Two-dimensional attractors of A-flows and fibered links on 3-manifolds

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Keywords: A-flow, attractor, fibered link

MSC2010 codes: 37D05

Introduction. Dynamical systems satisfying an Axiom A (in short, A-systems) were introduced by S.Smale. By definition, a non-wandering set of A-system is the topological closure of periodic orbits endowed with a hyperbolic structure. Due to Smale's Spectral Decomposition Theorem, the non-wandering set of any A-system is a disjoint union of closed, invariant, and topologically transitive sets called *basic sets*. E.Zeeman proved that any n-manifold, $n \geq 3$, supporting nonsingular flows supports an A-flow with a one-dimensional nontrivial basic set. It is natural to consider the existence of two-dimensional (automatically non-trivial) basic sets on n-manifolds beginning with closed 3-manifolds M^3. We prove that any closed orientable 3-manifolds supports A-flows with two-dimensional attractors. Our main attention concerns to embedding of non-mixing attractors and its basins (stable manifolds) in M^3.

Main results. Let f^t be an A-flow on a closed orientable 3-manifold M^3 and Λ_a a two-dimensional non-mixing attractor of f^t. The stable manifold (in short, a basin) $W^s(\Lambda_a)$ of Λ_a is an open subset of M^3 consisting of the trajectories whose ω-limit sets belong to Λ_a. First, we construct a special compactification of $W^s(\Lambda_a)$ called a casing by a collection of circles that form a fiber link in the casing.

Theorem 1. Let f^t be an A-flow on an orientable closed 3-manifold M^3 such that the non-wandering set $NW(f^t)$ contains a 2-dimensional non-mixing attractor Λ_a. Then there is a compactification $M(\Lambda_a) = W^s(\Lambda_a) \cup_{i=1}^k l_i$ of the basin $W^s(\Lambda_a)$ by the family of circles $l_1, \ldots, l_k$ such that

- $M(\Lambda_a)$ is a closed orientable 3-manifold;
- the flow $f^t|_{W^s(\Lambda_a)}$ is extended continuously to the nonsingular flow $\tilde{f}^t$ on $M(\Lambda_a)$ with the non-wandering set $NW(\tilde{f}^t) = \Lambda_a \cup_{i=1}^k l_i$ where $l_1, \ldots, l_k$ are repelling isolated periodic trajectories of $\tilde{f}^t$;
- the family $L = \{l_1, \ldots, l_k\} \subset M(\Lambda_a)$ is a fibered link in $M(\Lambda_a)$.

The second result of the paper, in a sense, is reverse to the first one.

Theorem 2. Let $\{l_1, \ldots, l_k\} \subset M^3$ be a fibered link in a closed orientable 3-manifold M^3. Then there is a nonsingular A-flow f^t on M^3 such that the non-wandering set $NW(f^t)$ contains a 2-dimensional non-mixing attractor and the repelling isolated periodic trajectories $l_1, \ldots, l_k$.

Corollary. Given any closed orientable 3-manifold M^3, there is a nonsingular A-flow f^t on M^3 such that the non-wandering set $NW(f^t)$ contains a two-dimensional attractor.

Acknowledgments. This work is supported by the Russian Science Foundation under grant 17-11-01041, except Theorem 2 supported by Laboratory of Dynamical Systems and Applications of National Research University Higher School of Economics, of the Ministry of science and higher education of the RF, grant ag. No 075-15-2019-1931.

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