Two-dimensional attractors of A-flows and fibered links on 3-manifolds
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Introduction. Dynamical systems satisfying an Axiom A (in short, A-systems) were introduced by S.Smale. By definition, a non-wandering set of A-system is the topological closure of periodic orbits endowed with a hyperbolic structure. Due to Smale’s Spectral Decomposition Theorem, the non-wandering set of any A-system is a disjoint union of closed, invariant, and topologically transitive sets called basic sets. E.Zeeman proved that any n-manifold, \( n \geq 3 \), supporting nonsingular flows supports an A-flow with a one-dimensional nontrivial basic set. It is natural to consider the existence of two-dimensional (automatically non-trivial) basic sets on \( n \)-manifolds beginning with closed 3-manifolds \( M^3 \). We prove that any closed orientable 3-manifolds supports A-flows with two-dimensional attractors. Our main attention concerns to embedding of non-mixing attractors and its basins (stable manifolds) in \( M^3 \).

Main results. Let \( f^t \) be an A-flow on a closed orientable 3-manifold \( M^3 \) and \( \Lambda_a \) a two-dimensional non-mixing attractor of \( f^t \). The stable manifold (in short, a basin) \( W^s(\Lambda_a) \) of \( \Lambda_a \) is an open subset of \( M^3 \) consisting of the trajectories whose \( \omega \)-limit sets belong to \( \Lambda_a \). First, we construct a special compactification of \( W^s(\Lambda_a) \) called a casing by a collection of circles that form a fiber link in the casing.

**Theorem 1.** Let \( f^t \) be an A-flow on an orientable closed 3-manifold \( M^3 \) such that the non-wandering set \( NW(f^t) \) contains a 2-dimensional non-mixing attractor \( \Lambda_a \). Then there is a compactification \( M(\Lambda_a) = W^s(\Lambda_a) \cup_{i=1}^k l_i \) of the basin \( W^s(\Lambda_a) \) by the family of circles \( l_1, \ldots, l_k \) such that

- \( M(\Lambda_a) \) is a closed orientable 3-manifold;
- the flow \( f^t|_{W^s(\Lambda_a)} \) is extended continuously to the nonsingular flow \( \tilde{f}^t \) on \( M(\Lambda_a) \) with the non-wandering set \( NW(\tilde{f}^t) = \Lambda_a \cup_{i=1}^k l_i \) where \( l_1, \ldots, l_k \) are repelling isolated periodic trajectories of \( \tilde{f}^t \);
- the family \( L = \{l_1, \ldots, l_k\} \subset M(\Lambda_a) \) is a fibered link in \( M(\Lambda_a) \).

The second result of the paper, in a sense, is reverse to the first one.

**Theorem 2.** Let \( \{l_1, \ldots, l_k\} \subset M^3 \) be a fibered link in a closed orientable 3-manifold \( M^3 \). Then there is a nonsingular A-flow \( f^t \) on \( M^3 \) such that the non-wandering set \( NW(f^t) \) contains a 2-dimensional non-mixing attractor and the repelling isolated periodic trajectories \( l_1, \ldots, l_k \).

**Corollary.** Given any closed orientable 3-manifold \( M^3 \), there is a nonsingular A-flow \( f^t \) on \( M^3 \) such that the non-wandering set \( NW(f^t) \) contains a two-dimensional attractor.

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