Study of Coordinated Scheduling and Transportation based on Serial-batching

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Outline

1. Basic batch scheduling
2. Supply Chain Scheduling with Non-identical Job Sizes and Release Times
3. Supply Chain Scheduling with Multiple Manufacturers
4. Supply Chain Scheduling with Arbitrary Machine Breakdown

1 Basic batch scheduling

Background

Extrusion Factory:

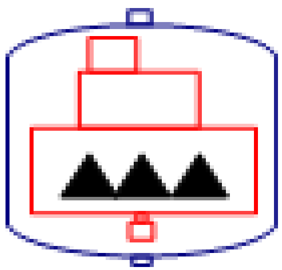
Aging Factory :

1 Basic batch scheduling

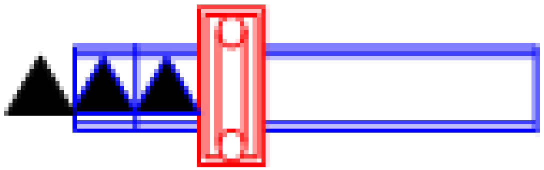
Background

Parallel batch: The jobs are processed simultaneously in their belonged batch

Serial batch: The jobs are processed one after another in their belonged batch



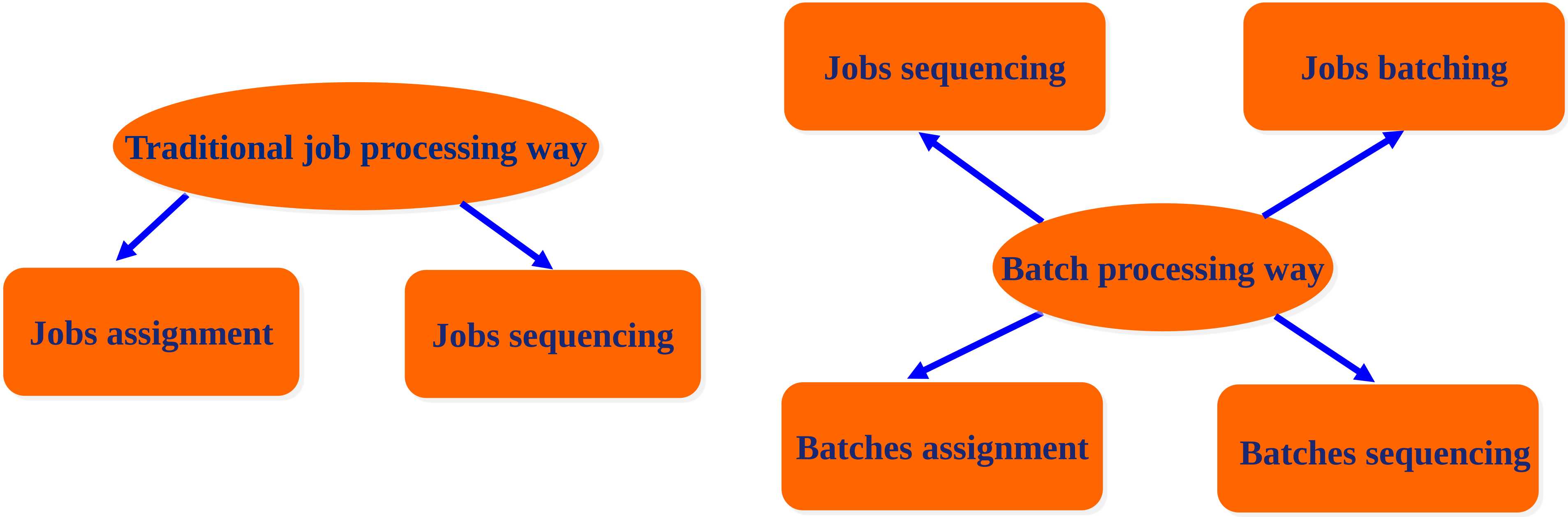
Parallel-batch processing machine



Serial-batch processing machine

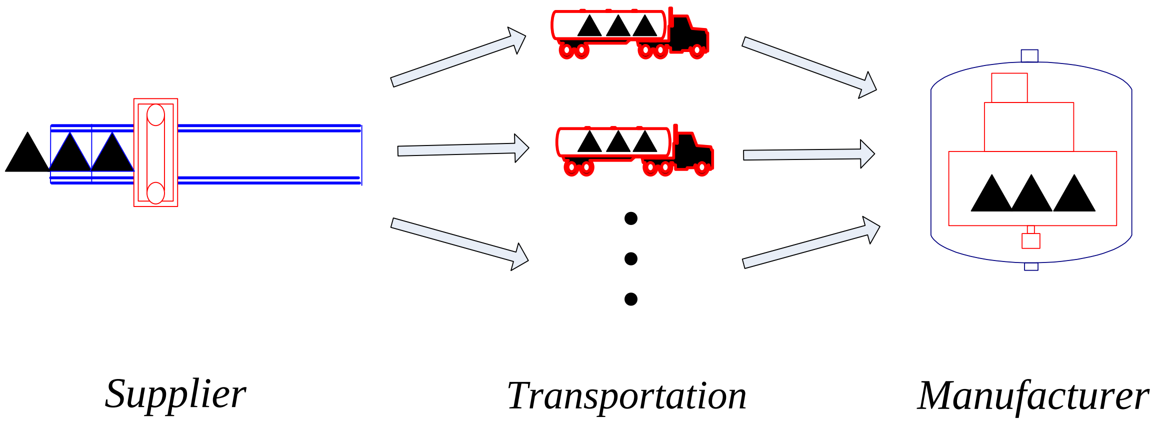
1 Basic batch scheduling

Difference between traditional job processing way and batch processing way



2 Supply Chain Scheduling with Non-identical Job Sizes and Release Times

$$S \rightarrow M \left| r_i, \sum_{J_i \epsilon b_k} s_i \leq c, s-batch, p-batch, T \right| C_{max}$$



(1) The jobs arriving dynamically at the supplier are to be processed on a serial batching machine in the form of serial bathes.

(2) After completed in the supplier, a batch is transported from the supplier to the manufacturer immediately for further processing.

(3) After one batch arriving at the manufacturer, the jobs in the batch are processed on a parallel batching machine in the form of parallel batches.

2 Supply Chain Scheduling with Non-identical Job Sizes and Release Times

Mixed integer programming model (1)

Minimize $C_{\max}$ (1)

Subject to

$$\sum_{k=1}^{L} x_{ik} = 1, \quad i=1,2,\ldots,n \tag{2}$$

$$\sum_{i=1}^{n} s_i \cdot x_{ik} \le c, \quad k=1,2,\ldots,L \tag{3}$$

$$S_{1k} \ge r_i \cdot x_{ik}, \quad i=1,2,\ldots,n, \quad k=1,2,\ldots,L \tag{4}$$

$$C_{1k} = S_{1k} + s + \sum_{i=1}^{n} x_{ik} \cdot p_i, \quad k=1,2,\ldots,L \tag{5}$$

$$S_{2k} \ge C_{1k} + T, \quad k=1,2,\ldots,L \tag{6}$$

$$C_{2k} = S_{2k} + P, \quad k=1,2,\ldots,L \tag{7}$$

$$C_{1k} - C_{1f} + P^f + s - (1 - y_{kf})M \le 0, \quad k=1,2,\ldots,L, \quad f=1,2,\ldots,L, \quad k \ne f \tag{8}$$

$$z_{kf} + z_{fk} = 1, \quad k=1,2,\ldots,L, \quad f=1,2,\ldots,L, \quad k \ne f \tag{9}$$

$$C_{\max} \ge C_{2k}, \quad k=1,2,\ldots,L \tag{10}$$

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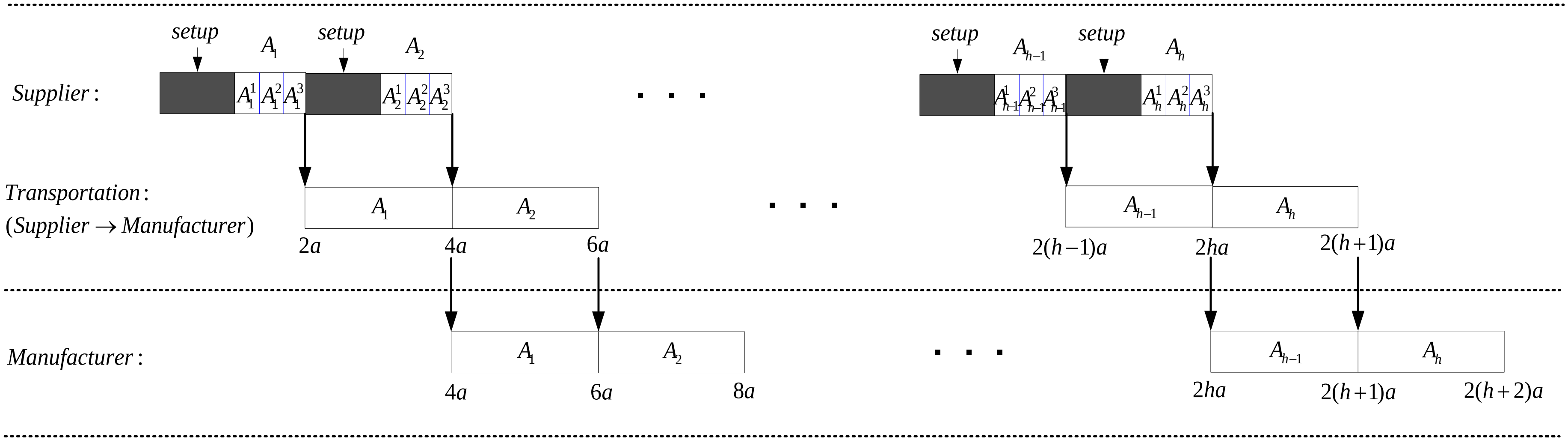
2 Supply Chain Scheduling with Non-identical Job Sizes and Release Times

Theorem 2.1.1 ***The problem*** $S \to M\left|r_i, \sum_{J_i \epsilon b_k} s_i \le c, s - batch, p - batch, T\right|C_{max}$ ***is strongly NP-hard.***

Proof: 3-PARTITION problem.



2 Supply Chain Scheduling with Non-identical Job Sizes and Release Times

General problem

Lemma 2.1.1. *The solution of any schedule remains unchanged when any two jobs are swapped in a batch.*

Corollary 2.1.1. *If the jobs within a batch are processed on the supplier's machine in non-decreasing order of release time, then the updated schedule still remains optimal.*

Jobs sequencing argument

2 Supply Chain Scheduling with Non-identical Job Sizes and Release Times

General problem

Lemma 2.1.2. *There exists an optimal schedule such that all batches are processed on the supplier's machine in non-decreasing order of ready time, which is equal to the largest release time of jobs contained in the batch.*

Batches sequencing argument

2 Supply Chain Scheduling with Non-identical Job Sizes and Release Times

General problem

Lemma 2.1.3. *Given a schedule* G , $G = \{\cdots, b_k, b_{k+1}, \cdots\}$ *,if* $b_{k+1} = \{J_j\}(j = 1,2,\cdots,n)$, $S_{1k} < r_j < S_{1k} + s$*, and* $\sum_{J_i \in b_k} s_i + s_j \leq c$ *, then the solution of the schedule can be improved after* J_j *is placed into* b_k .

Lemma 2.1.4. *Suppose any two neighbor batches* b_k *and* b_f *are processed on the supplier's machine in an optimal schedule* π^*. *If* $S_{1k} > S_{1f}$*, then* $S_{2k} > S_{2f}$.

Jobs batching argument

2 Supply Chain Scheduling with Non-identical Job Sizes and Release Times

Identical-release-time case

Lemma 2.1.5. *For the identical-release-time case, if there exist L^* batches in an optimal schedule π^*, then* $C_{max}(\pi^*) = r + L^* \cdot s + \sum_{i=1}^{n} p_i + T + P$.

Corollary 2.1.3. *For the identical-release-time case, if there exist L batches in a feasible schedule π, then* $C_{max}(\pi) \geq r + L \cdot s + \sum_{i=1}^{n} p_i + T + P$.

Corollary 2.1.4. *For the identical-release-time case,* $C_{max} \geq r + \left\lceil \sum_{i=1}^{n} \frac{s_i}{c} \right\rceil \cdot s + \sum_{i=1}^{n} p_i + T + P$

Relationship between the number of batches and the makespan

2 Supply Chain Scheduling with Non-identical Job Sizes and Release Times

Identical-release-time case

Lemma 2.1.6. *For the identical-release-time case, there exists a feasible schedule π with no additional idle time during the production and transportation.* ***The less the number of batches is, the better the solution is.***

Relationship between the number of batches and the makespan

2 Supply Chain Scheduling with Non-identical Job Sizes and Release Times

Identical-release-time case

Corollary 2.1.5. *For the identical-release-time case, the optimal schedule should have the minimum number of batches among all feasible schedules.*

Lemma 2.1.7. *For the identical-release-time case, considering any two batches in an optimal schedule, the solution remains unchanged after these two batches are swapped on the supplier's machine.*

Relationship between the number of batches and the makespan

2 Supply Chain Scheduling with Non-identical Job Sizes and Release Times

A two-phase heuristic algorithm

Phase 1:

Step 1: Set $k=1$, $B=\phi$, $S^1=0$, $S_{11}=r_1$, $P^1=0$, $t=r_1$, $U(t)=J$, $A(t)=\{J_i \mid r_i=t\}$, and $K(t)=U(t)\setminus A(t)$. Index jobs in $U(t)$, $A(t)$, and $K(t)$ as rule *ERT-DS-LPT*, respectively.

Step 2: If $t \geq r_{max}$, go to phase 2. Otherwise, go to step 3.

Step 3: Judge whether each job in $A(t)$ is placed into b_k as the job sequence until the last job. If $S^k+s_i \leq c$, then $b_k=b_k \cup \{J_i\}$, $S^k=S^k+s_i$, and $P^k=P^k+p_i$. Update $U(t)$, $A(t)$, and $K(t)$.

Step 4: If $S^k=c$, go to step 7. Otherwise, go to step 5.

Step 5: Judge whether each job in $K(t)$ is placed into b_k as the job sequence. If there exists a job satisfying that $S^k+s_i \leq c$ and $r_i \leq \alpha s+t$, where $0 \leq \alpha \leq 1$, then go to step 6. Otherwise, go to step 7.

Step 6: Set $b_k=b_k \cup \{J_i\}$, $S^k=S^k+s_i$, $P^k=P^k+p_i$, and $S_{1k}=t$. Update $U(t)$, and go to step 8.

Step 7: Process batch b_k, $B=B \cup b_k$. Set $t=t+s+P^k$, and $C_{1k}=t$. If $U(t)=\phi$, then stop. Otherwise, $k=k+1$, $b_k=\phi$, $S^k=0$, $P^k=0$, and go to step 8.

Step 8: Set $t=\max\{\min\{r_i \mid J_i \in U(t)\}, t\}$, $S_{1k}=t$. Update $A(t)$ and $K(t)$, and go to step 2.

Based on structural properties

2 Supply Chain Scheduling with Non-identical Job Sizes and Release Times

A two-phase heuristic algorithm

Phase 2:

Step 1: Index jobs in $U(t)$ as rule *DS-LPT*. Set $Q=k$, and $k=k+1$. Place the first job of $U(t)$ into b_k, and update $U(t)$.

Step 2: If $U(t)=\phi$, stop. Otherwise, go to step 3.

Step 3: $i=i+1$. For the *i*-th job concerned, place it into the lowest indexed batch, in which the sum size of the jobs does not exceed $c-s_i$ and the index is larger than Q. Update $U(t)$, go to step 2.

After all the jobs are formed into batches in phase 1 and phase 2, the generated batches are processed as their generating sequence in both phases based on Lemmas 2, 4, and 7.

2 Supply Chain Scheduling with Non-identical Job Sizes and Release Times

Computational experiments

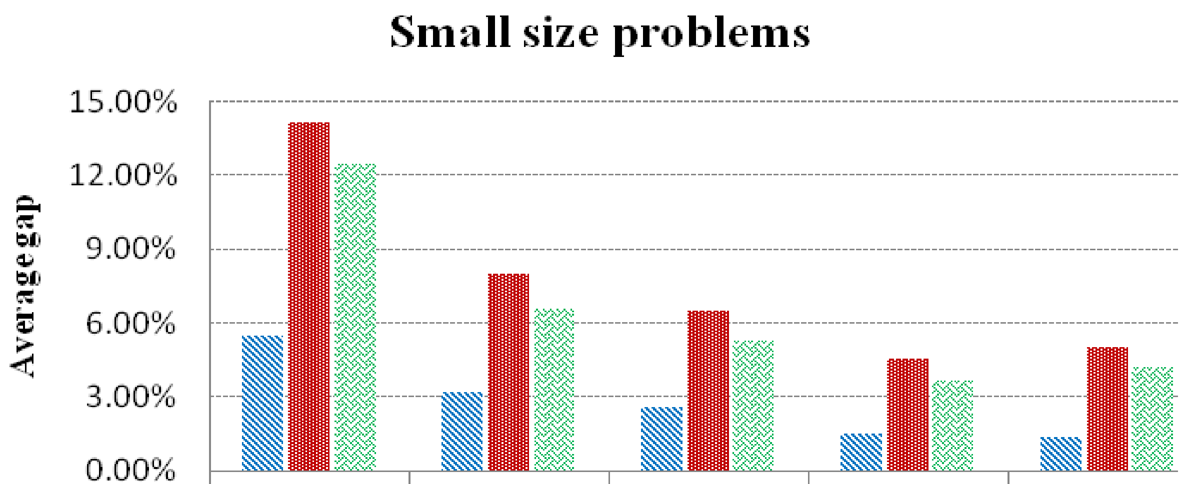
$$LB = \max\{LB_1, LB_2\} = \max\{r_{\min} + \left\lceil \sum_{i=1}^{n} s_i / c \right\rceil \cdot s + \sum_{i=1}^{n} p_i + T + P, r_{\min} + s + \min_{i=1,2,\ldots,n}\{p_i\} + T + \left\lceil \sum_{i=1}^{n} s_i / c \right\rceil \cdot P\}$$

$$Gap_H = \frac{C_{\max}^H - LB}{LB} \times 100\%$$

Performance Evaluation Index

2 Supply Chain Scheduling with Non-identical Job Sizes and Release Times

Computational experiments

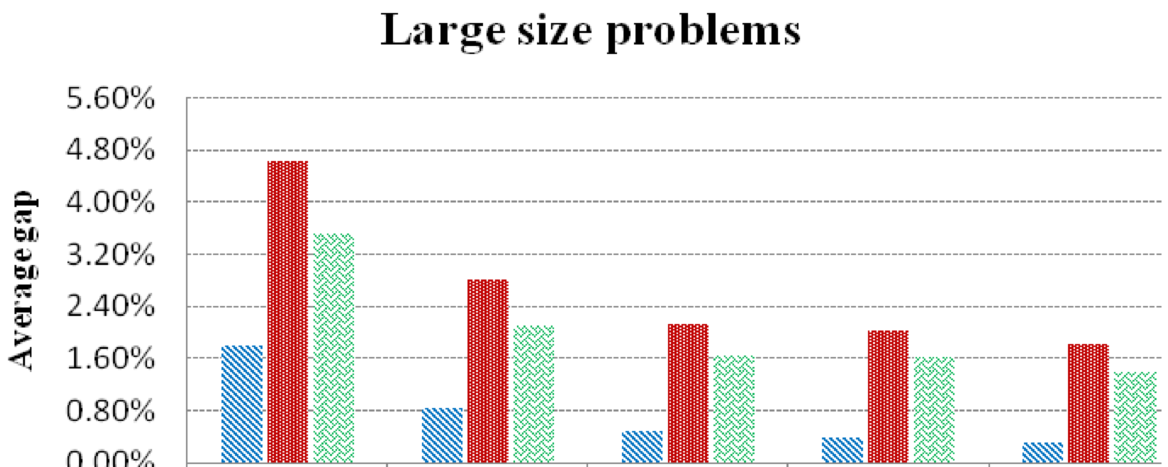


	20	40	60	80	100
TP-H	5.46%	3.17%	2.56%	1.47%	1.31%
MFF	14.09%	7.94%	6.45%	4.50%	4.97%
MBF	12.43%	6.55%	5.25%	3.63%	4.19%

Comparison of the average gap percentage on small-size problems by different heuristics ($C = 25$)

2 Supply Chain Scheduling with Non-identical Job Sizes and Release Times

Computational experiments

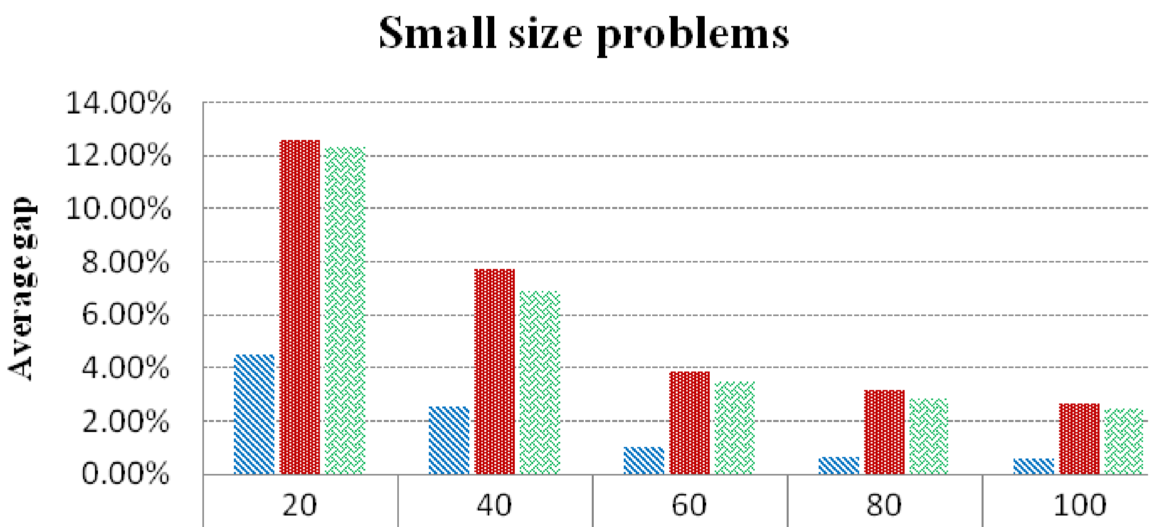


	300	500	700	900	1100
TP-H	1.80%	0.83%	0.47%	0.37%	0.29%
MFF	4.61%	2.79%	2.12%	2.01%	1.81%
MBF	3.50%	2.10%	1.63%	1.61%	1.37%

Comparison of the average gap percentage on large-size problems by different heuristics ($C = 25$)

2 Supply Chain Scheduling with Non-identical Job Sizes and Release Times

Computational experiments

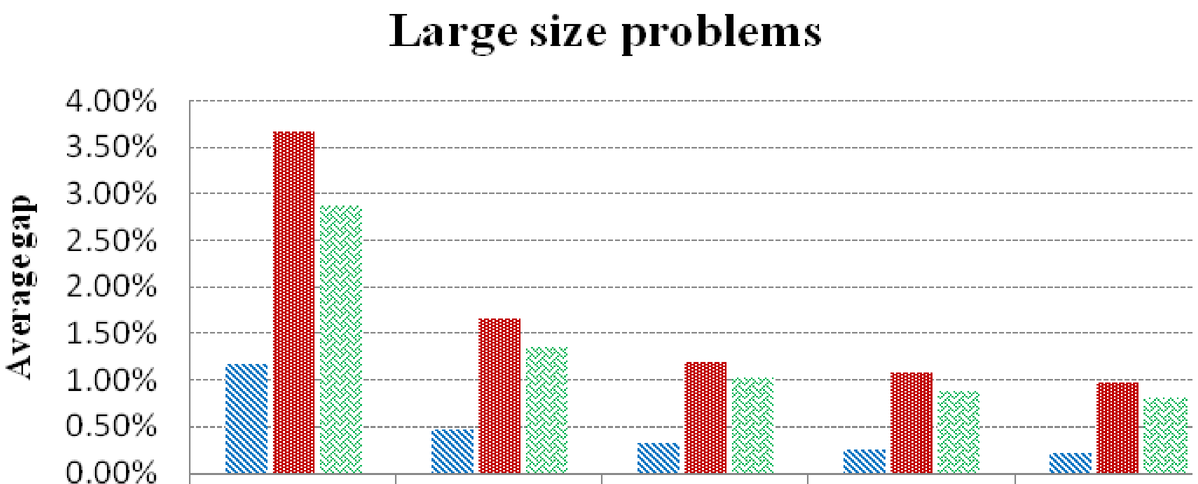


	20	40	60	80	100
TP-H	4.48%	2.48%	0.96%	0.62%	0.52%
MFF	12.57%	7.67%	3.82%	3.11%	2.61%
MBF	12.31%	6.85%	3.43%	2.80%	2.41%

Comparison of the average gap percentage on small-size problems by different heuristics ($C = 35$)

2 Supply Chain Scheduling with Non-identical Job Sizes and Release Times

Computational experiments

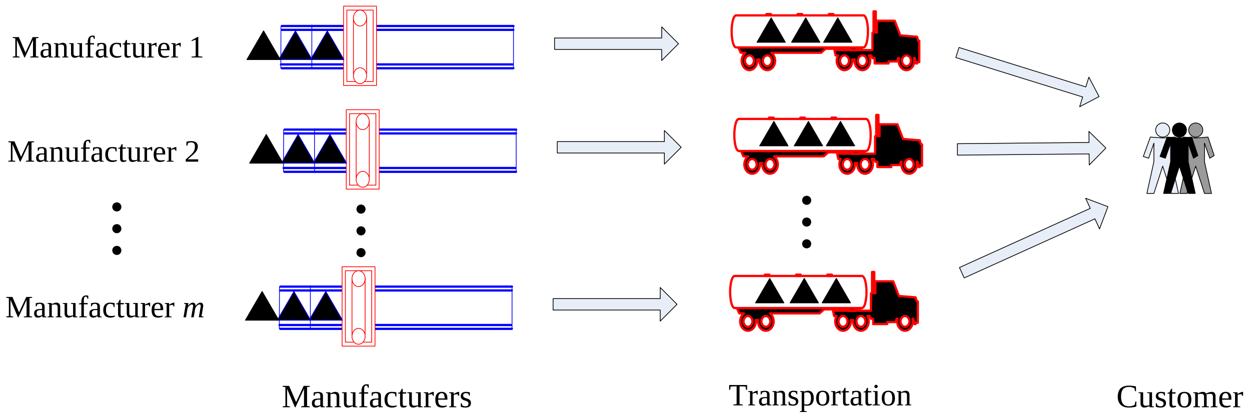


	300	500	700	900	1100
TP-H	1.16%	0.47%	0.32%	0.24%	0.21%
MFF	3.66%	1.65%	1.18%	1.07%	0.97%
MBF	2.86%	1.35%	1.03%	0.87%	0.81%

Comparison of the average gap percentage on small-size problems by different heuristics ($C = 35$)

3 Supply Chain Scheduling with Multiple Manufacturers

$$M \rightarrow C, m \left| c_j = c, t_k = T_j, \sum_{J_i \epsilon b_k} s_i \leq c \right| C_{max}$$



(1) **In the manufacturing stage, jobs are processed on the serial batching machines of multiple manufacturers.**

(2) **In the transportation stage, all batches are transported by vehicles from multiple manufactures to the customer for further processing..**

3 Supply Chain Scheduling with Multiple Manufacturers

Mixed integer programming model (2)

$$\text{Minimize: } C_{\max} \tag{1}$$

Subject to

$$\sum_{k=1}^{h} x_{ik} = 1,\ i=1,2,\ldots,N \tag{2}$$

$$\sum_{i=1}^{N} s_i \cdot x_{ik} \le c,\ k=1,2,\ldots,h \tag{3}$$

$$\sum_{k=1}^{h}\sum_{i=1}^{n_k} x_{ik} \cdot y_{il} = m_l,\ l=1,2,\ldots,n \tag{4}$$

$$\sum_{k=1}^{h}\sum_{i=1}^{N} x_{ik} = N \tag{5}$$

$$\sum_{j=1}^{m} w_{kj} = 1,\ k=1,2,\ldots,h \tag{6}$$

$$h_j = \sum_{k=1}^{h} w_{kj},\ j=1,2,\ldots,m \tag{7}$$

$$h = \sum_{j=1}^{m} h_j \tag{8}$$

$$\sum_{r=1}^{g} d_{kr} = 1,\ k=1,2,\ldots,h \tag{9}$$

$$S_{1(k+1)j} \ge C_{1kj} + t,\quad k=1,2,\ldots,h\text{-}1,\ j=1,2,\ldots,m \tag{10}$$

$$C_{1kj} = S_{1kj} + \sum_{i=1}^{N} x_{ik} \cdot p_i,\ k=1,2,\ldots,h,\ j=1,2,\ldots,m \tag{11}$$

$$S_{2kj} = C_{1kj},\ k=1,2,\ldots,h,\ j=1,2,\ldots,m \tag{12}$$

$$C_{2kj} = S_{1kj} + T_j,\ k=1,2,\ldots,h,\ j=1,2,\ldots,m \tag{13}$$

$$C^j = t \cdot h_j + \sum_{k=1}^{k} w_{kj} \cdot P_k,\ j=1,2,\ldots,m \tag{14}$$

$$C_{1kj} - C_{1fj} + P_f + t - (1 - z_{kfj})M \le 0,\ k=1,2,\ldots,h,\ f=1,2,\ldots,h,\ j=1,2,\ldots,m,\ k \ne f \tag{15}$$

$$C_{\max} \ge C^j + T_j,\quad j=1,2,\ldots,m \tag{16}$$

$$x_{ik} \in \{0,1\},\ \forall i,k \tag{17}$$

$$y_{il} \in \{0,1\},\ \forall i,l \tag{18}$$

$$z_{kfj} \in \{0,1\},\ \forall k,f,j \tag{19}$$

$$w_{kj} \in \{0,1\},\ \forall k,j \tag{20}$$

$$d_{kr} \in \{0,1\},\ \forall k,r \tag{21}$$

3 Supply Chain Scheduling with Multiple Manufacturers

Lemma 2.2.1. *There exists a schedule $\pi = (b_1, b_2, \cdots, b_f, \cdots, b_g, \cdots, b_h)$ for the problem ψ, in which the solutions remain unchanged when: (1) any two jobs in a batch are swapped; (2) any two batches processed on the same manufacturer's machine are swapped; (3) any two jobs in different batches processed on the same manufacturer's machine are swapped.*

Jobs sequencing and batches sequencing argument

3 Supply Chain Scheduling with Multiple Manufacturers

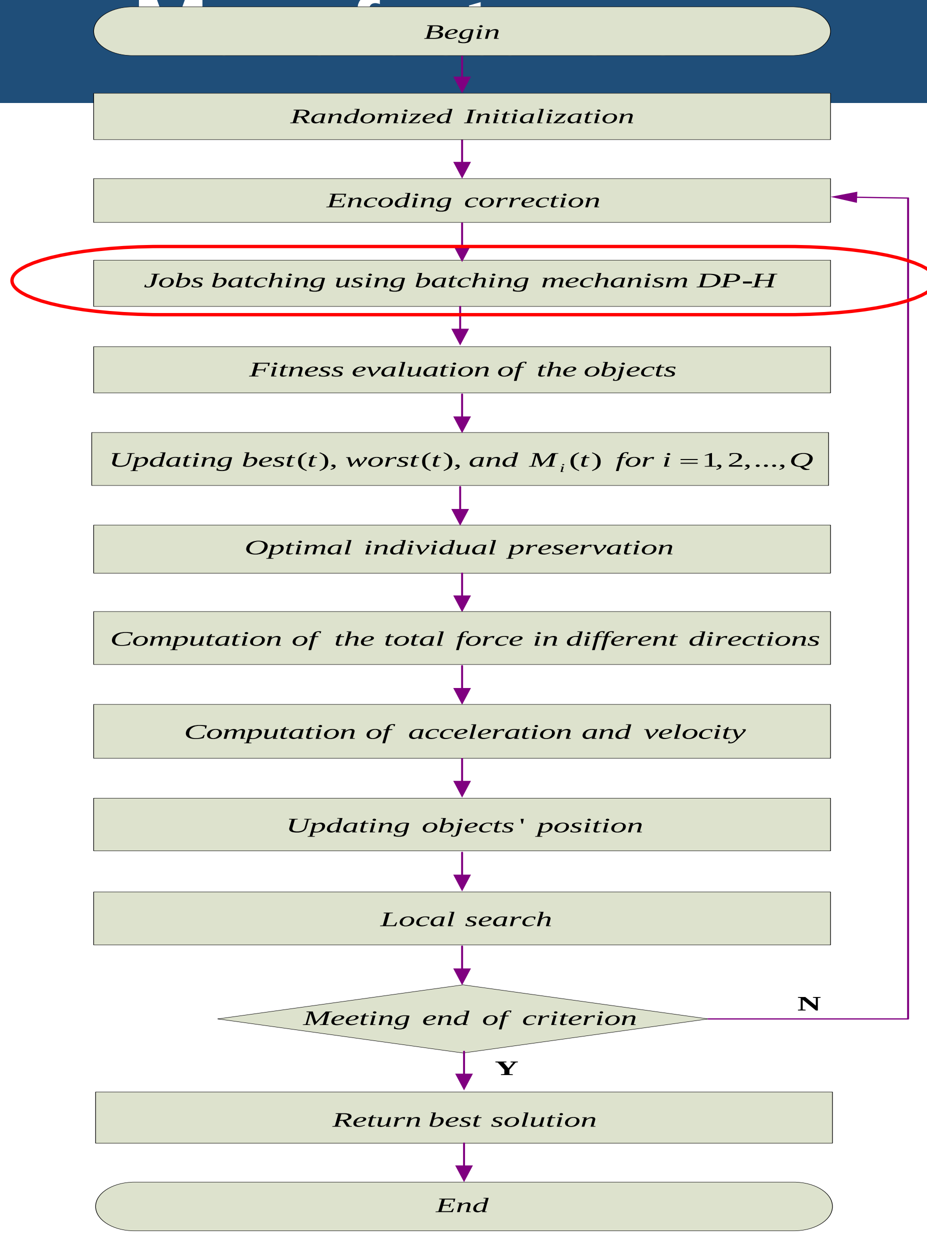
Corollary 2.2.1. *The maximum completion time of all jobs on the machine of the manufacturer is* $C^j = \sum_{k=1}^{h_j} P_k + t \cdot h_j (j = 1,2,\cdots,m)$.

Lemma 2.2.2. *The sum of maximum completion time of all jobs on the machines of all manufacturers is* $\sum_{j=1}^{m} C^j = t \cdot h + \sum_{i=1}^{N} p_i$.

Relationship between the number of batches and the makespan

3 Supply Chain Scheduling with Multiple

Modified Gravitational Search Algorithm



3 Supply Chain Scheduling with Multiple Manufacturers

Batching mechanism DP-H

Step 1: Initialize the job set J^j_{total} $(j=1,2,\ldots,m)$ of all jobs to be processed in each manufacturer according to the position sequence values of the objects. The number of the jobs in J^j_{total} is represented by n^j_{total} $(j=1,2,\ldots,m)$, $\sum_{j=1}^{m} n^j_{total} = N$. Calculate the jobs number n^j_l $(l=1,2,\ldots,n, j=1,2,\ldots,m)$ of each type to be processed in the j-th manufacturer. $q=0$, $j=1$.

Step 2: $q=q+1$. Apply dynamic programming algorithm to obtain an optimal combination O_q from J^j_{total} as a batch. The function value of the maximum used capacity of the machine capacity v within the first i jobs is denoted by $g(i,v)$, which is attained from those partial schedules associated with state (i,v) for $1 \le i \le n^j_{total}$ and $1 \le v \le c$. The details of dynamic programming algorithm are as follows:

· Initialization:

For each i from 1 up to n^j_{total} do

For each v from 1 up to c do

$g(i,v)=0$

· Recursive equations:

For each i from 1 up to n^j_{total} do

For each v from 1 up to c do

$$g(i,v)=\begin{cases} g(i-1,v), & s_i > v \\ \max\{g(i-1,v), g(i-1,v-s_i)+s_i\}, & else \end{cases}$$

The optimal solution value is equal to $g(n^j_{total}, c)$, and the corresponding schedule can be found by backtracking.

Step 3: Calculate the jobs number of each type in O_q, respectively. Denote the number as k^l_q $(l=1,2,\ldots,n)$.

Step 4: Calculate the execution number d_q of the combination O_q.

$$d_q = \min_{l=1,2,\ldots,n} \lfloor n^j_l / k^l_q \rfloor$$

Step 5: Update J^j_{total}, n^j_{total}, and n^j_l $(l=1,2,\ldots,n)$.

$J^j_{total} = J^j_{total} / J^q$, where the set J^q represents the jobs of all combinations of O_q.

$$n^j_{total} = n^j_{total} - d_q \sum_{l=1}^{n} k^l_q$$

For each l from 1 up to n do

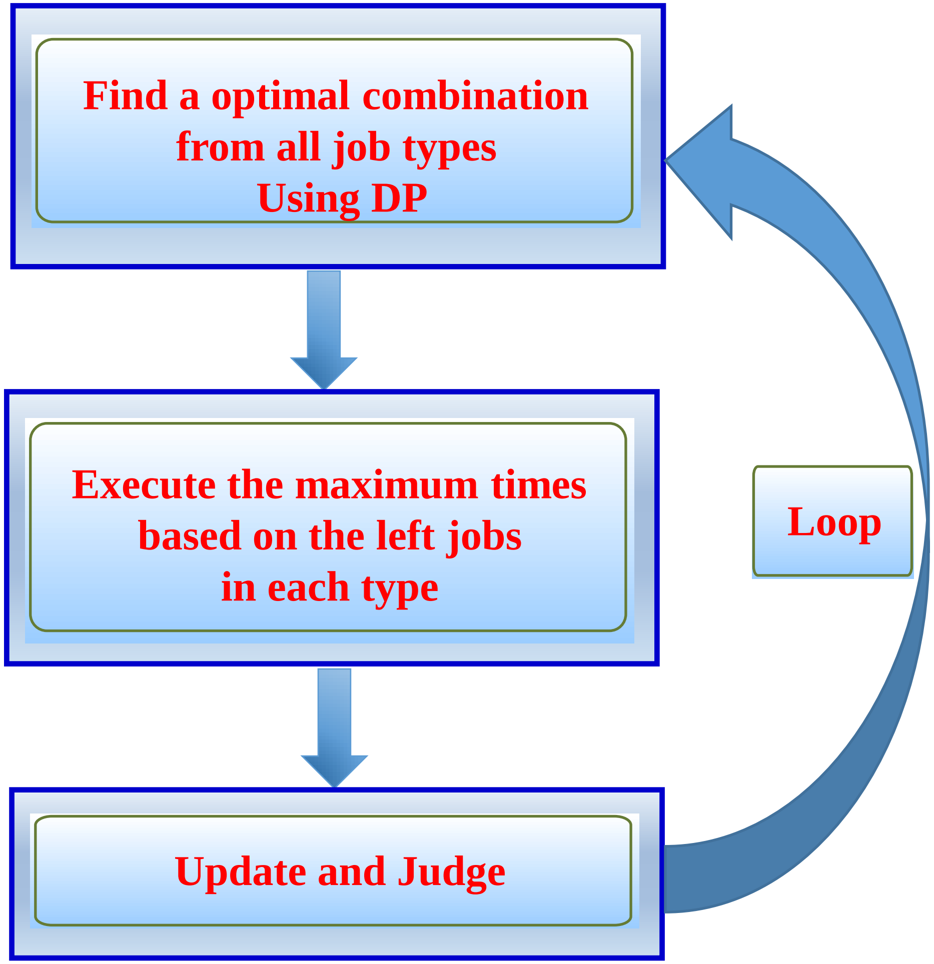
$n^j_l = n^j_l - d_q \cdot k^l_q$

Step 6: If $J^j_{total} = \phi$, go to step 7; otherwise, go to step 2.

Step 7: Output the result matrix φ_j of the batches, where the first column Q_x $(x=1,2,\ldots,q)$ denotes the jobs combination of the x-th iteration and the second column d_x $(x=1,2,\ldots,q)$ represents the number of the jobs combination of the x-th iteration.

3 Supply Chain Scheduling with Multiple Manufacturers

Batching mechanism DP-H



3 Supply Chain Scheduling with Multiple Manufacturers

Computational experiments

$$LB = \frac{\sum_{i=1}^{N} p_i + t \cdot \left\lceil \sum_{i=1}^{N} s_i / c \right\rceil + \min_{j=1,2,\cdots,m} T_j}{m}$$

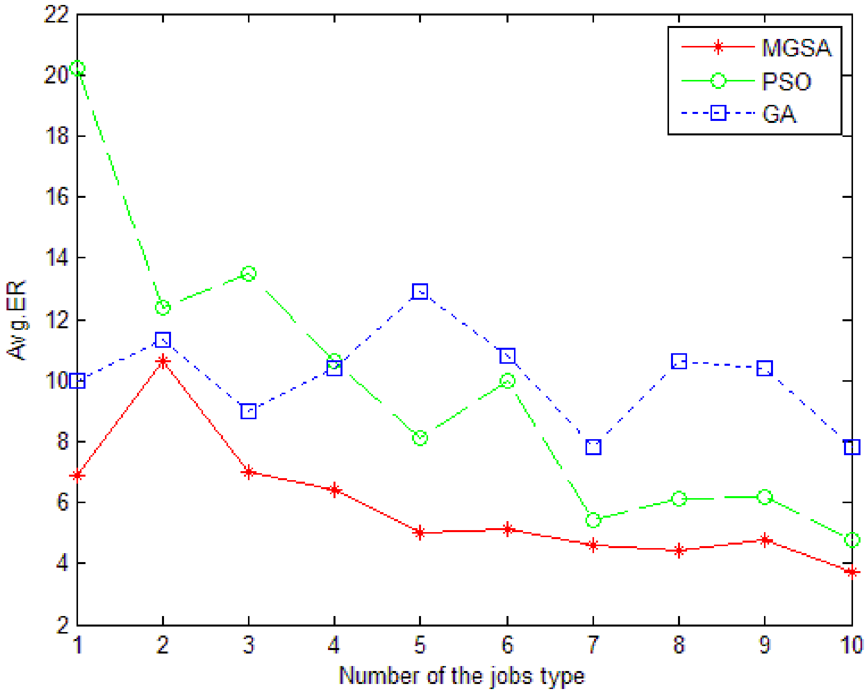
$$Gap_H = \frac{C_{\max}^{H} - LB}{LB} \times 100\%$$

↓

Performance Evaluation Index

3 Supply Chain Scheduling with Multiple Manufacturers

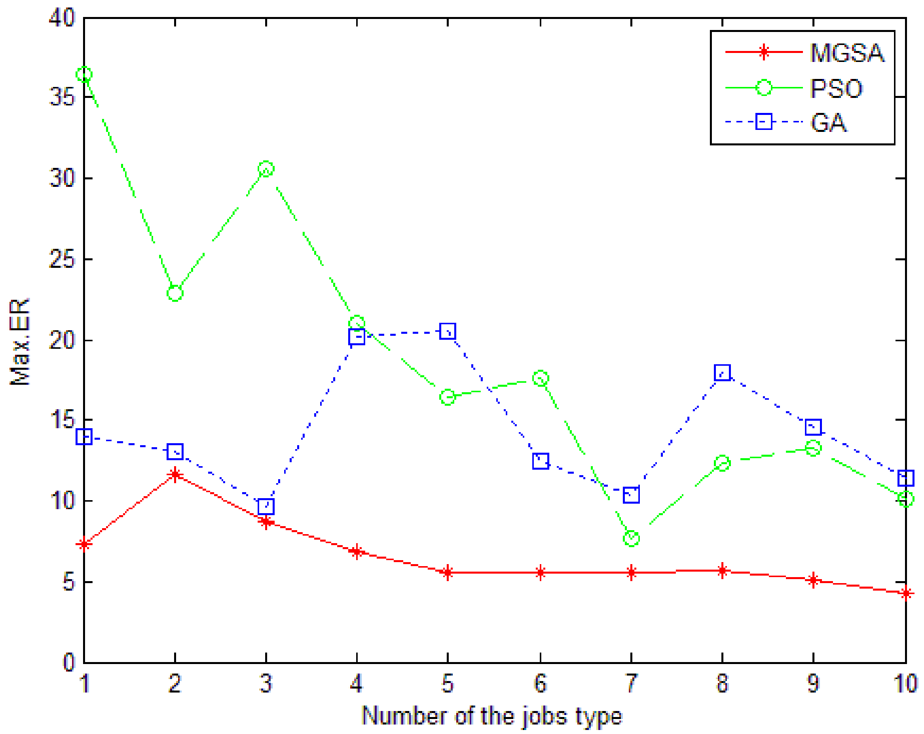
Computational experiments



The average error ratios for each problem with different number of jobs type

3 Supply Chain Scheduling with Multiple Manufacturers

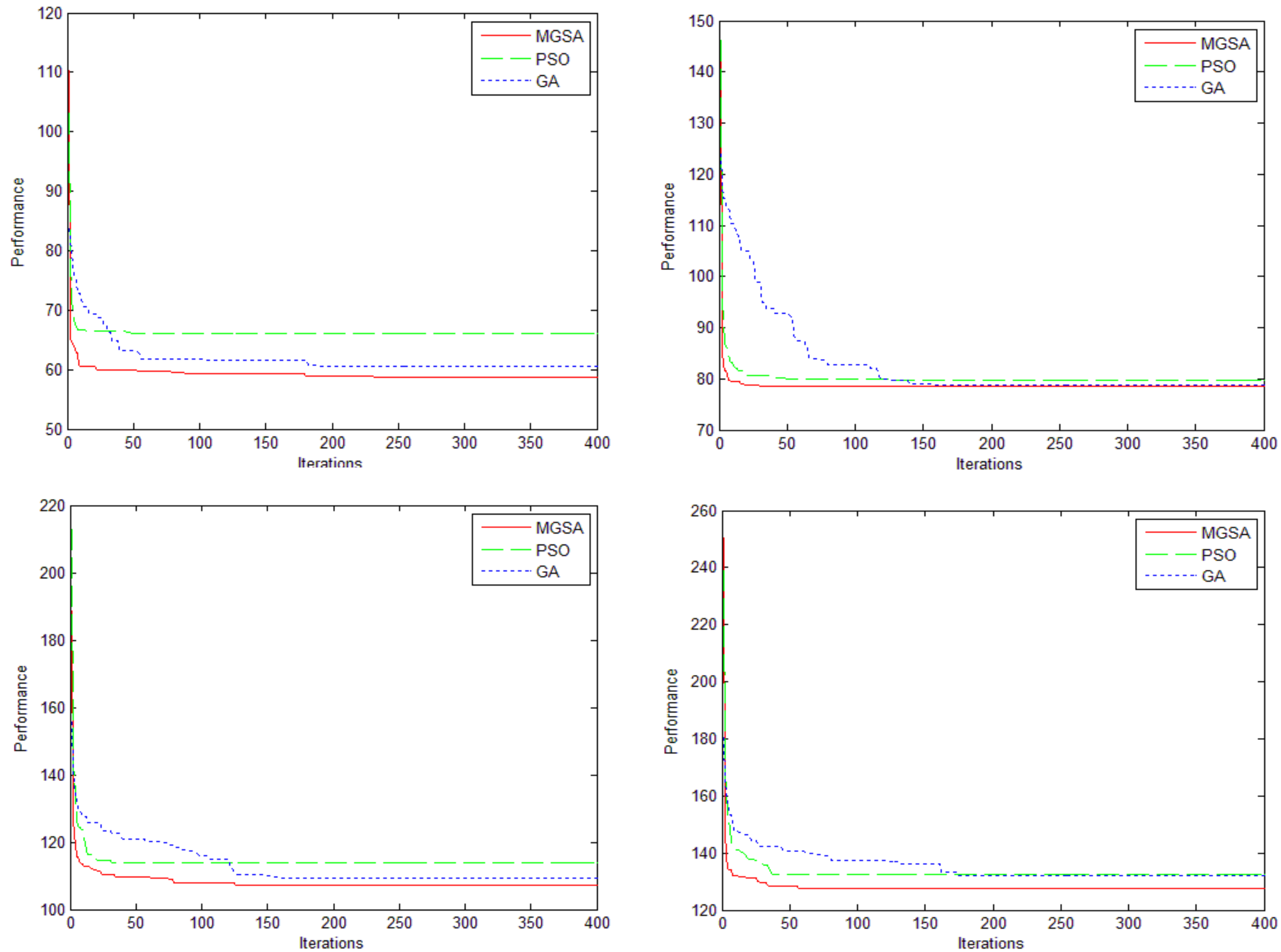
Computational experiments



The maximum error ratios for each problem with different number of jobs type

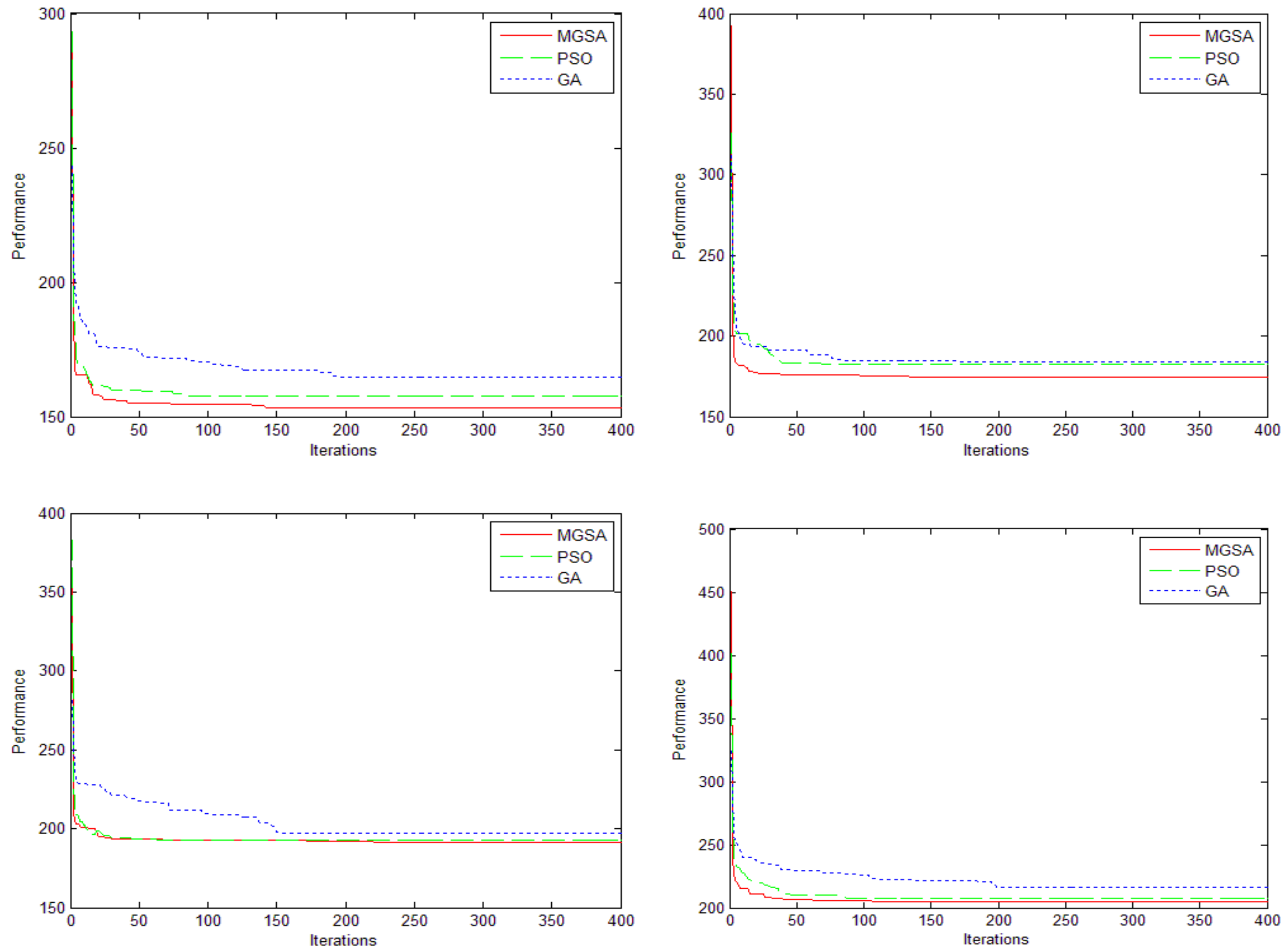
3 Supply Chain Scheduling with Multiple Manufacturers

Computational experiments



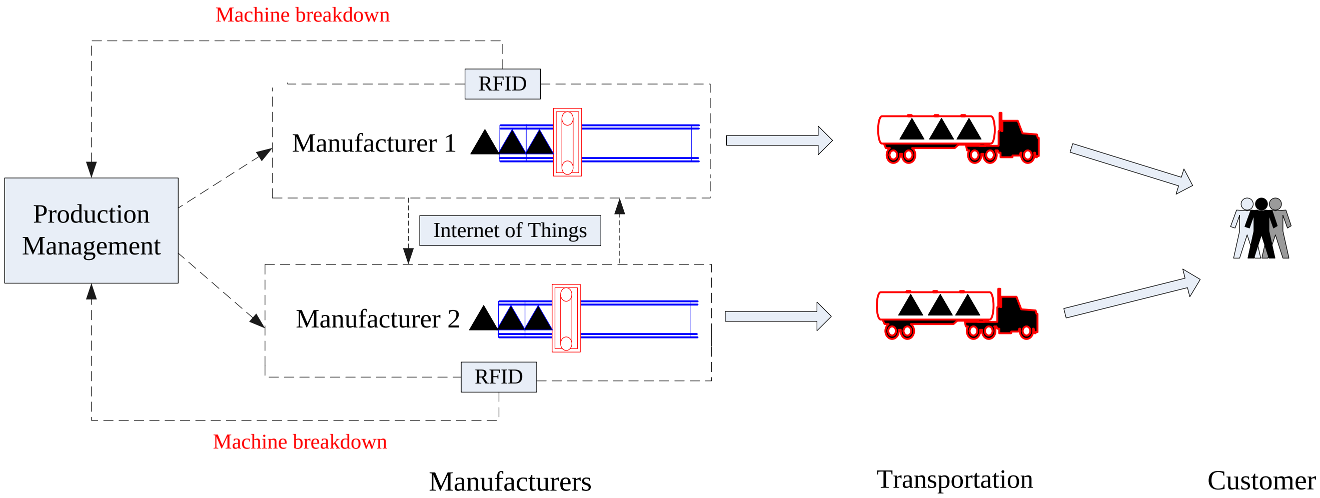
3 Supply Chain Scheduling with Multiple Manufacturers

Computational experiments



4 Supply Chain Scheduling with Arbitrary Machine Breakdown

$$S \rightarrow M \left| r_i, \sum_{J_i \epsilon b_k} s_i \leq c, s - batch, ua, T \right| C_{max}$$



(1) There is a set of jobs to be processed in two parallel manufacturers and then transported to a customer.

(2) Machine breakdown may occur during the scheduling period, and the information of the breakdown will be transmitted to the center of production management immediately by RFID based on IoT.

4 Supply Chain Scheduling with Arbitrary Machine Breakdown

Mixed integer programming model (3)

Minimize

$$C_{max} \qquad (1)$$

Subject to

$$\sum_{k=1}^{h} x_{ik} = 1, \qquad i = 1,2,\cdots,n \qquad (2)$$

$$\sum_{i=1}^{n} s_i \cdot x_{ik} \leq c, \qquad k = 1,2,\cdots,h \qquad (3)$$

$$\sum_{j=1}^{2} z_{kj} = 1, \qquad k = 1,2,\cdots,h \qquad (4)$$

$$h_j = \sum_{k=1}^{h} z_{kj}, \qquad j = 1,2 \qquad (5)$$

$$h = \sum_{j=1}^{2} h_j \qquad (6)$$

4 Supply Chain Scheduling with Arbitrary Machine Breakdown

Mixed integer programming model (3)

$$S_{1kj} \geq \max_{J_i \in b_k}\{r_i\} + g_k s, \quad k = 1,2,\cdots,h, \; j = 1,2 \tag{7}$$

$$C_{1kj} \geq S_{1kj} + \sum_{i=1}^{n} x_{ik} \cdot p_i, \quad k = 1,2,\cdots,h, \; j = 1,2 \tag{8}$$

$$v_{kjl}\left(d_{jl} - S_{1kj}\right)\left(C_{1kj} - e_{jl}\right) \geq 0, \quad k = 1,2,\cdots,h, \; j = 1,2, \; l = 1,2,\cdots,m_j \tag{9}$$

$$C_{2kj} = C_{1kj} + g_k T, \quad k = 1,2,\cdots,h, \; j = 1,2 \tag{10}$$

$$g_f C_{1kj} - C_{1fj} + P_f + g_f s - \left(1 - y_{kfj}\right)M \leq 0,$$

$$k = 1,2,\cdots,h, \quad f = 1,2,\cdots,h, \quad j = 1,2, \; k \neq f \tag{11}$$

$$C^j \geq s \cdot h_j + \sum_{k=1}^{h} z_{kj} \cdot P_k + \sum_{l=1}^{m_j}\left(e_{jl} - d_{jl}\right), \quad j = 1,2 \tag{12}$$

$$C_{max} \geq C^j + T, \quad j = 1,2 \tag{13}$$

$$x_{ik}, y_{kfj}, z_{kj}, v_{kjl} \in \{0,1\}, \quad \forall i,k,f,j,l \tag{14}$$

4 Supply Chain Scheduling with Arbitrary Machine Breakdown

Lemma 2.3.1. *For all schedules, the solution remains unchanged when any two jobs in a batch are swapped.*

Lemma 2.3.2. *There exists an optimal schedule such that all jobs in each batch are processed in non-decreasing order of their arrival times on the machines of both manufacturers* m_1 *and* m_2.

Jobs and batches sequencing argument

4 Supply Chain Scheduling with Arbitrary Machine Breakdown

Lemma 2.3.3 *In a schedule of the problem* ψ, *each batch is processed in non-decreasing order of ready time, which is equal to the arrival time of the last job in the batch. There exists a batch* $b_k = \{J_i, \cdots, J_{i+x}, J_{i+x+1}, \cdots, J_{i+n_k-1}\}$ *processed in the manufacturer* $m_j (j = 1,2)$, *where* $n_k \geq 2$ *and* $r_i \geq C_{1(k-1)j}$. *If* $r_{i+n_k-1} > r_{i+x} + s$, *then the solution can be improved.*

The improvement condition

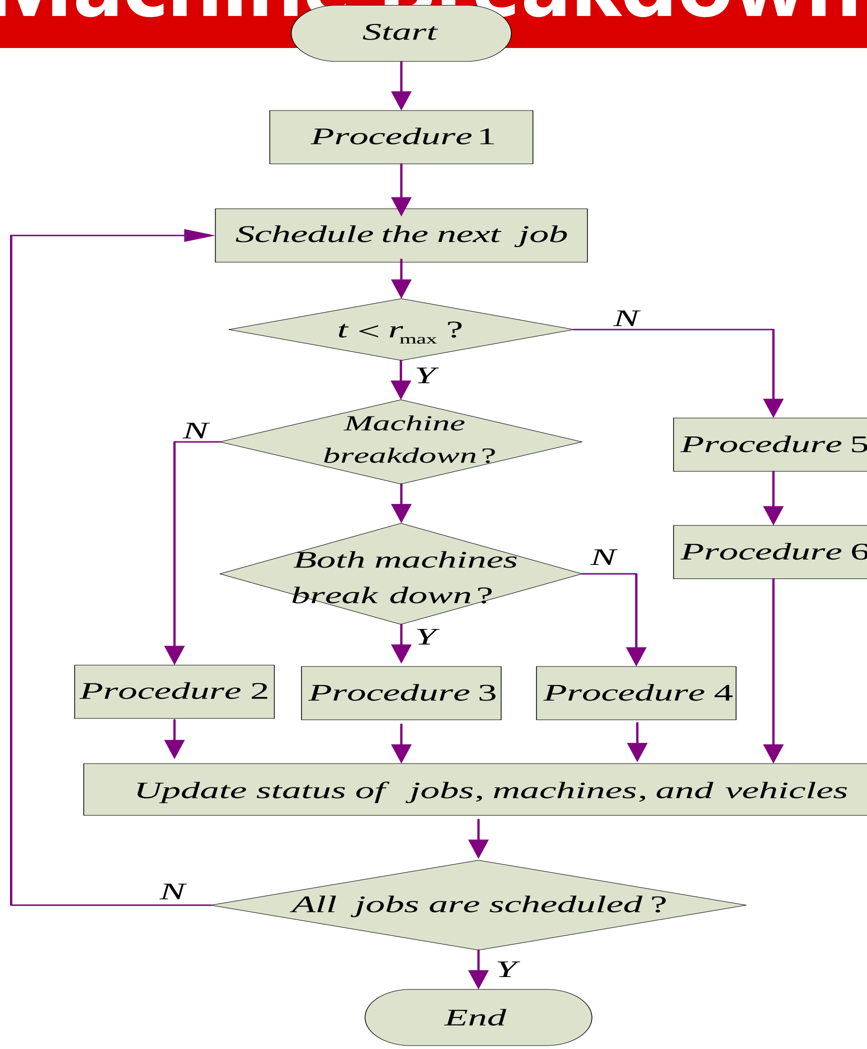
4 Supply Chain Scheduling with Arbitrary Machine Breakdown

Lemma 2.3.4. *The maximum completion time of all jobs on the machine of the manufacturer j is* $C^j \geq \sum_{k=1}^{h_j} P_k + s \cdot h_j + \min_{i=1,\cdots,n}\{r_i\} + \sum_{l=1}^{m_j}(e_{jl} - d_{jl})(j = 1,2)$.

Lemma 2.3.5. *The sum of maximum completion time of all jobs on the machines of both manufacturers is* $\sum_{j=1}^{2} C^j \geq \sum_{i=1}^{n} p_i + s \cdot h + 2\min_{i=1,\cdots,n}(r_i) + \sum_{j=1}^{2}\sum_{l=1}^{m_j}(e_{jl} - d_{jl})(j = 1,2)$.

The result of the job completion times

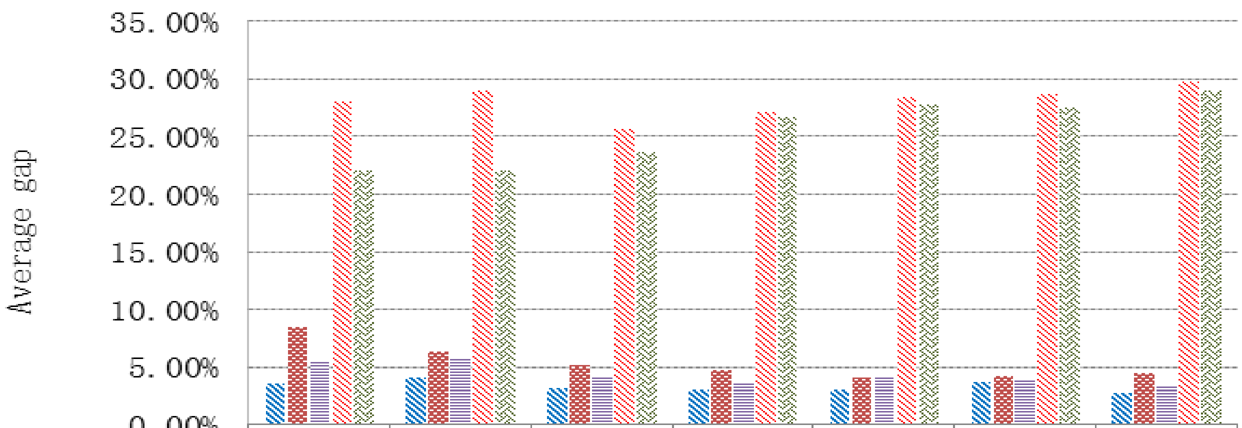
4 Supply Chain Scheduling with Arbitrary Machine Breakdown



Flow chart of the proposed heuristic algorithm

4 Supply Chain Scheduling with Arbitrary Machine Breakdown

Computational experiments



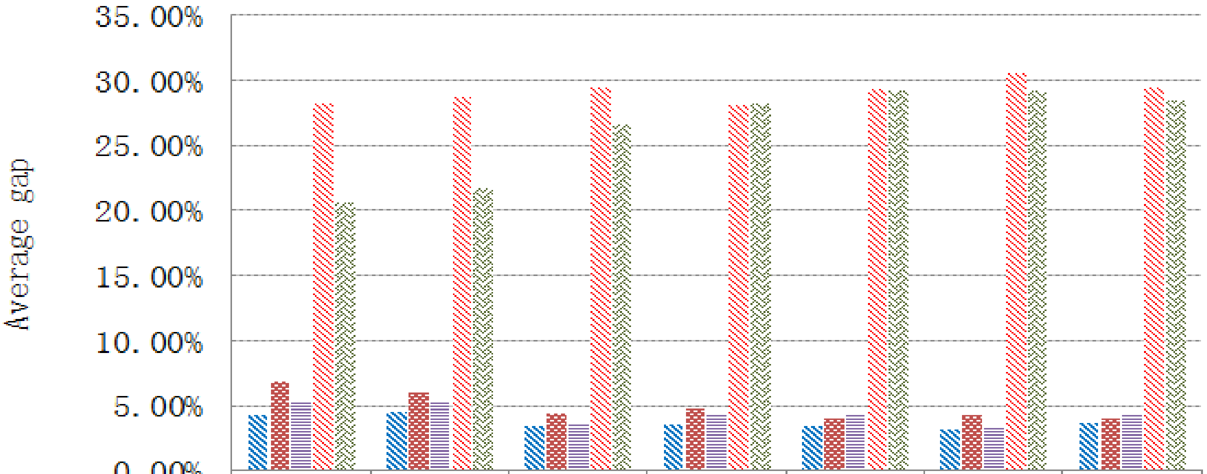
	50	100	200	400	600	800	1000
Heristic H	3. 61%	4. 09%	3. 17%	3. 03%	3. 10%	3. 74%	2. 81%
OS	8. 59%	6. 43%	5. 15%	4. 75%	4. 14%	4. 22%	4. 60%
FIFO	5. 60%	5. 78%	4. 16%	3. 66%	4. 23%	3. 86%	3. 54%
SPT	28. 02%	29. 04%	25. 70%	27. 12%	28. 55%	28. 74%	29. 77%
LPT	22. 04%	22. 16%	23. 65%	26. 73%	27. 86%	27. 49%	28. 98%

Computational results for Ag=0.05 and MTTR=12

MTBF and MTTR denoting the mean time of the intervals between the machine breakdowns and repairing the machines, and Ag= MTTR/(MTTR+MTBF)

4 Supply Chain Scheduling with Arbitrary Machine Breakdown

Computational experiments

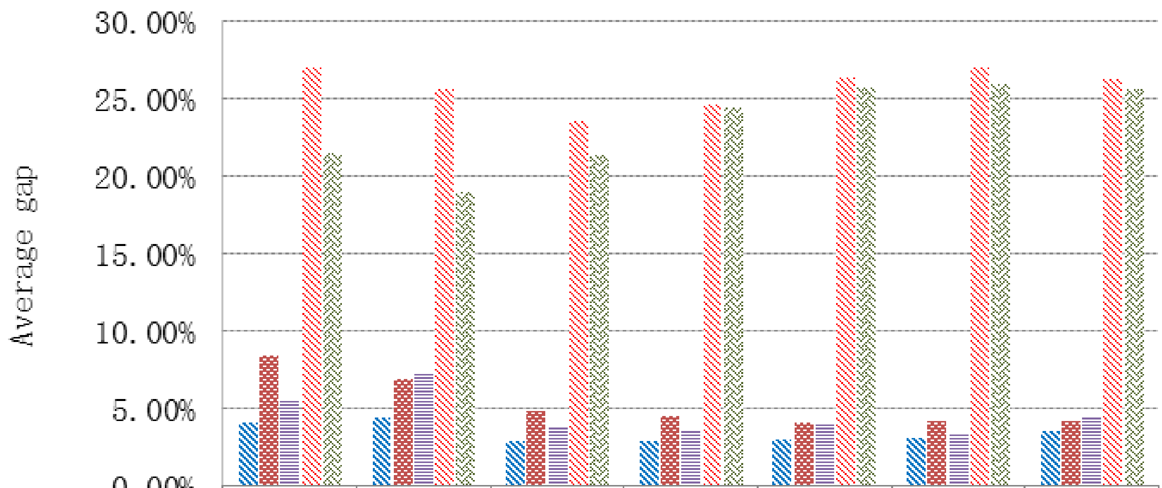


	50	100	200	400	600	800	1000
Heristic H	4.31%	4.52%	3.40%	3.52%	3.51%	3.24%	3.65%
OS	6.87%	6.07%	4.44%	4.83%	4.13%	4.29%	4.10%
FIFO	5.47%	5.43%	3.72%	4.37%	4.44%	3.42%	4.41%
SPT	28.29%	28.68%	29.46%	28.18%	29.31%	30.61%	29.43%
LPT	20.61%	21.71%	26.65%	28.25%	29.29%	29.29%	28.50%

Computational results for Ag=0.05 and MTTR=36

4 Supply Chain Scheduling with Arbitrary Machine Breakdown

Computational experiments

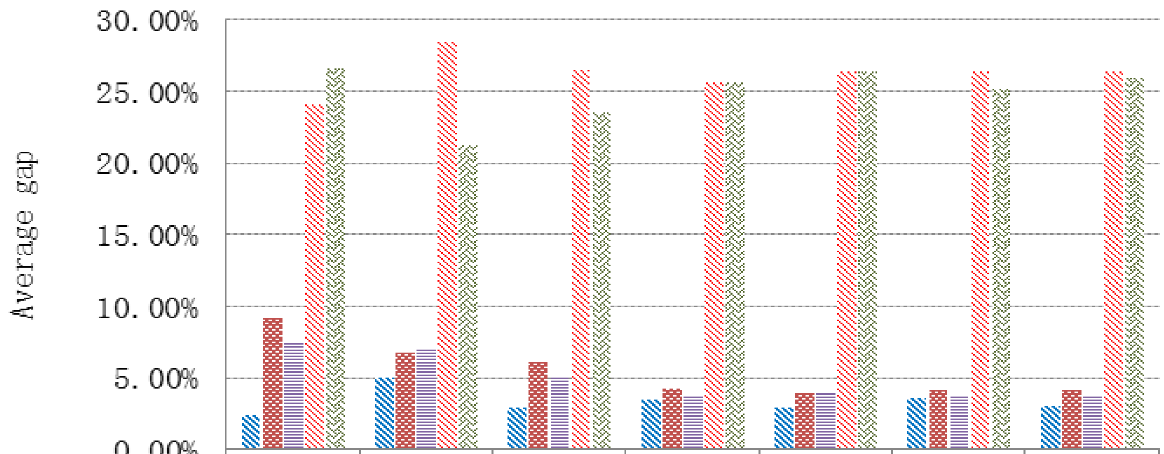


	50	100	200	400	600	800	1000
Heristic H	4.12%	4.46%	3.00%	2.98%	3.01%	3.21%	3.59%
OS	8.47%	6.95%	4.92%	4.61%	4.10%	4.27%	4.30%
FIFO	5.57%	7.31%	3.96%	3.56%	4.19%	3.43%	4.60%
SPT	27.12%	25.61%	23.59%	24.70%	26.38%	27.07%	26.35%
LPT	21.55%	19.03%	21.43%	24.41%	25.71%	25.98%	25.69%

Computational results for Ag=0.1 and MTTR=12

4 Supply Chain Scheduling with Arbitrary Machine Breakdown

Computational experiments



	50	100	200	400	600	800	1000
Heristic H	2. 44%	5. 03%	2. 93%	3. 54%	3. 01%	3. 65%	3. 07%
OS	9. 18%	6. 75%	6. 10%	4. 33%	4. 00%	4. 17%	4. 19%
FIFO	7. 55%	7. 12%	5. 13%	3. 89%	4. 08%	3. 82%	3. 87%
SPT	24. 04%	28. 47%	26. 54%	25. 60%	26. 40%	26. 38%	26. 36%
LPT	26. 60%	21. 23%	23. 55%	25. 66%	26. 38%	25. 17%	25. 93%

Computational results for Ag=0.1 and MTTR=36


Thank you!