



## Local eventual positivity of operator semigroups

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**Introduction.** The theory of positive  $C_0$ -semigroups on  $L^p$ -spaces, or more generally, on Banach lattices, is well-known and has a plethora of applications (see [4]). Recently, the study of semigroups that only become (and stay) positive after a sufficiently large time was initiated in [1] and [2], where the authors also presented a variety of applications.

Another related notion that appeared several years earlier is of *local eventual positivity*. This loosely means that the solution of the corresponding Cauchy problem becomes eventually positive in a part of the domain. For example, it has been shown that (see [3]) for a homogeneous biharmonic heat equation in  $\mathbb{R}^d$ , if a positive initial datum has compact support, then the solution of the corresponding Cauchy problem becomes positive on compact sets for large times and that such eventual positivity does not occur on the whole domain.

In this talk, we discuss the local eventual positivity of strongly continuous semigroups on  $L^p$ -spaces. In particular, if  $(e^{tA})_{t \geq 0}$  is a  $C_0$ -semigroup on some  $L^p$ -space  $F$ , and  $S$  and  $T$  are bounded operators on  $F$ , then given a function  $f \in F$  which is positive almost everywhere, we provide sufficient conditions for existence of  $t_0 \geq 0$ , so that the functions  $Se^{tA}Tf$  are also positive almost everywhere for  $t \geq t_0$ . Moreover, we also look at some conditions that guarantee that the time  $t_0$  is independent of the initial datum  $f$ . We will see that these results can be applied to concrete differential equations, for instance, the Dirichlet bi-Laplacian. Additionally, we will consider some spectral and convergence implications of local eventual positivity.

While we restrict ourselves to  $L^p$ -spaces during the talk, the results mentioned can be generalized to general Banach lattices.

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