Around Baillon's theorem on maximal regularity

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Maximal regularity of an abstract Cauchy problem

$$\frac{\mathrm{d}}{\mathrm{dt}}x(t) = Ax(t) + f(t), \quad t > 0, \quad x(0) = 0,$$

which refers to the property that the regularity of the inhomogeneity f is preserved by $\frac{\mathrm{d}}{\mathrm{dt}}x$ and Ax, where A generates a strongly continuous semigroup on a Banach space X, is omnipresent in the study of (parabolic) evolution equations. In contrast to regularity with respect to L^p, for $p \in (1, \infty)$, Baillon's theorem [1] states that it is rare when considered with respect to supremum norms; it only occurs when the corresponding semigroup is uniformly continuous or the geometry of the state space is sufficiently coarse, that is, X contains c_0. In this talk we show that the latter alternative can be excluded for a refined notion, which is based on an analogy to the concept of admissibility, a notion appearing in infinite-dimensional systems theory. We moreover comment on the "dual" situation of maximal regularity with respect to L^1.

References:

[1] J.-B. Baillon. Caractère borné de certains générateurs de semi-groupes linéaires dans les espaces de Banach. // C. R. Acad. Sci. Paris Sér. A-B. 1980. Vol. 290 No. 16 P. A757–A760.

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