Approximation of fractional equations in Banach spaces

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In this talk we have a deal with the well-posedness and approximation for nonhomogeneous fractional differential equations in Banach spaces E:

$$(\mathbf{D}_t^\alpha u)(t) = Au(t) + f(t), \quad t \in [0,T]; \quad u(0) = x,$$

where $\mathbf{D}_t^\alpha$ is the Caputo-Dzhrbashyan derivative $0 < \alpha < 1$, the operator A generates analytic C_0-semigroup, the function $f(\cdot) : [0,T] \to E$ is smooth enough.

The same way as in [1-2] we get the necessary and sufficient condition for the coercive well-posedness of nonhomogeneous fractional Cauchy problems in the spaces $C_0^\beta([0,T];E)$, $L^p[0,T];E$. Then using implicit difference scheme and explicit difference scheme, we deal with the full discretization of the solutions of nonhomogeneous and semilinear fractional differential equations in time variables and as in [3] we get the stability of the schemes and the order of convergence.

We discuss also the full discretization of autonomous semilinear problem.

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