Around Baillon’s theorem on maximal regularity
F. L. Schwenninger, B. Jacob, J. Wintermayr

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Maximal regularity of an abstract Cauchy problem
\[
\frac{d}{dt} x(t) = Ax(t) + f(t), \quad t > 0, \quad x(0) = 0,
\]
which refers to the property that the regularity of the inhomogeneity \(f\) is preserved by \(\frac{d}{dt}x\) and \(Ax\), where \(A\) generates a strongly continuous semigroup on a Banach space \(X\), is omnipresent in the study of (parabolic) evolution equations. In contrast to regularity with respect to \(L^p\), for \(p \in (1, \infty)\), Baillon’s theorem [1] states that it is rare when considered with respect to supremum norms; it only occurs when the corresponding semigroup is uniformly continuous or the geometry of the state space is sufficiently coarse, that is, \(X\) contains \(c_0\). In this talk we show that the latter alternative can be excluded for a refined notion, which is based on an analogy to the concept of admissibility, a notion appearing in infinite-dimensional systems theory. We moreover comment on the “dual” situation of maximal regularity with respect to \(L^1\).

References: