Exponential Stability for Port-Hamiltonian Systems

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We consider port-Hamiltonian Systems; i.e., Cauchy problems of the form

$$\partial_t u + P_1\partial_x \mathcal{H}u + P_0\mathcal{H}u = 0,$$
$$u(0) = u_0,$$

where $P_0 = -P_0^* \in \mathbb{R}^{n\times n}$, $P_1 = P_1^* \in \mathbb{R}^{n\times n}$ and $\mathcal{H} \in L_\infty([a,b];\mathbb{R}^{n\times n})$ such that there exists $c > 0$ with

$$cI_{n\times n} \leq \mathcal{H}(x) = \mathcal{H}(x)^* \quad (x \in [a,b] \text{ a.e.}).$$

In order to solve the above problem, one is interested in m-accretive realisations of the spatial operator $P_1\partial_x\mathcal{H} + P_0\mathcal{H}$ in the weighted L_2-space

$$L_{2,\mathcal{H}}([a,b])^n = (L_2([a,b])^n, \langle\cdot,\mathcal{H}\cdot\rangle_{L_2([a,b])^n})$$

by imposing suitable boundary conditions; that is, one considers restrictions $A \subseteq P_1\partial_x\mathcal{H} + P_0\mathcal{H}$ with domain

$$\operatorname{dom}(A) = \left\{ v \in L_2([a,b])^n\,;\, \mathcal{H}v \in H^1([a,b])^n, W_B \begin{pmatrix} (\mathcal{H}v)(b) \\ (\mathcal{H}v)(a) \end{pmatrix} = 0 \right\}$$

where $W_B \in \mathbb{R}^{N\times 2N}$ is a suitable matrix. Such m-accretive realisations are well-studied and can be characterised by properties of the matrix W_B, see e.g. [1, Theorem 1].
Assuming now the m-accretivity of a restriction A, the question arises whether the associated C_0-semigroup $(\mathrm{e}^{-tA})_{t\geq 0}$ is exponentially stable. The latest theorem in this direction seems to be the following:

Theorem ([3, Theorem 3.5]) If there exists $c \in \{a, b\}$ and $\kappa > 0$ such that

$$\langle v, Av\rangle_{L_{2,\mathcal{H}}([a,b])^n} \geq \kappa \|(\mathcal{H}v)(c)\|^2 \quad (v \in \operatorname{dom}(A))$$

and $\mathcal{H}$ is of bounded variation, then $-A$ generates an exponentially stable C_0-semigroup.

Similar results were known before under stronger regularity assumptions (e.g. Lipschitz-continuity) on the mapping $\mathcal{H}$. The reason for this additional regularity lies in the method of proof for these result, which is based on a-priori estimates for solutions of Port-Hamiltonian systems in terms of their boundary values, see e.g. [2, Lemma 9.1.2].

We will show that we can remove this additional regularity assumption for $\mathcal{H}$ and prove an analogous result for $\mathcal{H} \in L_\infty([a,b])^n$. In fact, we will provide a characterisation result for exponential stability in terms of the matrices P_0, P_1, W_B and the function $\mathcal{H}$.

The talk is based on an ongoing project together with Rainer Picard (TU Dresden), Marcus Waurick (TU Freiberg) and Bruce Watson (Univ. of Witwatersrand).

References:

[1] B. Jacob, K. Morris, H. Zwart. C_0-semigroups for hyperbolic partial differential equations on a one-dimensional spatial domain. // J. Evol. Equ., 15:2 (2015) 493-502

[2] B. Jacob, H. Zwart: Linear port-Hamiltonian systems on infinite-dimensional spaces. — vol. 223 of Operator Theory: Advances and Applications, Birkhäuser/Springer Basel AG, 2012.

[3] J. Schmid. Stabilization of port-Hamiltonian systems with discontinuous energy densities// arXiv:1809.00857 [math.AP] (2018)

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