Trace decreasing semigroup for an open quantum system interacting with a repeatedly measured ancilla

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Problem setting. We consider a quantum system dynamics caused by successive selective measurements of an ancilla coupled to the system. This scenario is similar to the Zeno effect because the ancilla remains effectively frozen; however, the system dynamics is non-trivial. For the finite measurement rate τ^{-1} and the system-probe interaction strength γ we derive analytical evolution equations for the system in the stroboscopic limit $\tau \to 0$ and $\gamma^2\tau = \text{const}$. We prove that the induced dynamics of the subnormalized density operator is a semigroup provided the system-ancilla interaction is unitary [1]. Dynamics of the normalized density operator is given by a non-linear equation [1]. The obtained semigroup dynamics can be considered as a deviation from the Zeno subspace dynamics on a longer timescale $T \sim (\gamma^2\tau)^{-1} \gg \gamma^{-1}$.

Main result. Importantly, the induced conditional dynamics of the main system is described by the effective non-Hermitian Hamiltonian. By physical examples of multi-spin models we show that the effective non-Hermitian Hamiltonian may drive the system to a maximally entangled stationary state [2]. In addition, we report a new recipe to construct a physical scenario where the quantum dynamics of a physical system represented by a given non-Hermitian Hamiltonian H_{eff} may be simulated [2]:

Theorem. The trace decreasing semigroup dynamics with the generator $-i[H_{\text{eff}}, \cdot]$ results from the stroboscopic measurements on an ancilla, which is originally prepared in the state $|0_A\rangle\langle 0_A|$ and interacts with the system through self-adjoint Hamiltonian

$$H = \frac{1}{2}(H_{\text{eff}} + H_{\text{eff}}^\dagger) \otimes |0_A\rangle\langle 0_A| + \sqrt{cI + \frac{i}{\tau}(H_{\text{eff}} - H_{\text{eff}}^\dagger)} \otimes (|0_A\rangle\langle 1_A| + |1_A\rangle\langle 0_A|)\,,$$

where $c = \max(0, -M)$ and M is the minimum eigenvalue of the operator $\frac{i}{\tau}(H_{\text{eff}} - H_{\text{eff}}^\dagger)$.

Discussion. The developed formalism can be generalized to the case of time-dependent system-ancilla Hamiltonian and the case of non-selective measurements [3]; however, the semigroup dynamics for the system does not take place in general. We can also consider a situation, in which the measurement basis changes in time, which is illustrated by nonselective measurements in the basis of diabatic states of the Landau-Zener model.

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