Equilibrium and Optimality in International Trade Models under Monopolistic Competition: the Unified Approach

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Highlights

- The homogeneous model of international trade under the monopolistic competition (MC) of producers.
- The idea of Sergey Kokovin (NRU HSE): the search for equilibrium is equivalent to the problem of optimization, but revenue, not utility.
- The case of two countries:
  - in equilibrium, the elasticity of production costs is convex combination of the elasticities of revenue in individual consumption;
  - in optimality, the elasticity of production costs is convex combination of the elasticities of sub-utility of individual consumption.
- These generalize the well-known facts in closed economy under MC:
  - in equilibrium, the elasticity of revenue equals the elasticity of costs;
  - in optimality, the elasticity of utility equals the elasticity of costs.
- In symmetric market equilibrium, the "inverse" elasticities of production costs is a convex combination of the "inverse" elasticities of normalized revenue in individual consumption. This can be generalized in the case of international trade of several countries.

Outline

Motivation, goal

- The concept of **monopolistic competition:** E.Chamberlin (1933, 1962).
- Market **equilibrium**:
  - A.Dixit & J.Stiglitz (AER, 1977) - 3029 citations in WoS, Nobel Prize in Economics (2001, only Stiglitz)
  - P.Krugman (JIE, 1979, Trade) - 1191 citations in WoS, (AER, 1980, Pattern of Trade) - 1647 citations in WoS, Nobel Prize in Economics (2008)
  - M.Melitz (Econometrica, 2003, Heterogeneous) - 3924 citations in WoS, no Nobel Prize in Economics (yet?)
- Social **optimality**: S.Dhingra & J.Morrow (2019 Journal of Political Economy, 12 citations in WoS), etc.
- In the presented paper: **unified approach** to market equilibrium and social optimality

MC Trade Model assumptions (as in Dixit-Stiglitz-Krugman)

- Consumers: ***identical***, each endowed with ***one*** unit of labor.
- Labor is the only production factor; Consumption, output, prices etc. are measured in ***labor***.
- Firms are ***identical***, but produce "***varieties***" ("almost the same") of good.
- Each firm produces one variety as a price-maker, but its demand is influenced by other varieties.
- Each variety is produced by ***one*** firm that produces a ***single*** variety.
- Each demand function results from ***additive utility*** function
- ***Number (mass) of firms is big enough*** to ignore firm's influence on the whole industry/economy.
- ***Free entry*** drives all profits to zero.
- Labor supply/demand in each country is balanced, trade is balanced.
- Let be two countries, H ("big") and F ("small").

Consumers

- L^H : the number of consumers in country H.
- $L^F \leq L^H$: the number of consumers in country F.
- N^H : the endogenous mass of firms in country H.
- N^F : the endogenous mass of firms in country F.
- $\forall\, k, l \in \{H, F\}$:
    - $x_i^{kl} \equiv x^{kl}(i)$: the amount of the variety produced in country k by firm $i \in [0, N^k]$ and consumed by a consumer in country l (***individual*** consumption);
    - $p_i^{kl} \equiv p^{kl}(i)$: the price of the unit of the variety produced in country k by firm $i \in [0, N^k]$ and consumed by a consumer in country l.
- $w^H = w$: the wage rate in country H.
- w^F : the wage rate in country F, normalizing to one, $w^F = 1$.
- $u(\cdot)$: a sub-utility function, $u(0) = 0$, $u'(\xi) > 0$, $u''(\xi) < 0$.

Inverse demand functions

- In country H, the problem of representative consumer is

$$\int_0^{N^H} u\left(x_i^{HH}\right) di + \int_0^{N^F} u\left(x_i^{FH}\right) di \to \max$$

s.t.

$$\int_0^{N^H} p_i^{HH} x_i^{HH} di + \int_0^{N^F} p_i^{FH} x_i^{FH} di \leq w$$

- In country F, the problem of representative consumer is

$$\int_0^{N^F} u\left(x_i^{FF}\right) di + \int_0^{N^H} u\left(x_i^{HF}\right) di \to \max$$

s.t.

$$\int_0^{N^F} p_i^{FF} x_i^{FF} di + \int_0^{N^H} p_i^{HF} x_i^{HF} di \leq 1$$

- $\Longrightarrow$ the inverse demand functions:

$$p_i^{kl}\left(x_i^{kl}, \lambda^l\right) = \frac{u'\left(x_i^{kl}\right)}{\lambda^l}, \; i \in [0, N^k], \; k, l \in \{H, F\}$$

where λ^H, λ^F are Lagrange multipliers.

Producers

- $\tau \geq 1$ trade cost of "iceberg" type (to sell 1 in another country, the firm has to produce $\tau \cdot 1$)
- The **size** of firm $i \in [0, N^H]$ in country H : $Q_i^H = L^H x_i^{HH} + \tau L^F x_i^{HF}$
- The **size** of firm $i \in [0, N^F]$ in country F : $Q_i^F = L^F x_i^{FF} + \tau L^H x_i^{FH}$
- **Production costs** for each firm in each country: the ***strictly increasing*** and ***twice differentiable*** function V
- **Profit** of firm i in country H:

$$\pi_i^H = L^H R_{\lambda^H}\left(x_i^{HH}\right) + L^F R_{\lambda^F}\left(x_i^{HF}\right) - wV\left(Q_i^H\right), \; i \in [0, N^H]$$

profit of firm i in country F:

$$\pi_i^F = L^F R_{\lambda^F}\left(x_i^{FF}\right) + L^H R_{\lambda^H}\left(x_i^{FH}\right) - V\left(Q_i^F\right), \; i \in [0, N^F]$$

- where $R_\lambda(\xi) := \frac{u'(\xi)\cdot\xi}{\lambda}$ - **"Revenue per consumer"**
- **Labor balance** in country H : $\int_0^{N^H} V\left(Q_i^H\right) di = L^H$
- **Labor balance** in country F : $\int_0^{N^F} V\left(Q_i^F\right) di = L^F$

Symmetric case: consumption, inverse demand, firm's size, profit, Labor Balance

- Recall: **consumers** are **identical**, **producers** are **identical**.
- $\Longrightarrow$ consider the **symmetric** case, i.e., omit index i in consumptions: $x^{kl}(i) = x^{kl}\ \forall i. \Longrightarrow$
- inverse demand: $p^{kl}\left(x^{kl}, \lambda^{l}\right) = \frac{u'\left(x^{kl}\right)}{\lambda^{l}}$, $k, l \in \{H, F\}$
- sizes of the firms:

$$Q^{H} = L^{H}x^{HH} + \tau L^{F}x^{HF}, \quad Q^{F} = L^{F}x^{FF} + \tau L^{H}x^{FH}$$

- profits:

$$\pi^{H} = L^{H}R_{\lambda^{H}}\left(x^{HH}\right) + L^{F}R_{\lambda^{F}}\left(x^{HF}\right) - wV\left(Q^{H}\right)$$

$$\pi^{F} = L^{F}R_{\lambda^{F}}\left(x^{FF}\right) + L^{H}R_{\lambda^{H}}\left(x^{FH}\right) - V\left(Q^{F}\right)$$

- Labor Balances: $N^{H}V\left(Q^{H}\right) = L^{H}, \quad N^{F}V\left(Q^{F}\right) = L^{F}$

Symmetric case: Trade Balance, Social Welfare

- Trade Balance (“export equals import”):

$$L^F N^H p^{HF}\left(x^{HF},\lambda^F\right)x^{HF} = L^H N^F p^{FH}\left(x^{FH},\lambda^H\right)x^{FH},$$

i.e.,

$$R_{\lambda^F}\left(x^{HF}\right)V\left(Q^F\right) = R_{\lambda^H}\left(x^{FH}\right)V\left(Q^H\right)$$

- Social Welfare (total utility):

$$U = L^H\cdot\left(N^H u\left(x^{HH}\right) + N^F u\left(x^{FH}\right)\right) + L^F\cdot\left(N^F u\left(x^{FF}\right) + N^H u\left(x^{HF}\right)\right) =$$

$$\left(L^H u\left(x^{HH}\right) + L^F u\left(x^{HF}\right)\right)N^H + \left(L^F u\left(x^{FF}\right) + L^H u\left(x^{FH}\right)\right)N^F,$$

i.e.,

$$U = L^H\cdot\frac{L^H u\left(x^{HH}\right) + L^F u\left(x^{HF}\right)}{V\left(Q^H\right)} + L^F\cdot\frac{L^F u\left(x^{FF}\right) + L^H u\left(x^{FH}\right)}{V\left(Q^F\right)}$$

Symmetric Equilibrium: preliminaries

- For equilibrium, producers choose inverse demand and maximize profits
- $\Longrightarrow$ First Order Conditions (FOC) for producers:

$$\frac{\partial \pi^H}{\partial x^{HH}} = 0, \quad \frac{\partial \pi^H}{\partial x^{HF}} = 0, \quad \frac{\partial \pi^F}{\partial x^{FF}} = 0, \quad \frac{\partial \pi^F}{\partial x^{FH}} = 0$$

and Second Order Conditions (SOC) for producers:

$$\frac{\partial^2 \pi^H}{\partial x^{HH} \partial x^{HH}} < 0, \quad \frac{\partial^2 \pi^H}{\partial x^{HH} \partial x^{HH}} \cdot \frac{\partial^2 \pi^H}{\partial x^{HF} \partial x^{HF}} - \left(\frac{\partial^2 \pi^H}{\partial x^{HH} \partial x^{HF}} \right)^2 > 0$$

$$\frac{\partial^2 \pi^F}{\partial x^{FF} \partial x^{FF}} < 0, \quad \frac{\partial^2 \pi^F}{\partial x^{FF} \partial x^{FF}} \cdot \frac{\partial^2 \pi^F}{\partial x^{FH} \partial x^{FH}} - \left(\frac{\partial^2 \pi^F}{\partial x^{FF} \partial x^{FH}} \right)^2 > 0$$

- Firms enter into the market until their profit remains positive. $\Longrightarrow$ free entry implies the zero-profit condition

$$\pi^H = 0, \qquad \pi^F = 0$$

Symmetric Equilibrium: definition

- By definition, the **symmetric market equilibrium** is a bundle

$$\left(x_{equ}^{HH}, x_{equ}^{HF}, x_{equ}^{FF}, x_{equ}^{FH}, p_{equ}^{HH}, p_{equ}^{HF}, p_{equ}^{FF}, p_{equ}^{FH}, \lambda_{equ}^{H}, \lambda_{equ}^{F}, N_{equ}^{H}, N_{equ}^{F}, w_{equ}\right)$$

satisfying

- rational consumption conditions ($\Longrightarrow$inverse demand)
- rational production conditions (FOC and SOC for producers)
- free entry (zero-profit) condition
- labor balance conditions
- the trade balance condition

Symmetric Optimality

- For optimality, the Social Welfare is maximized. $\Longrightarrow$
  - First Order Conditions (FOC)

$$\frac{\partial U}{\partial x^{HH}} = 0, \quad \frac{\partial U}{\partial x^{HF}} = 0, \quad \frac{\partial U}{\partial x^{FF}} = 0, \quad \frac{\partial U}{\partial x^{FH}} = 0$$

  - and Second Order Conditions (SOC)

$$\frac{\partial^2 U}{\partial x^{HH} \partial x^{HH}} < 0, \quad \frac{\partial^2 U}{\partial x^{HH} \partial x^{HH}} \cdot \frac{\partial^2 U}{\partial x^{HF} \partial x^{HF}} - \left(\frac{\partial^2 U}{\partial x^{HH} \partial x^{HF}} \right)^2 > 0$$

$$\frac{\partial^2 U}{\partial x^{FF} \partial x^{FF}} < 0, \quad \frac{\partial^2 U}{\partial x^{FF} \partial x^{FF}} \cdot \frac{\partial^2 U}{\partial x^{FH} \partial x^{FH}} - \left(\frac{\partial^2 U}{\partial x^{FF} \partial x^{FH}} \right)^2 > 0$$

- The **symmetric social optimality** is a bundle

$$\left(x^{HH}_{opt}, x^{HF}_{opt}, x^{FF}_{opt}, x^{FH}_{opt}, N^{H}_{opt}, N^{F}_{opt} \right)$$

satisfying

- rational welfare conditions (FOC and SOC for welfare);
- labor balance conditions

Results: preliminaries (I)

- As usual, let

$$\mathscr{E}_g(\xi) = \frac{g'(\xi)\xi}{g(\xi)}$$

be the elasticity of function g. Note that

$$\mathscr{E}_{R_{\lambda^H}}(\xi) = \mathscr{E}_{R_{\lambda^F}}(\xi) = \mathscr{E}_R(\xi)$$

where $R(\xi) = u'(\xi)\xi$ is "normalized" revenue.

- Let

$$q^H_{equ} = L^H x^{HH}_{equ}, \quad q^F_{equ} = L^F x^{FF}_{equ}, \quad q^H_{opt} = L^H x^{HH}_{opt}, \quad q^F_{opt} = L^F x^{FF}_{opt}$$

be total domestic consumptions while

$$Q^H_{equ} = L^H x^{HH}_{equ} + \tau L^F \cdot x^{HF}_{equ}, \quad Q^F_{equ} = L^F x^{FF}_{equ} + \tau L^H x^{FH}_{equ}$$

$$Q^H_{opt} = L^H x^{HH}_{opt} + \tau L^F \cdot x^{HF}_{opt}, \quad Q^F_{opt} = L^F x^{FF}_{opt} + \tau L^H x^{FH}_{opt}$$

be firm sizes.

Results: preliminaries (II)

- Let us introduce

$$s^H\left(x^{HH}, x^{HF}, A^H, A^F\right) := \frac{L^H A^H\left(x^{HH}\right)}{L^H A^H\left(x^{HH}\right) + L^F A^F\left(x^{HF}\right)}$$

$$s^F\left(x^{FF}, x^{FH}, A^H, A^F\right) := \frac{L^F A^F\left(x^{FF}\right)}{L^F A^F\left(x^{FF}\right) + L^H A^H\left(x^{FH}\right)}$$

- Let us denote

$$s^H_{equ} = s^H\left(x^{HH}_{equ}, x^{HF}_{equ}, R_{\lambda^H_{equ}}, R_{\lambda^F_{equ}}\right), \quad s^F_{equ} = s^F\left(x^{FF}_{equ}, x^{FH}_{equ}, R_{\lambda^H_{equ}}, R_{\lambda^F_{equ}}\right)$$

$$s^H_{opt} = s^H\left(x^{HH}_{opt}, x^{HF}_{opt}, u, u\right), \quad s^F_{opt} = s^F\left(x^{FF}_{opt}, x^{FH}_{opt}, u, u\right)$$

- In propositions below, we assume that market equilibrium and social optimality exist (and, moreover, are unique).The questions of the existence and uniqueness of equilibrium and optimality are separate problems (often not quite simple).

Results: Proposition 1, equilibrium

- **Proposition 1, equilibrium.** In symmetric market equilibrium, the elasticities of normalized revenue in individual consumptions and the elasticities of production costs satisfy the conditions

$$s^H_{equ} \cdot \mathscr{E}_R\left(x^{HH}_{equ}\right) = \frac{q^H_{equ}}{Q^H_{equ}} \cdot \mathscr{E}_V\left(Q^H_{equ}\right)$$

$$\left(1 - s^H_{equ}\right) \cdot \mathscr{E}_R\left(x^{HF}_{equ}\right) = \left(1 - \frac{q^H_{equ}}{Q^H_{equ}}\right) \cdot \mathscr{E}_V\left(Q^H_{equ}\right)$$

$$s^F_{equ} \cdot \mathscr{E}_R\left(x^{FF}_{equ}\right) = \frac{q^F_{equ}}{Q^F_{equ}} \cdot \mathscr{E}_V\left(Q^F_{equ}\right)$$

$$\left(1 - s^F_{equ}\right) \cdot \mathscr{E}_R\left(x^{FH}_{equ}\right) = \left(1 - \frac{q^F_{equ}}{Q^F_{equ}}\right) \cdot \mathscr{E}_V\left(Q^F_{equ}\right)$$

Results: Proposition 1, optimality

- **Proposition 1, optimality.** In symmetric social optimality, the elasticities of sub-utility of individual consumptions and the elasticities of production costs satisfy the conditions

$$s^H_{opt} \cdot \mathcal{E}_u\left(x^{HH}_{opt}\right) = \frac{q^H_{opt}}{Q^H_{opt}} \cdot \mathcal{E}_V\left(Q^H_{opt}\right)$$

$$\left(1 - s^H_{opt}\right) \cdot \mathcal{E}_u\left(x^{HF}_{opt}\right) = \left(1 - \frac{q^H_{opt}}{Q^H_{opt}}\right) \cdot \mathcal{E}_V\left(Q^H_{opt}\right)$$

$$\left(1 - s^F_{opt}\right) \cdot \mathcal{E}_u\left(x^{FF}_{opt}\right) = \frac{q^F_{opt}}{Q^F_{opt}} \cdot \mathcal{E}_V\left(Q^F_{opt}\right)$$

$$s^F_{opt} \cdot \mathcal{E}_u\left(x^{FH}_{opt}\right) = \left(1 - \frac{q^F_{opt}}{Q^F_{opt}}\right) \cdot \mathcal{E}_V\left(Q^F_{opt}\right)$$

Results: Corollary

- The well-known facts in closed economy monopolistic competition: "in equilibrium, the elasticity of revenue equals the elasticity of total costs" and "in optimality, the elasticity of utility equals the elasticity of total costs". Now,
- **Corollary, equilibrium.** In symmetric market equilibrium, the elasticity of production costs is a convex combination of the elasticities of normalized revenue in individual consumption, i.e.,

$$s^H_{equ} \cdot \mathcal{E}_R\left(x^{HH}_{equ}\right) + \left(1 - s^H_{equ}\right) \cdot \mathcal{E}_R\left(x^{HF}_{equ}\right) = \mathcal{E}_V\left(Q^H_{equ}\right)$$
$$s^F_{equ} \cdot \mathcal{E}_R\left(x^{FF}_{equ}\right) + \left(1 - s^F_{equ}\right) \cdot \mathcal{E}_R\left(x^{FH}_{equ}\right) = \mathcal{E}_V\left(Q^F_{equ}\right)$$

- **Corollary, optimality.** In symmetric social optimality, the elasticity of production costs is a convex combination of the elasticities of sub-utility of individual consumption, i.e.,

$$s^H_{opt} \cdot \mathcal{E}_u\left(x^{HH}_{opt}\right) + \left(1 - s^H_{opt}\right) \cdot \mathcal{E}_u\left(x^{HF}_{opt}\right) = \mathcal{E}_V\left(Q^H_{opt}\right)$$
$$s^F_{opt} \cdot \mathcal{E}_u\left(x^{FF}_{opt}\right) + \left(1 - s^F_{opt}\right) \cdot \mathcal{E}_u\left(x^{FH}_{opt}\right) = \mathcal{E}_V\left(Q^F_{opt}\right)$$

Results: Proposition 2

- Let us note that the obvious disadvantage of the formulas in Proposition 1 and Corollary is the poor interpretability of the coefficients. We hope that the proposition below does not have this disadvantage.
- **Proposition 2.** In symmetric market equilibrium, the elasticities of normalized revenue in individual consumptions and the elasticities of production costs satisfy the conditions

$$\frac{q^H_{equ}}{Q^H_{equ}} \cdot \frac{1}{\mathcal{E}_R\left(x^{HH}_{equ}\right)} + \left(1 - \frac{q^H_{equ}}{Q^H_{equ}}\right) \cdot \frac{1}{\mathcal{E}_R\left(x^{HF}_{equ}\right)} = \frac{1}{\mathcal{E}_V\left(Q^H_{equ}\right)}$$

$$\frac{q^F_{equ}}{Q^F_{equ}} \cdot \frac{1}{\mathcal{E}_R\left(x^{FF}_{equ}\right)} + \left(1 - \frac{q^F_{equ}}{Q^F_{equ}}\right) \cdot \frac{1}{\mathcal{E}_R\left(x^{FH}_{equ}\right)} = \frac{1}{\mathcal{E}_V\left(Q^F_{equ}\right)}$$

- Moreover, Proposition 2 can be generalized to the case of several countries.

The case of several countries: preliminaries

Let, for K countries, $I = \{1, \ldots, K\}$.Let, for $k \in I$ and $l \in I$,

- N^kbe the mass of firms in country k,
- x_i^{kl}be the amount of variety produced in country k by firm $i \in [0, N^k]$ and consumed in country l by a consumer,
- p^{kl}be the corresponding prices,
- L^kbe the number of consumer in country k,
- $q_i^{kl} = \tau^{kl} L^l x_i^{kl}$be the total output for firm $i \in [0, N^k]$ in country k for selling in country$l$, $\tau^{kl} \geq 1$, $\tau^{kk} = 1$,
- w^k be the wage in country k.

The case of several countries: consumer

- The problem of representative consumer in country $k \in I$ is

$$\sum_{l \in I} \int_0^{N^l} u\left(x_i^{lk}\right) di \to max$$

s.t.

$$\sum_{l \in I} \int_0^{N^l} p_i^{lk} x_i^{lk} di \leq w^k$$

- For symmetric case, FOC is

$$u'\left(x^{lk}\right) - \lambda^k p^{lk} = 0$$

whereλ^k is the corresponding Lagrange multiplier.

The case of several countries: producer

- For a producer in country $k \in I$, total output is

$$Q^k = \sum_{l \in I} q^{kl}$$

Let us substitute the inverse demand function

$$p^{kl} = \frac{u'\left(x^{kl}\right)}{\lambda^l}$$

in profits. Then the profit of a producer in country k is

$$\pi^k = \sum_{l \in I} \frac{L^l}{\lambda^l} \cdot R\left(x^{kl}\right) - w^k \cdot V\left(Q^k\right)$$

(Let us recall that $R(\xi) = u'(\xi) \cdot \xi$ is "normalized" revenue.)

The case of several countries: producer, equilibrium

- To define symmetric equilibrium, as usual, we write FOC and SOC, free entry conditions, labor and trade balances. It turns out that first second order conditions and free entry conditions allow to generalize Proposition 2.
- Let x_{equ}^{kl} be equilibrium consumption and

$$q_{equ}^{kl} = \tau^{kl} L^{l} x_{equ}^{kl}$$

Proposition 3. For country $k \in I$, in symmetric market equilibrium, the elasticities of normalized revenue in individual consumptions and the elasticities of production costs satisfy the condition

$$\sum_{l \in I} \frac{q_{equ}^{kl}}{Q_{equ}^{k}} \cdot \frac{1}{\mathcal{E}_R \left(x_{equ}^{kl} \right)} = \frac{1}{\mathcal{E}_V \left(Q_{equ}^{k} \right)}$$

Conclusion (I)

- We study, in the monopolistic competition framework, the homogeneous model of international trade with additively separable utility function for each consumer.
- One of the most interesting topic in these studies is the so-called "comparative statics", i.e., the influence of the models' parameters (market size, transport costs, etc.) on the equilibrium and optimal variables: consumption, firm sizes, market sizes, social welfare, etc.
- Instead, we study a unified approach to both market equilibrium and social optimality.

Conclusion (II)

For the case of international trade between ***two*** countries,

- in symmetric market equilibrium, the elasticities of production costs and the elasticities of normalized revenue in individual consumptions; moreover, the elasticity of production costs is a convex combination of the elasticities of normalized revenue in individual consumption;
- in symmetric social optimality, the elasticities of production costs and the elasticities of sub-utility of individual consumptions; moreover, the elasticity of production costs is a convex combination of the elasticities of sub-utility of individual consumption;
- in symmetric market equilibrium, the "inverse" elasticities of production costs is a convex combination of the "inverse" elasticities of normalized revenue in individual consumption; moreover, the coefficients of this convex combination have a clear meaning: they are the ratio of total domestic consumption to the size of the firm.
- The last result generalizes to the case of international trade between several countries.

Conclusion (III)

- Therefore, we generalize the well-known facts in closed economy monopolistic competition: "in equilibrium, the elasticity of revenue equals the elasticity of total costs" and and "in optimality, the elasticity of utility the elasticity of total cost". It can allow to clarify the nature of these concepts.
- Finally, it would be also glad to know whether the best choice for the two economies can gives the best choice for each economy separately. This can be the topic of future research.

THANK YOU FOR THE ATTENTION!