Subordination for Semigroups in locally convex Spaces

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Introduction: Subordination in the sene of Bochner. Let X be a Banach space and $(T(t))_{t\geq 0}$ a bounded C_0-semigroup on X with generator $-A$. Given a so-called Bernstein function f, there exists a unique vaguely continuous convolution semigroup of sub-probability measures $(\mu_t)_{t\geq 0}$ on $[0,\infty)$ associated with f, and we can define a new semigroup $(S(t))_{t\geq 0}$ by

$$S(t)x := \int_{[0,\infty)} T(s)x \,\mathrm{d}\mu_t \quad (x \in X, t \geq 0).$$

It turns out that $(S(t))_{t\geq 0}$ is strongly continuous and its generator is given by $-f(A)$. This sketched construction is called *subordination in the sense of Bochner*, see e.g. [1, 2].

Subordination in locally convex spaces.

In many applications, in particual those coming from stochastic processes, there do appear semigroups which are not strongly continuous w.r.t. the norm topology of a Banach space, but admit continuous orbits only for a coarser locally convex topology.

In this talk we will show that subordination can also be performed for locally (sequentially) equicontinuous, equibounded C_0-semigroups on (sequentially) complete Hausdorff locally convex spaces. This generalisation gives rise to applications on bi-continuous semigroups [3] as well as transition semigroups for C_b-Feller processes. In the context of stochastic processes, subordination corresponds to a random time change. Thus, we obtain an analytic description of the semigroup induced by the time-changed process.

Remark. Subordination can be viewed as a functional calculus technique, which can thus also be used in locally convex spaces.

The results are contained in [4].

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