Robust Optimization and Learning in Approximate Dynamic Programming Using Kernels

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Motivations and Objectives -Motivations-

Decision making is ubiquitous in science and engineering. Dynamic Programming (DP) provides a structured approach, but the CURSE OF DIMENSIONALITY makes finding an optimal policy:

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- lacking computation efficiency
- requiring off-line calculations
- ignoring uncertainty within the problem parameters

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Outline

- The agent needs to take an action given the actual state and the reward from taking that action.
- The action taken does not need to increase the immediate reward but the expected cumulative reward.
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Markov Decision Processes

An MDP is defined by:

- S, a state space.
- A, an action space.
- $\mathcal{P}$, a transition probability distribution, where: $\mathcal{P}^a_{ss'} = Pr\{s_{t+1} = s' | s_t = s, a_t = a\}$.
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Value Functions

To evaluate how good a particular policy is, the agent needs to evaluate $\mathcal{J}_\pi(s)$ and $Q^\pi(s, a)$.

- The state-value function:

$$\mathcal{J}_\pi(s) = E_\pi \left\{ \sum_{k=t}^{\infty} \gamma^{k-t} g_{s_k s_{k+1}}^{\pi(s_k)} | s_t = s \right\}$$

  Where $(0 < \gamma < 1)$.

- the action-value function:

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Value Functions -continued-

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- The optimal policy π^* is the one that maximizes $\mathcal{J}^\pi$

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- Many real life problems have states that are similar or closed to each other
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- We need to construct a subset $\overline{S}$ using clustering. The value function will be calculated for the elements in $\overline{S}$ only
- $\{S_1, S_2, \ldots, S_l\}$ be a partition of S
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Reduced States' Set

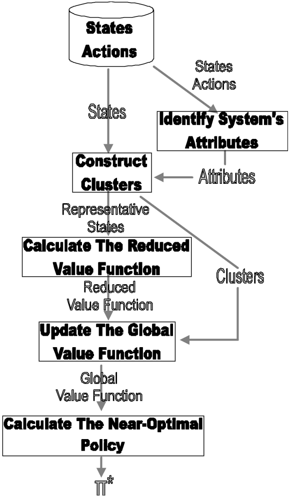

Figure: Reduced States' set diagram

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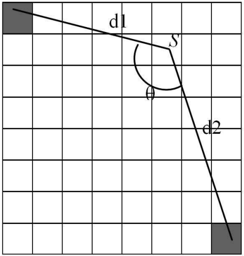
- At each intersection we can go on all 4 directions.
- The only available exits from the city are the north-west and south-east blocks.



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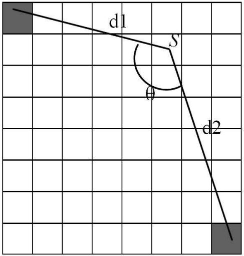

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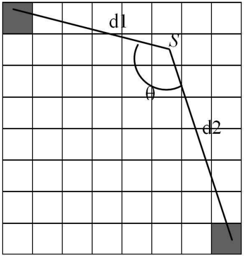

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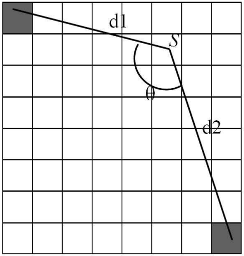
- 254 states and 2 terminal ones.
- The reward to go from one block to another is -1.
- The reward to get to one of the terminal states is 0.
- 3 attributes: d_1, d_2, and θ.



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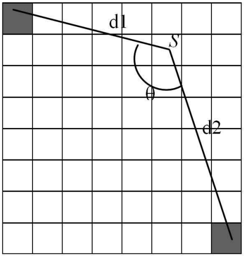

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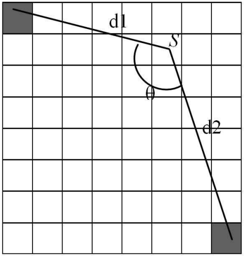

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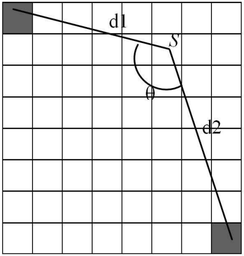

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- The accuracy rate is introduced to compare the different policies obtained:
$A_r = \frac{\text{number of accurate states}}{\text{number of states (256)}}$

	Optimal	Reduced	Reduced
# of clusters	256	73	56
Time (sec)	211	70	33
# of accurate states	256	256	196
A_r	1	1	.77

Table: Comparing Computation Time and Accuracy

Curse of dimensionality

- Consider finite sets with fixed cardinality for the states and actions.
- Need huge amount of resources to find an optimal policy.
- Need off-line algorithms.
- However
  In real world problems, the environment might change and include new states or actions.
- A policy that was optimal for a previous environment might not be optimal anymore.
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On-line Reinforcement Learning using Kernels

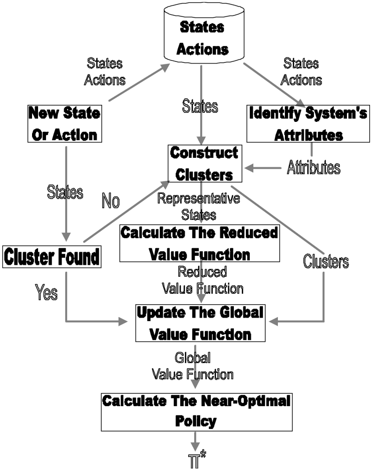

Figure: On-line Reduced States' set diagram

Consider the same city example with the following changes:

- Some blocks in the city might be inaccessible or some of the actions might become impossible to take.
- Due to the congestion, one of the emergency exits might be temporarily disabled.
- The emergency services might succeed in increasing the number of emergency exits.
- To make the problem feasible, we assume that during the whole horizon, there is at least one terminal state.
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Events	Optimal	Reduced			Reduced		
	time (sec)	time (sec)	clusters	A_r	time (sec)	clusters	A_r
- 1 block	220	75	74	1	36	57	.769
- 1 block	220	80	75	1	1	57	.762
- 1 exit	350	170	75	1	62	57	.773
- 1 block	345	1	75	1	1	57	.754
+ 1 block	347	1	75	1	1	57	.754
+ 1 exit	219	76	75	1	35	57	.773
+ 1 block	222	72	74	1	33	56	.773
+ 1 block	221	73	74	1	1	56	.766
Total	2144	548	-	-	170	-	-

Table: Comparing Computation Time and Accuracy for On line Reduced states' set

- We used a Modified Asynchronous Policy Iteration:
- The value iterations are executed according to the following updates:

$$\mathcal{J}_{k+1}(s) = \begin{cases} (\mathcal{T}_{\pi_k}\mathcal{J}_k)(s), \text{ if } s \in \overline{S}, \\ \mathcal{J}_{k+1}(s_i), \text{ s.t. } s \in S_i, \text{ otherwise,} \end{cases}$$

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- Let $s \in \overline{S}_i$ and $s \notin \overline{S}$, let $s' \in \overline{S}_j$.
- Let $\mathcal{P}_{ss'}^{\pi_k(s)} > 0$ and $s'' \in \overline{S}_j$ such that:

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- then $\exists\ \epsilon \geq 0$ such that:
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- Conduct a set of experiments on the class of shortest path problems.
- Compare the time it takes to find an optimal policy using our algorithm and without reducing the states' set cardinality.
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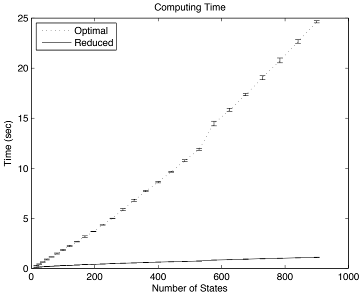


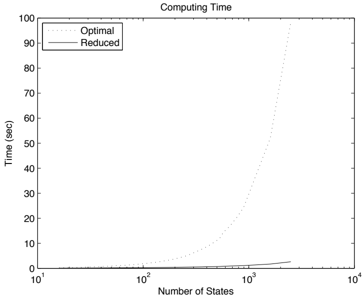


Figure: Optimal Vs Reduced Computation Times

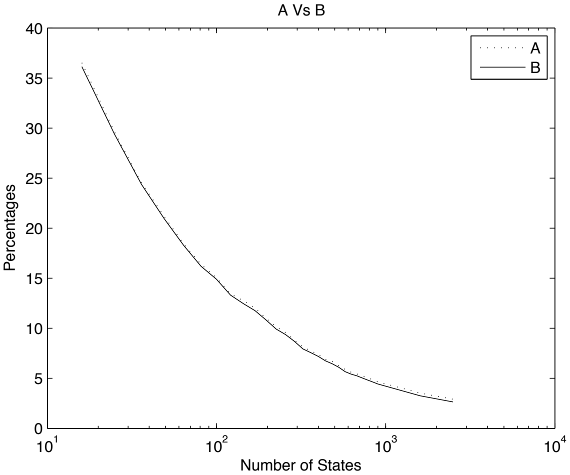


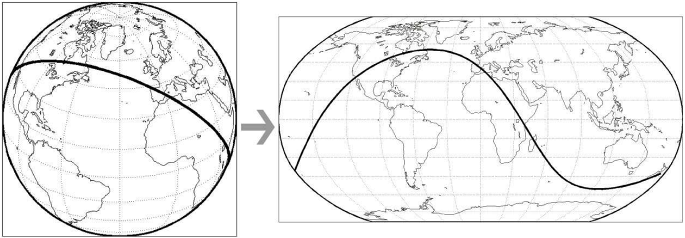
Figure: The Impact of the Clustering Time

- Kernel methods play a major role in Machine Learning.
- They provide a simple framework for manipulating nonlinear relationships.
- Instantaneous adaptation of former linear algorithms.
- Require modest computational resources.

- A kernel is a continuous symmetric real-valued function defined on compact subsets of $\mathbb{R}^n$, $k : (x, y) \mapsto k(x, y)$.
- A Mercer kernel is a nonnegative definite kernel.
- The domain of a Mercer kernel is called the input space.
- The quantity $k(x, y)$ can be used to represent measures of angle and measures of distance.
- Angles and distances are between inputs mapped in a higher dimensional Hilbert space.
- The Hilbert space is called the feature space.

- Mercer's theorem suggests a particular decomposition of Mercer kernels.
- A Mercer kernel can be expressed as a dot product between two inputs mapped in the feature space, $k(x, y) = \langle \phi(x) \cdot \phi(y) \rangle$.
- Explicit knowledge of the map ϕ and the feature space is not required. The only thing of importance is the kernel itself.
- With k we can calculate angles and distances between elements in the feature space without knowing the map ϕ.

- Kernel methods simplify the representation of nonlinear patterns in the input space.
- The intersection between hyperplanes and the manifold has a non-trivial reciprocal image in the input space.
- Instead of searching complex patterns in the input space, we use kernel methods.



Objectives

- Use Linear approximate dynamic programming to speed up the algorithm.
- Incorporate robustness to get stable policies.
- Use kernels to define the concept of the neighborhood of a state.
- Combine all the above techniques to generate an on-line robust RL algorithm.

RORLK: Algorithm Diagram

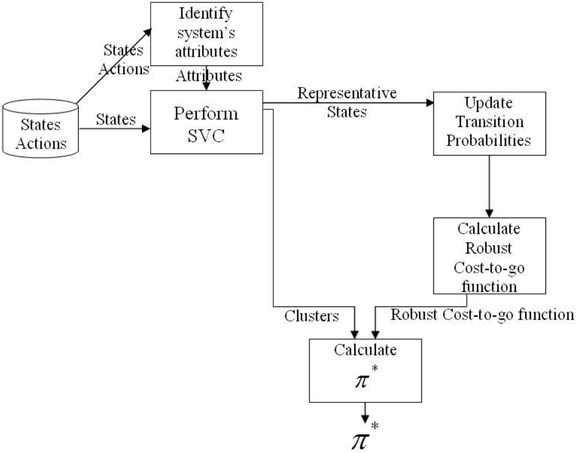


Figure: Robust On-line RL using Kernels diagram

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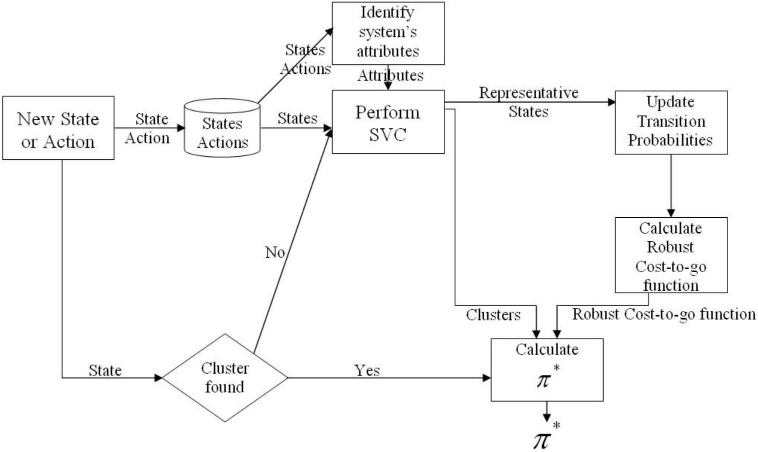
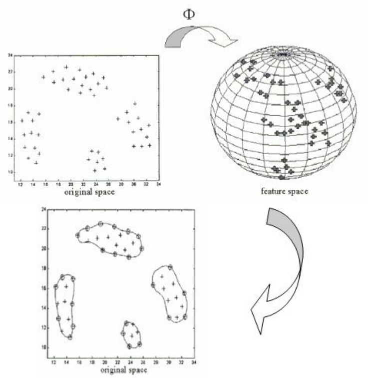


Figure: Robust On-line RL using Kernels diagram

- This approach aims to decrease the complexity of the problem by reducing the number of states.
- Many real life problems have states that are similar or closed to each other.
- This similarity yields the same or similar value function.
- $\Rightarrow$ calculate the value function just for one of the similar states.
- To determine similar states we will use clustering.

- Clustering is a part of data mining that consists of grouping a set of data according to various attributes.
- Many methods have been developed.
- The effectiveness of each approach depends on the nature of the data.
- Ben-Hur (2001) derived SVC from SVM.

- Map data points to a feature space.
- Search for the smallest sphere enclosing all data points.
- Map the sphere back and generate clusters.



- $\{X_i\}_{1 \leq i \leq n}$ a data set of n points in the input space.
- $X_i \in \mathbb{R}^d$ and d is the number of attributes.
- We need to find the smallest sphere with radius R.
- The optimization formulation is the following:

$$
\begin{array}{ll}
min_{R,a,\xi_i} & R^2 + C\sum_i \xi_i \\
\text{Subject to:} & \\
 & \|\Phi(X_i) - a\|^2 \leq R^2 + \xi_i, \quad i = 1 \cdots n \\
 & \xi_i \geq 0
\end{array}
$$

- To solve this optimization problem, we right the Lagrangian:

$$L(R, a, \mu, \beta, \xi) = R^2 - \sum_i \left(R^2 + \xi_i - \|\Phi(X_i) - a\|^2\right) \beta_i - \sum_i \xi_i \mu_i + C \sum_i \xi_i$$

  Where $\beta_i \geq 0$ and $\mu_i \geq 0$ are the Lagrange multipliers.

- Using the Karush-Kuhn-Tucker complementary slackness conditions, the Lagrangian becomes:

$$L(\beta) = \sum_i \Phi(X_i)^2 \beta_i - \sum_{ij} \beta_i \beta_j \Phi(X_i) \cdot \Phi(X_j)$$

  With the constraints $0 \leq \beta_i \leq C$.

- Replacing the dot product with the Gaussian kernel function:

  $K\left(X_i, X_j\right) = e^{-q\left\|X_i - X_j\right\|^2}$

  Where q is the width parameter.

- we get:

$$\begin{array}{ll} max_{\beta} & \sum_i K(X_i, X_i)\beta_i - \sum_{ij} \beta_i \beta_j K(X_i, X_j) \\ \text{Subject to:} & \\ & 0 \leq \beta_i \leq C, \quad i = 1 \cdots n \\ & \sum_i \beta_i = 1 \end{array}$$

- The distance of each image in the feature space from the sphere center *a*:

$$
\begin{aligned}
R^2(X) &= \|\Phi(X_i) - a\|^2 \\
&= K(X, X) - 2\sum_i \beta_i K(X_i, X) + \sum_{ij} \beta_i \beta_j K(X_i, X_j)
\end{aligned}
$$

- The radius of the sphere is as follows:

$$R_{Sphere} = \frac{\sum_{X_i \text{ is } SV} R(X_i)}{Number\ of\ SVs}$$

- clusters are defined as the connected components of the graph induced by A, where A is the adjacency matrix defined by:

$$A_{ij} = \begin{cases} 1, & \text{if for all } y \in (X_i, X_j), \; R(y) \leq R_{sphere} \\ 0, & \text{Otherwise} \end{cases}$$

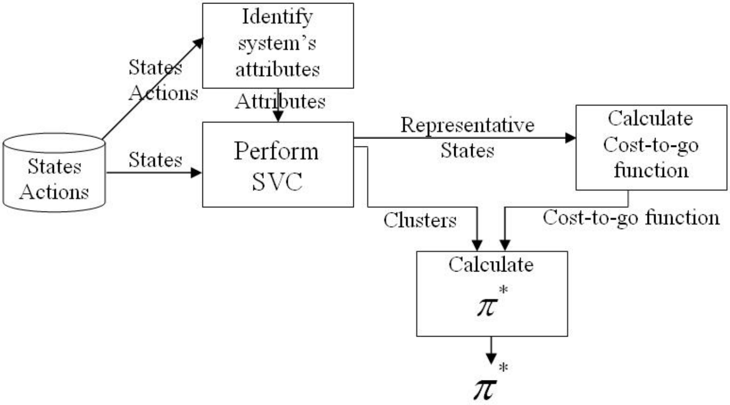


Figure: Reduced States' set diagram

Bellman's equation can be solved using the following Linear Dynamic Programming (LDP):

$$\max \quad c^T \mathcal{J}$$

$$\textit{subject to}: \; g_s^a + \gamma \sum_{s' \in S} \mathcal{P}_{ss'}^a \mathcal{J}(s') \geq \mathcal{J}(s), \; \forall s \in S, \forall a \in A(s)$$

Using the partitioning $S = \{S_1, S_2, \ldots, S_l\}$ we can reduced the LDP to:

$$\max \sum_{i=1}^{l} \overline{c}_i \mathcal{J}(\overline{s}_i)$$

$$\textit{subject to}: \; g^a_{\overline{s}_j} + \gamma \sum_{i=1}^{l} |S_i| \, \mathcal{P}^a_{s\overline{s}_i} \mathcal{J}(\overline{s}_i) \geq \mathcal{J}(\overline{s}_j), \forall j = 1 \cdots l, \forall a \in A(\overline{s}_j)$$

where $\overline{c}_i = \sum_{s \in S_i} c_s$

$\mathcal{J}^*$ is approximated using a linear combination of preselected basis functions:

$$\phi_k : S \mapsto \mathbb{R},\ k = 1, \ldots, K$$

$\Rightarrow$ Generate a weight vector $\tilde{r} \in \mathbb{R}^K$, such that:

$$\mathcal{J}^*(s) \approx \sum_{k=0}^{K} \phi_k(s)\tilde{r}$$

Using the vector $\tilde{r}$ we get the Linear Approximate DP (LADP):
$\max_{r} C^T \Phi r$
subject to

$$g^a_{ss''} + \gamma \sum_{s' \in S} \mathcal{P}^a_{ss'} (\Phi r)(s') \geq (\Phi r)(s), \ \forall s \in S, \forall a \in A(s)$$

where $\Phi = \begin{bmatrix} \phi_1(1) & \cdots & \phi_K(1) \\ \vdots & & \vdots \\ \phi_1(|S|) & \cdots & \phi_K(|S|) \end{bmatrix}$

Using the partitioning $S = \{S_1, S_2, \ldots, S_l\}$ we can reduced the LADP to:

max $\overline{c}^T \overline{\Phi} r$

$$\textit{subject to } g^a_{\overline{s}_j} + \gamma \sum_{i=1}^{l} |S_i| \, \mathcal{P}^a_{\overline{s}_j \overline{s}_i} \left(\overline{\Phi} r\right)(\overline{s}_i) \geq \left(\overline{\Phi} r\right)(\overline{s}_j), \forall j = 1 \cdots l, \forall a \in A(\overline{s}_j),$$

$$\text{where } \overline{\Phi} = \begin{bmatrix} \phi_1(\overline{s}_1) & \cdots & \phi_K(\overline{s}_1) \\ \vdots & & \vdots \\ \phi_1(\overline{s}_l) & \cdots & \phi_K(\overline{s}_l) \end{bmatrix}.$$

LADP Using Kernels

- The LADP yields good approximation to the value function
- How to choose the basis function and K
- We propose to use the partitioning $S = \{S_1, S_2, \ldots, S_l\}$ and set:

$$\phi_k(s) = \kappa(s, \bar{s}_k), \; \forall \; k = 1 \ldots l$$

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- $K = l$ is the number of clusters

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Experiments and Results

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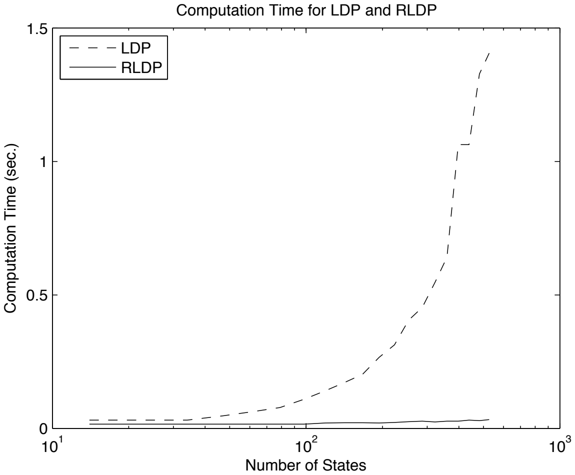

Figure: Computation Time -LDP Vs RLDP-

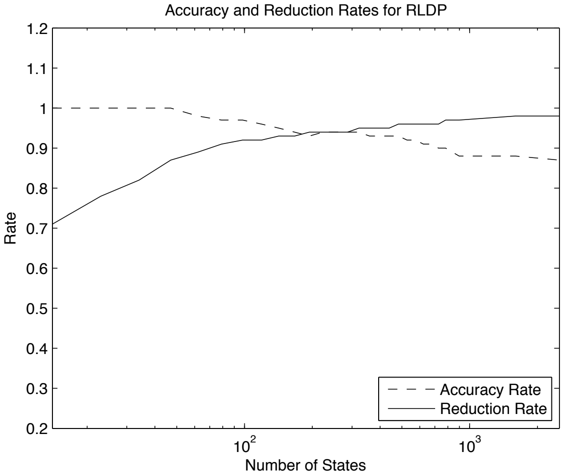

Figure: Accuracy and Reduction Rates -LDP Vs RLDP-

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$$\max \; c^T x$$

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Robust Linear Dynamic Programming

- Use Bertsimas and Sim's formulation to account for uncertainty within the transition probability matrices
- We assume that each entry $\mathcal{P}^a_{ss'}$ is modeled as a symmetric and bounded random variable $\tilde{\mathcal{P}}^a_{ss'}$, such that:

$$\tilde{\mathcal{P}}^a_{ss'} \in \left[\mathcal{P}^a_{ss'} - \hat{\mathcal{P}}^a_{ss'}, \mathcal{P}^a_{ss'} + \hat{\mathcal{P}}^a_{ij}\right]$$

- We set $\alpha^a_{s^w s'^{w'}} = (1 - \delta_{s^w s'^{w'}}) \frac{\delta_{s^w s'^{w'}} - \gamma \mathcal{P}^a_{s^w s'^{w'}}}{q + (1-q)\delta_{s^w s'^{w'}}}$ and $\hat{\alpha}^a_{s^w s'^{w'}} = (1 - \delta_{s^w s'^{w'}}) \frac{\gamma \hat{\mathcal{P}}^a_{s^w s'^{w'}}}{q + (1-q)\delta_{s^w s'^{w'}}}$

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- We set $\alpha^a_{s^w s'^{w'}} = (1 - \delta_{s^w s'^{w'}}) \frac{\delta_{s^w s'^{w'}} - \gamma \mathcal{P}^a_{s^w s'^{w'}}}{q + (1-q)\delta_{s^w s'^{w'}}}$ and $\hat{\alpha}^a_{s^w s'^{w'}} = (1 - \delta_{s^w s'^{w'}}) \frac{\gamma \hat{\mathcal{P}}^a_{s^w s'^{w'}}}{q + (1-q)\delta_{s^w s'^{w'}}}$

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The Robust Linear Dynamic Programming is the following:

$$
\begin{aligned}
\max \quad & c_\omega^T \mathcal{J}_\omega && (1)\\
\textit{subject to}: & \sum_{j=1}^{|S|}\sum_{v=1}^{q} \alpha^a_{s^w s^v_j}\mathcal{J}(s^v_j) + z^{aw}_s \Gamma^{aw}_s + \sum_{j=1}^{|S|}\sum_{v=1}^{q} p_{s^w s^v_j} \le g^a_{s^w}, \ \forall\ s^w, a && (2)\\
& z^{aw}_s + p_{s^w s^v_j} \ge \hat{\alpha}^a_{s^w s^v_j} y^v_j, \ \forall\ s^w, a, j, v && (3)\\
& -y^v_j \le \mathcal{J}(s^v_j) \le y^v_j, \ \forall\ j, v && (4)\\
& p_{s^w s^v_j} \ge 0, \ \forall\ s^w, j, v && (5)\\
& z \ge 0 && (6)\\
& y \ge 0. && (7)
\end{aligned}
$$

The Reduced Robust Linear Dynamic Programming is the following:

$$\max \quad \sum_{i=1}^{I} \frac{\overline{c}_i}{q} \sum_{v=1}^{q} \mathcal{J}(\overline{s}_i) \tag{8}$$

$$\textit{subject to}: \sum_{j=1}^{I} \sum_{v=1}^{q} |\overline{S}_j| \alpha^{a}_{\overline{s}_i^w \overline{s}_j^v} \mathcal{J}(\overline{s}_j^v) + z^{aw}_{\overline{s}_i} \Gamma^{aw}_{\overline{s}_i} + \sum_{j=1}^{I} \sum_{v=1}^{q} p_{s^w s_j^v} \leq g^{a}_{\overline{s}_i^w}, \ \forall\ \overline{s}_i^w, a \tag{9}$$

$$z^{aw}_{\overline{s}_i} + p_{\overline{s}_i^w \overline{s}_j^v} \geq \hat{\alpha}^{a}_{\overline{s}_i^w \overline{s}_j^v} y_j^v, \ \forall\ \overline{s}_i^w, a, j, v \tag{10}$$

$$-y_j^v \leq \mathcal{J}(\overline{s}_j^v) \leq y_j^v, \ \forall\ j, v \tag{11}$$

$$p_{\overline{s}_i^w \overline{s}_j^v} \geq 0, \ \forall\ \overline{s}_i^w, j, v \tag{12}$$

$$z \geq 0 \tag{13}$$

$$y \geq 0. \tag{14}$$

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- K is the strike price
- T is the expiration date
- x_t is the price of the underlying asset at t
- r is the risk free interest rate
- σ is the volatility
- The intrinsic value of a put option for the holder is:

$$f(x_t) = \begin{cases} K - x_t, & \text{if } x_t \leq K \text{ and } t \leq T \\ 0, & \text{otherwise} \end{cases}$$

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Example: Strategy for American Options

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- The value function is the expected value of the intrinsic value under the risk neutral assumption:

$$\mathcal{J}(x_t) = \max_{t \in [0,T]} E_{\pi} \left\{ e^{-rt} f(x_t) \right\}$$

- The evolution of x_t is simulated using the Binomial Options Pricing Model (BOPM)

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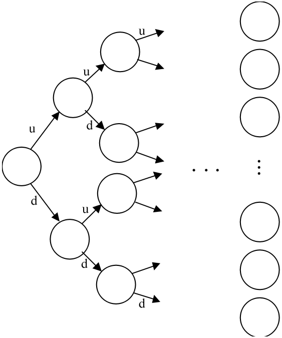

Figure: The Binomial Options Pricing Model

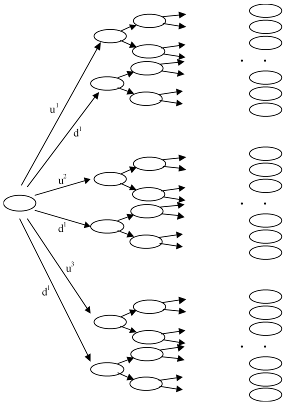

Figure: The Robust Binomial Options Pricing Model

The parameters used are:

	$K = 1$	$x_0 = .5$	$e^{-r} = .99$	
	Low Volatility	Moderate Volatility	High Volatility	Average
u	$\frac{13}{12}$	$\frac{8}{6}$	$\frac{9}{5}$	1.406
d	$\frac{12}{13}$	$\frac{6}{8}$	$\frac{5}{9}$	.711

Table: Parameters Used for the Exercising Strategy

The intrinsic value for this example is:

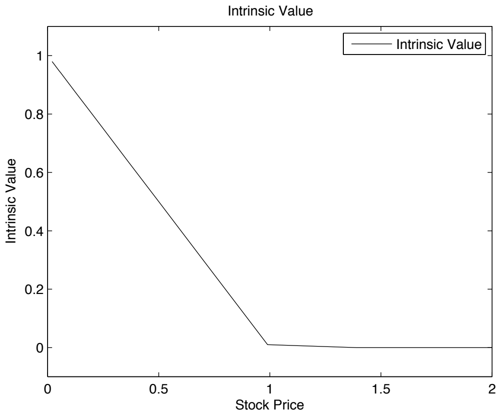

Figure: The Intrinsic Value for the Put Option

Time	Low Volatility	Moderate Volatility	High Volatility	Average
1	0.393	0.281	0.154	0.253
2	0.393	0.281	0.154	0.253
3	0.393	0.281	.278	0.253
4	0.393	0.281	.278	0.356
5	0.426	0.375	.278	0.356
6	0.426	0.375	.278	0.356
7	0.461	0.375	.278	0.356
8	0.5	0.5	0.5	0.5
9	0.7	0.7	0.7	0.7
10	1	1	1	1

Table: Exercising Strategies -Robust Vs Average-

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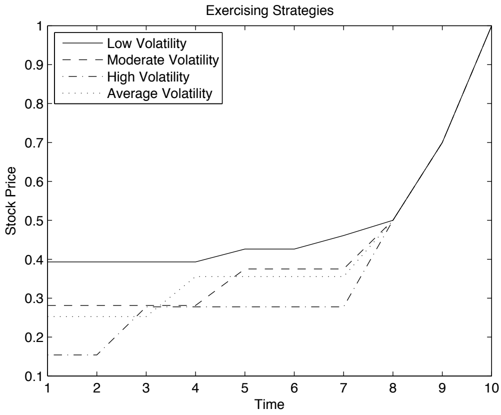

Figure: Exercising Strategies for the Robust Formulation and the Average

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- Coped with this problem using clustering
- Conducted a mathematical analysis to support the results obtained
- Developed an on line dynamic programming procedure
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Questions

Thank you