A linear program approach to global attractors

C. Schlosser,[1] **M. Korda.**[2]

Keywords: Global attractor, dynamical systems, infinite-dimensional linear programming, sum-of-squares, semidefinite programming, occupation measures

MSC2010 codes: 34D45, 90C22

Global attractors, i.e., sets to which all bounded sets converge asymptotically under the action of the dynamics, are typical and important objects in the (asymptotic) analysis of dynamical systems. Global attractors can be of complex nature so that computing/approximating them is (computationally) challenging. We try to approach the task of computing converging approximations of the global attractor by first embedding it into a linear setting. This is done by using the so-called occupation measures. Historically this idea dates back to the 1970ies when optimal control problems were reformulated as linear programs on measures. For a dynamical system $\dot{x} = f(x)$ with locally Lipschitz dynamics $f : \mathbb{R}^n \to \mathbb{R}^n$ and compact state constraint $X \subset \mathbb{R}^n$ we give the following linear program

$$\begin{array}{lll} p^* := \sup & \mu_0(X) & \\ \text{s.t.} & \mu_0, \hat{\mu}_0, \mu_+, \mu_- \in M(X) & \\ & \int_X \beta v^1 - \nabla v^1 \cdot f \, d\mu_+ = \int_X v^1 \, d\mu_0 & \forall v^1 \in \mathcal{C}^1(\mathbb{R}^n) \\ & \int_X \beta v^2 + \nabla v^2 \cdot f \, d\mu_- = \int_X v^2 \, d\mu_0 & \forall v^2 \in \mathcal{C}^1(\mathbb{R}^n) \\ & \mu_0 + \hat{\mu}_0 \quad = \lambda\big|_X & \end{array} \tag{1}$$

where $M(X)$ denotes the set of (non-negative) measures on $\mathbb{R}^n$ supported on X and $\lambda\big|_X$ denotes the Lebesgue measure restricted to X. We show that p^* is given by the Lebesgue measure of the global attractor. The main ingredients for the result are a well known characterization of global attractors in terms of maximal positively invariant sets in combination with a previous result on characterizing maximal positively invariant sets by linear programs on measures.

We can take advantage of the obtained linear structure because it allows to us to consider its dual problem where positivstellensätze from real algebraic geometry can be applied. Together with Lasserre's sum-of-squares hierarchy this leads to a sequence of semidefinite programs for which we show that their solutions provide converging outer approximations of the global attractor.

In this talk we will not go too much into the details of the arguments but we rather give an introduction to the methods used.

Acknowledgments. The authors are thankful to Matthew Kvalheim for early comments on the text.

References

[1] J.C. Robinson. Infinite-Dimensional Dynamical Systems. An Introduction to Dissipative Parabolic PDEs and the Theory of Global Attractors. — Cambridge University Press, 2001.

[2] M. Korda, D. Henrion, J.N. Jones. Convex computation of the maximum controlled invariant set for polynomial control systems. // SIAM J. Control Optim. 2014. V. 52. No. 5. P. 2944–2969.

[3] J. Rubio. Generalized curves and extremal points. // SIAM Journal on Control. 1975. V. 13. No. 1. P. 28–47.

[4] C. Schlosser, M. Korda. Converging outer approximations to global attractors using semidefinite programming. // arXiv preprint arXiv:2005.03346 (2020).

[1]CNRS; LAAS; 7 avenue du colonel Roche, France, F-31400 Toulouse. Email: cschlosser@laas.fr

[2]CNRS; LAAS; 7 avenue du colonel Roche, France, F-31400 Toulouse. Faculty of Electrical Engineering, Czech Technical University in Prague, Technická 2, Czech Republic, CZ-16626 Prague. Email: korda@laas.fr