Modular Korteweg - de Vries equation: Riemann, cnoidal and solitary waves

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The review paper by Oleg Rudenko [1] suggests several examples of elastic systems with so-called modular nonlinearities. In this study we consider the modular Korteweg - de Vries (KdV) equation in the form $u_t + 6 u u_x + u_{xxx} = 0$. This equation is not integrable by means of the Inverse Scattering Transform in the general case, but sign-defined functions which never change the sign satisfy the integrable KdV equation, and hence possess an exact solution. Firstly, we consider the dispersionless limit of the modular KdV equation and analyze the evolution of a simple nonlinear wave (Riemann wave) and its Fourier transform including the asymptotics when the wave tends to break [2]. Then, we study the structure of travelling waves. If the waves propagate on a pedestal and do not cross the zero level $u = 0$, they coincide with the well-known travelling wave solutions of the classic KdV equation in the form of cnoidal and solitary waves. If the pedestal is zero, the structure of sign-varying travelling waves is expressed through Jacobi elliptic functions. The interaction of solitary waves of different polarities is studied numerically using an implicit pseudo-spectral method. The simulation has revealed the inelastic character of the collision; in the course of the interaction the solitons can alter their amplitudes (the small soliton decreases and the large one grows) and emit small-amplitude waves. The inelastic effects are most pronounced when the solitons' amplitudes are close. When their amplitudes differ significantly, the maximum wave height which is attained during the absorb-emit interaction tends to the sum of the heights of the solitons with the polarity inherited from the large soliton, as predicted in the frameworks of different long-wave integrable models in [3, 4]. As a result of the collision the solitons may experience non-classic phase shifts as they both jump back.

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