Operator semigroups associated with stochastic processes within the framework of semigroup and Gelfand–Shilov classifications

I. V. Melnikova,[1] **U. A. Alekseeva,**[2] **V. A. Bovkun.**[3]

Keywords: semigroup of operators, pseudo-differential operator, Levy process, transition probability, Levy–Khintchine formula.

MSC2010 codes: 60G51, 60J35, 46F10, 47G30

Introduction. A wide class of processes arising in various fields of natural science, economics and social phenomena can be mathematically described by stochastic differential equations (SDE). Recently, great interest in the problems of financial mathematics has led to significant advances in this area.

The most studied is the class of diffusion SDEs with Wiener processes being the randomness sources. The solutions of such equations, due to the continuity properties of Wiener processes have continuous trajectories. Therefore, modeling based on diffusion-type equations is most suitable for describing processes that do not have jumps. Simulation based on Levy and more general Levy type processes allows one to study along with continuous, jump processes.

At the same time, both in applications and in fundamental science, often what is needed is not the random process itself, defined by SDE or a set of properties, but its probabilistic characteristic. The study of the relationship between SDEs and deterministic equations for probabilistic characteristics is one of the main directions of stochastic analysis.

Main results. The talk is devoted to solution properties of equations for probabilistic characteristics, defined by stochastic Levy processes. It is shown that, in contrast to the PDEs for probabilistic characteristics determined by Wiener processes, the equations determined by Levy processes and more general Markov processes are pseudo-differential.

The semigroup technique underlies the study of Cauchy problems for the obtained pseudo-differential equations. The central place is occupied by semigroups with kernels formed by transition probabilities of Markov processes and their important subclasses — Feller and Levy processes. The kernels are considered in spaces of tempered distributions.

The connection between the semigroup classification based on the spectral properties of generators, which are generally pseudo-differential operators, and the GelfandB—Shilov classification for differential systems based on generalized Fourier transform techniques, has been shown. The embedding scheme has been constructed.

References:

[1] Irina V. Melnikova, Alexei Filinkov: The Cauchy Problem: Three Approaches. — Chapman & Hall/CRC Monographs and Surveys in Pure and Applied Mathematics **120**, 2001.

[2] Irina V. Melnikova. Stochastic Cauchy Problems in Infinite Dimensions. Regularized and Generalized Solutions. CRC Press, 2016.

[3] I.V. Melnikova, U.A. Alekseeva. Semigroup Classification and GelfandB–Shilov Classification of Systems of Partial Differential Equations // Mathematical Notes. 2018. Vol. 104. No 6. P. 886-899.

[4] I.V. Melnikova, V.A. Bovkun, U.A. Alekseeva. Integro-differential equations, generated stochastic problems // Differential equations. 2021. Vol. 57. No. 3. P.1653-1663.

[1]Ural Federal University, Institute of Natural Sciences and Mathematics, Russia, Yekaterinburg. Email: Irina.Melnikova@urfu.ru

[2]Ural Federal University, Institute of Natural Sciences and Mathematics, Russia, Yekaterinburg. Email: Uliana.Alekseeva@urfu.ru

[3]Ural Federal University, Institute of Natural Sciences and Mathematics, Russia, Yekaterinburg. Email: Vadim.Bovkun@urfu.ru