} = 0$$

$$\frac{No.even\,closed\ walks - No.odd\ closed\ walks}{No.even\,closed\ walks + No.odd\ closed\ walks}$$

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Network Bipartivity

$$b_s = \frac{Tr(\cosh(A)) - Tr(\sinh(A))}{Tr(\cosh(A)) + Tr(\sinh(A))}$$

$$b_s = \frac{Tr(\exp(-A))}{Tr(\exp(A))}$$

$K_{n \to inf}$ ← $0 < b_s \leq 1$ → Bipartite

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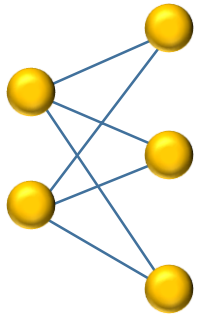
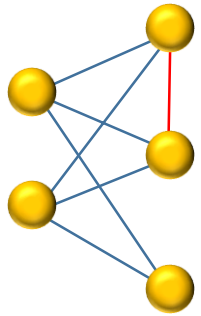
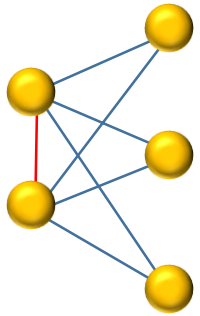
Network Bipartivity



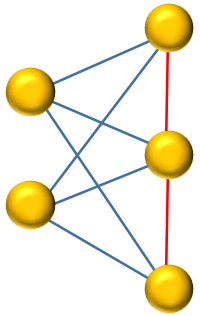
$b_s = 1.000$



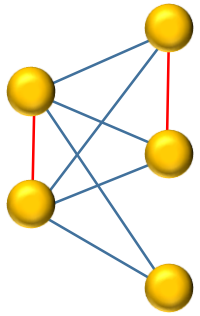
$b_s = 0.658$



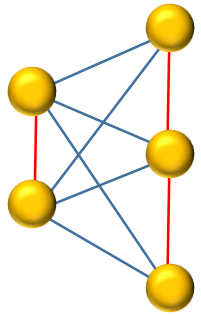
$b_s = 0.538$



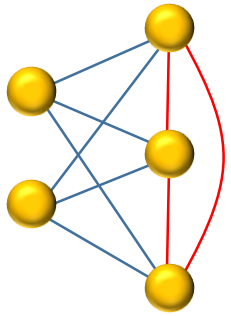
$b_s = 0.462$



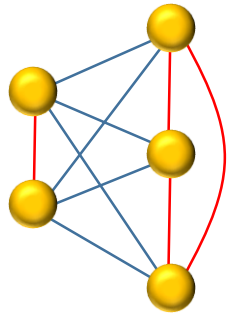
$b_s = 0.383$



$b_s = 0.289$



$b_s = 0.289$



$b_s = 0.194$

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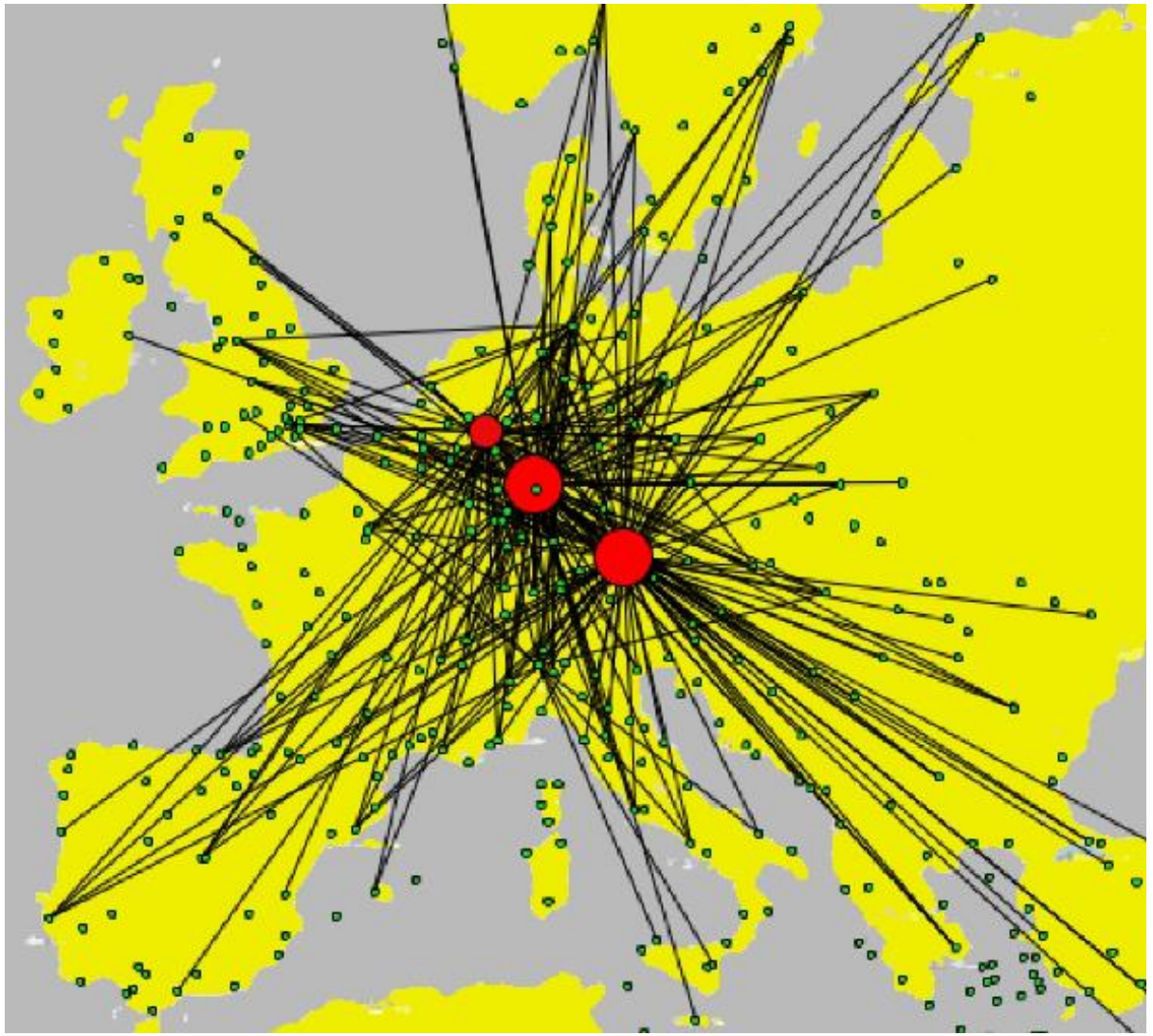
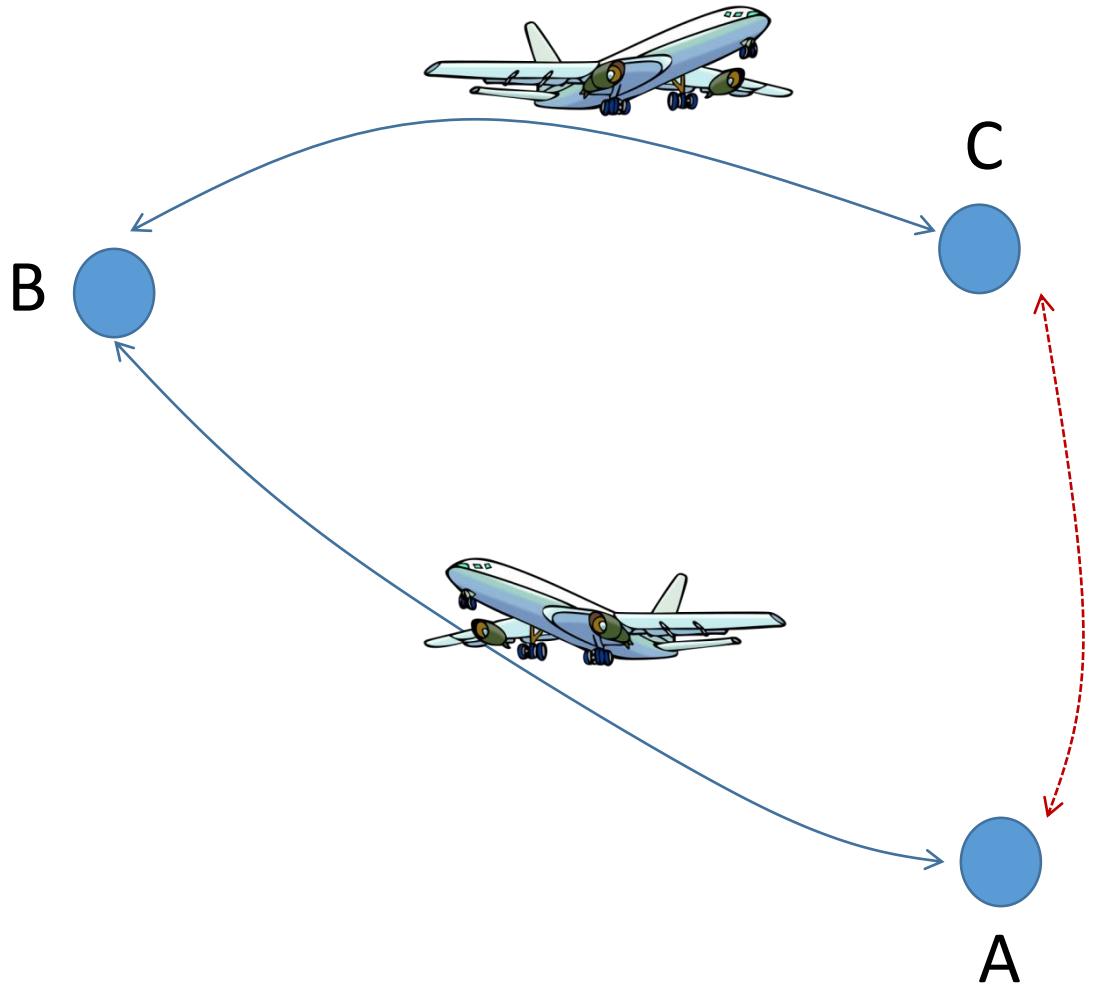
Application. Airline Networks



Inconvenience of bipartivity



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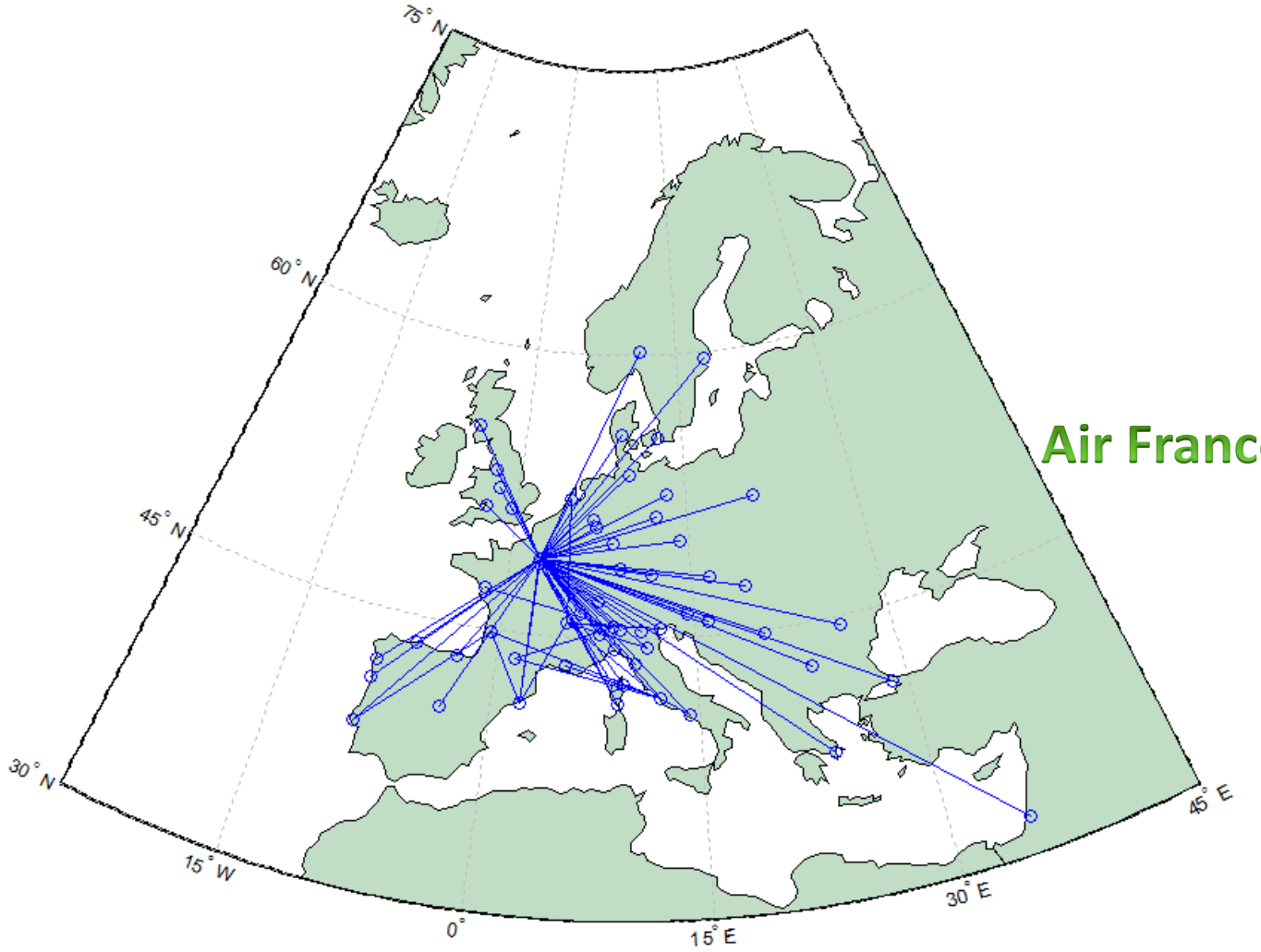
Bipartivity & Airline Networks

Traditional Airlines



Air France

$$K\left(G, \beta = 1\right) = 0.8773$$

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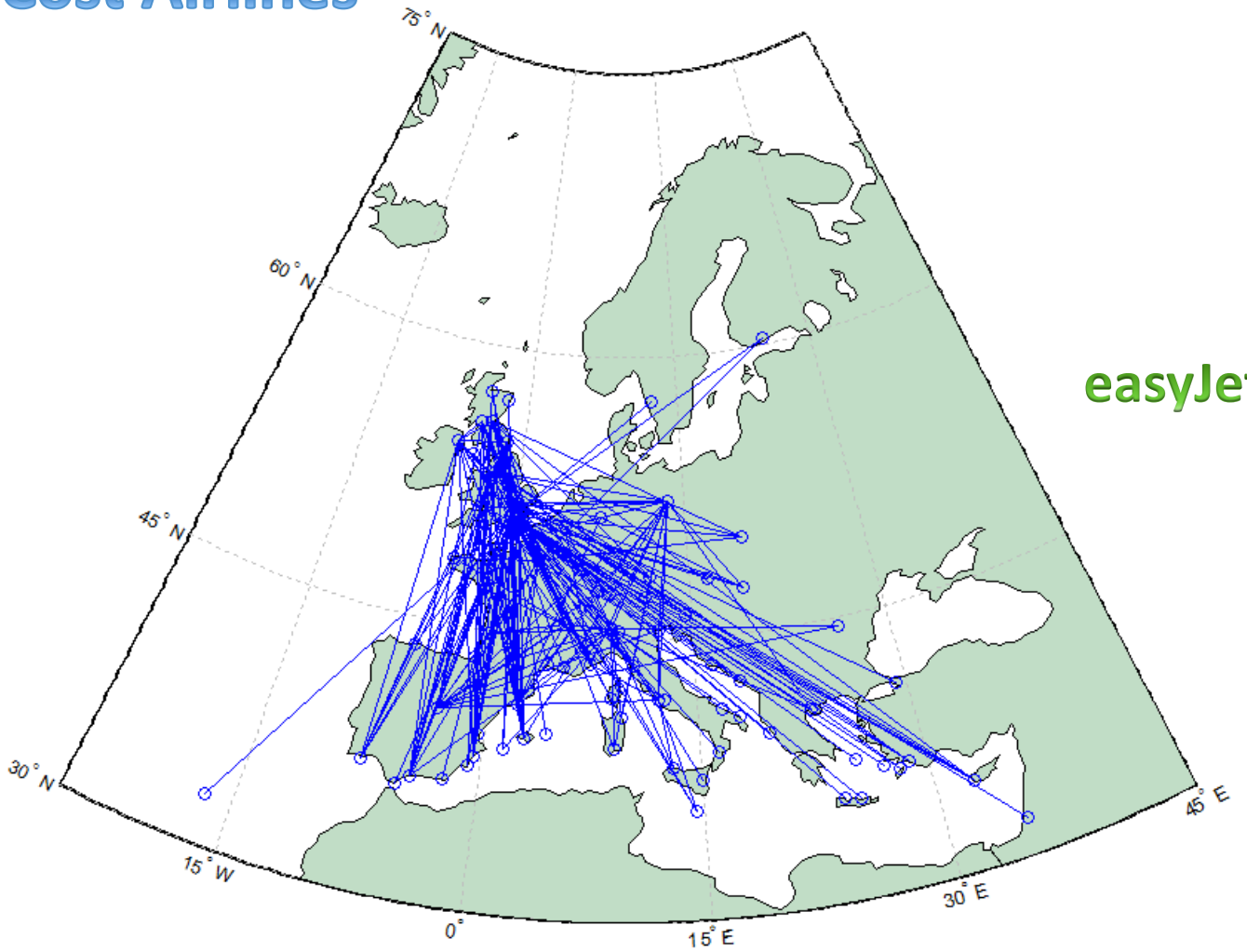
Bipartivity & Airline Networks

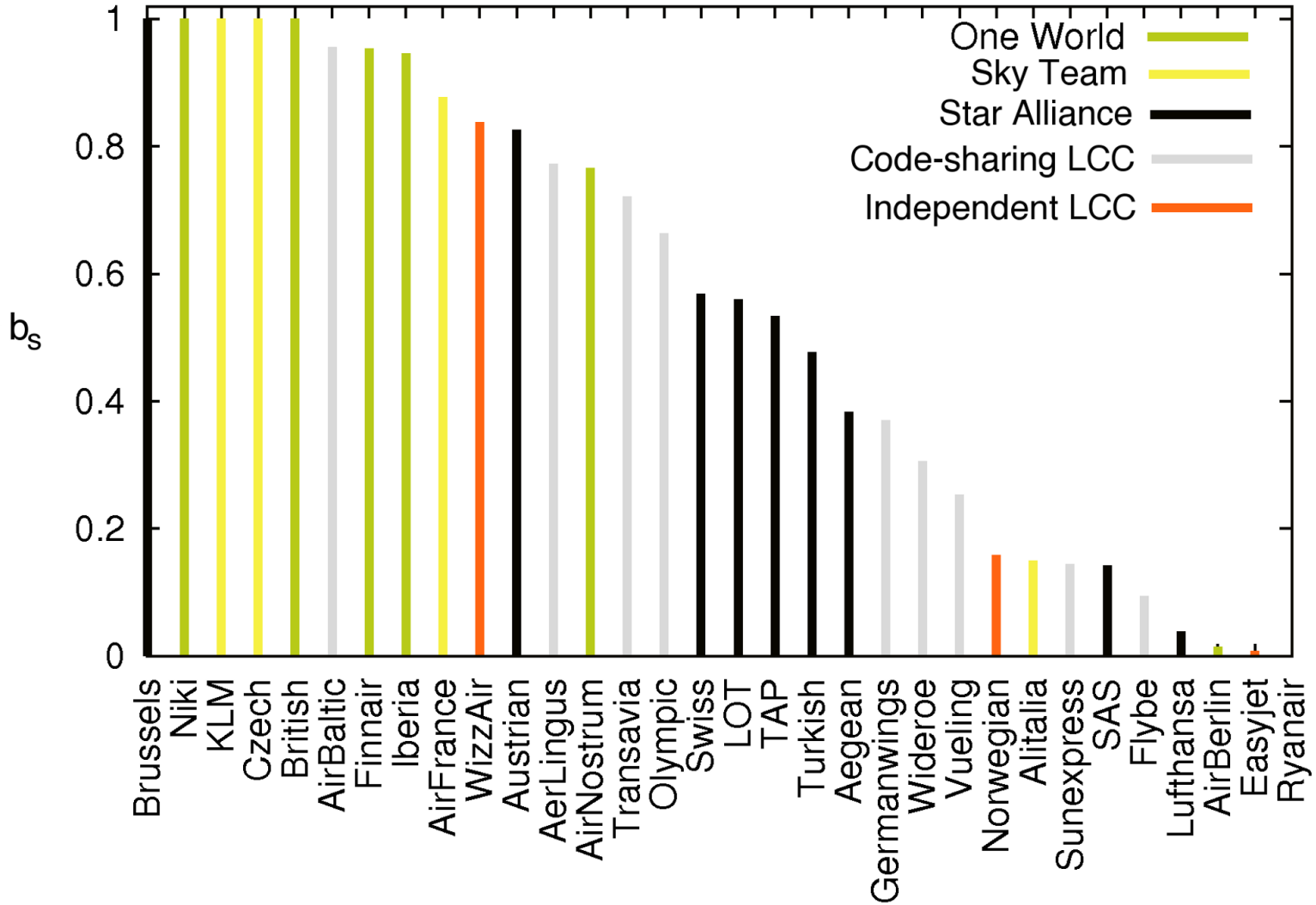
Low-Cost Airlines



easyJets

$$K\left(G, \beta = 1\right) = 0.0083$$

Bipartivity & Airline Networks



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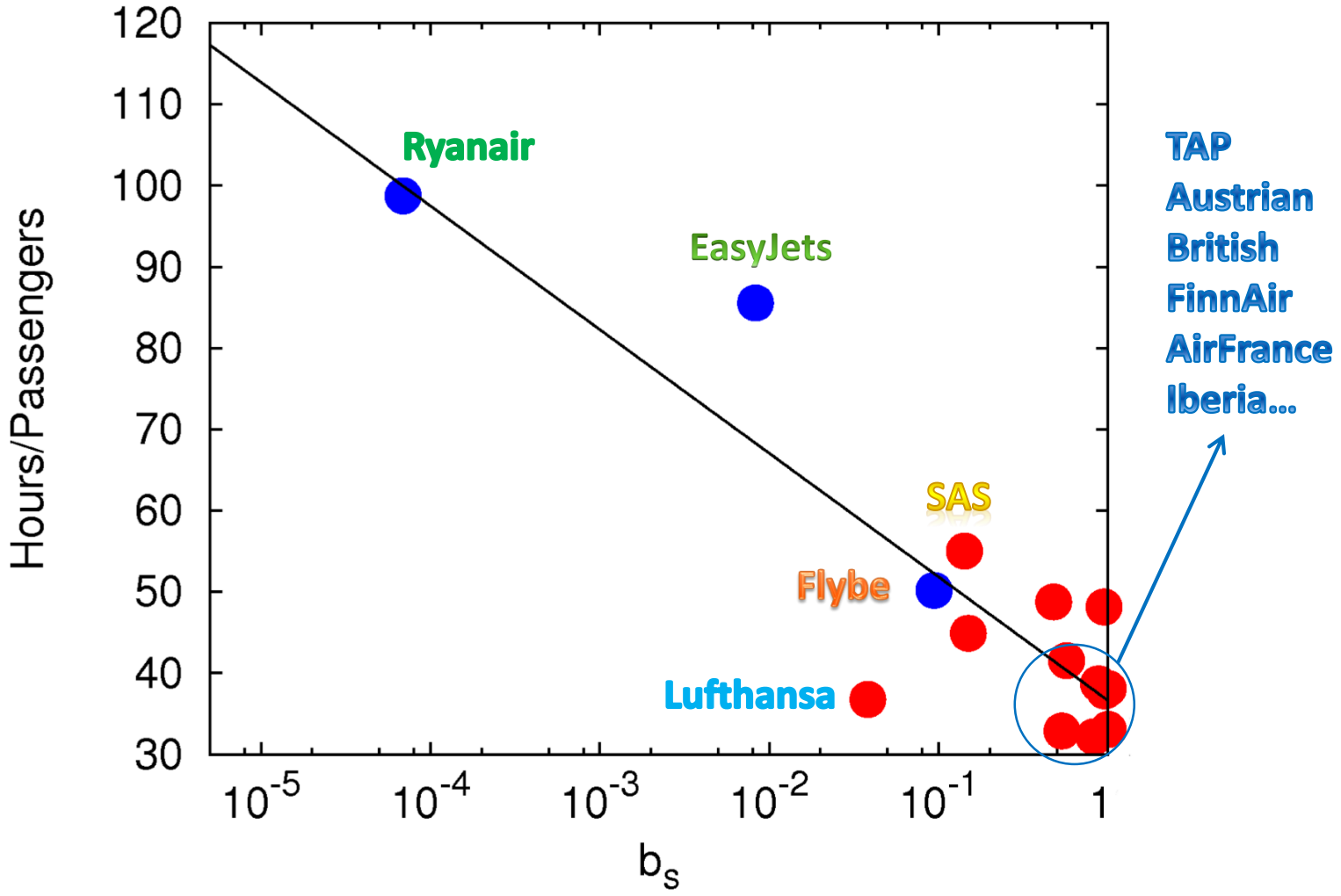
Bipartivity & Airline Networks

$$Eff = -6.61 \ln K + 36.59$$



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Improving Efficiency

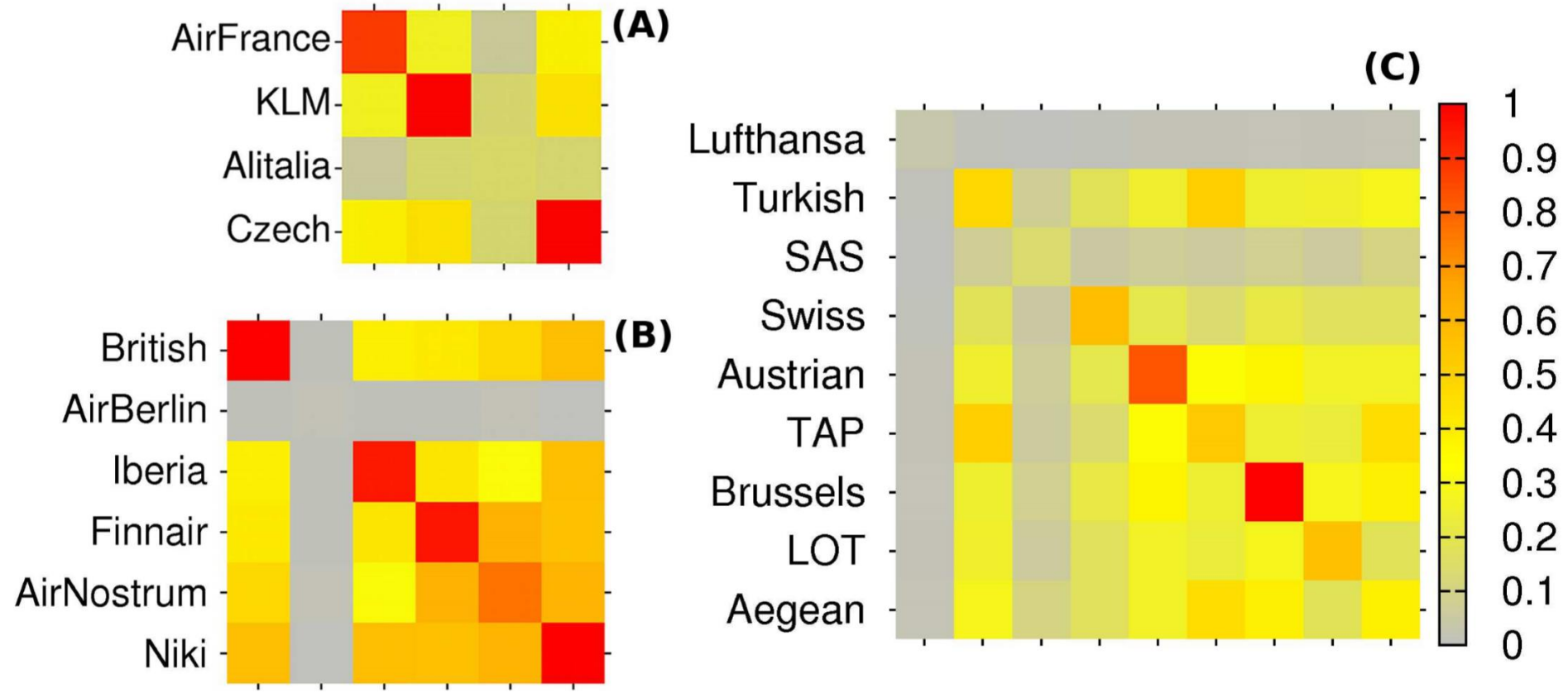

Figure 3: Bipartibity of all the possible two-airline combinations within each of the alliances of traditional carriers. (A) Sky Team, (B) One World and (c) Star Alliance.

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Improving Efficiency

1) Merging companies

Air France

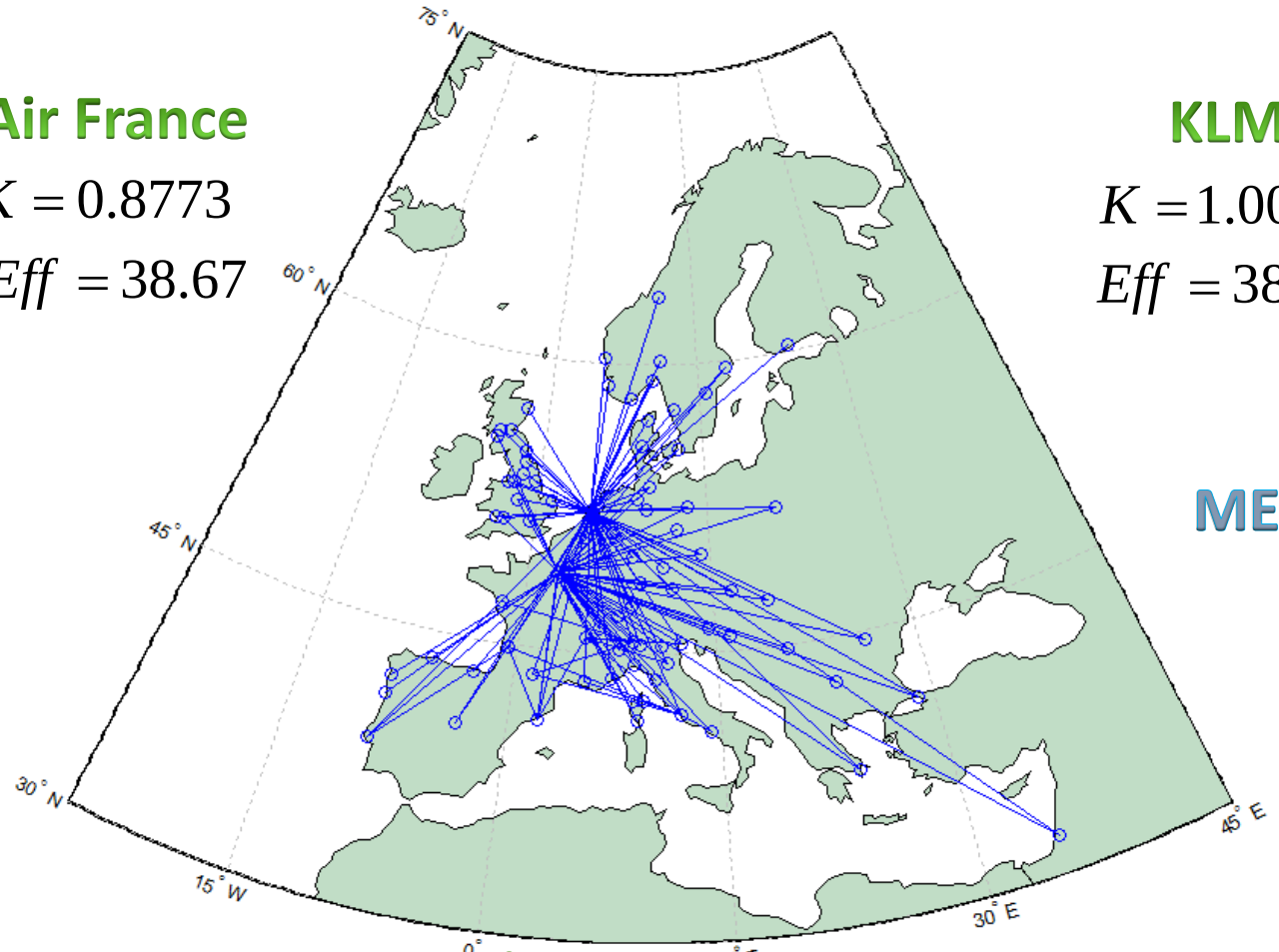
$K = 0.8773$

$Eff = 38.67$

KLM

$K = 1.000$

$Eff = 38.07$



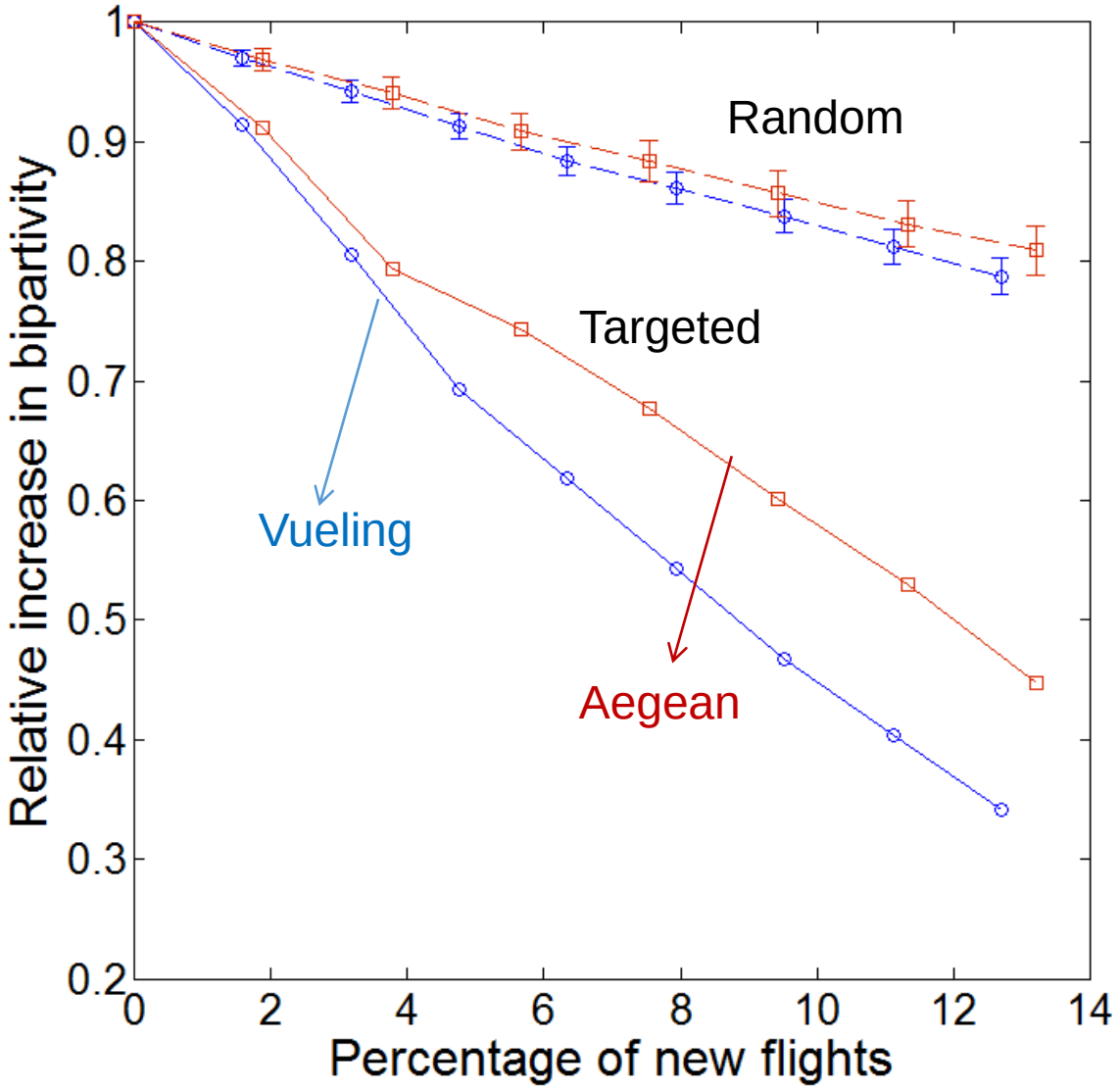
MERGED in 2003

Air France + KLM

$K = 0.1140$ $\langle Eff \rangle = 50.85$

Improving Efficiency

2) Creating new flights



Improving Efficiency

3) Increasing the frequency of flights)