Coalescing stochastic flows on metric graphs

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Introduction. Let $\psi = \{\psi_{s,t} : -\infty < s \leq t < \infty\}$ be a stochastic flow on a locally compact separable metric space M, that is a family of measurable random mappings of M such that $\psi_{s,t}(\psi_{r,s}(x)) = \psi_{r,t}(x)$ a.s., $\psi_{s,s}(x) = x$ a.s., for any sequence $t_1 < t_2 < \ldots < t_n$ mappings $\psi_{t_1,t_2}, \ldots, \psi_{t_{n-1},t_n}$ are independent, for any $s < t$ mappings $\psi_{s,t}$ and $\psi_{0,t-s}$ are equally distributed and for all $f \in C_0(M)$, $s \leq t$ and $x \in M$,

$$\lim_{(u,v)\to(s,t)} \sup_{y\in M} E(f(\psi_{u,v}(y)) - f(\psi_{s,t}(y)))^2 = 0,$$

$$\lim_{y\to x} E(f(\psi_{s,t}(y)) - f(\psi_{s,t}(x)))^2 = 0, \lim_{x\to\infty} E(f(\psi_{s,t}(x)))^2 = 0.$$

It is known (see [1]) that the relation

$$P^{(n)}(x, B) = \mathbb{P}((\psi_{0,t}(x_1), \ldots, \psi_{0,t}(x_n)) \in B), n \geq 1, x \in M^n, B \in \mathcal{B}(M^n),$$

establishes a one-to-one correspondence between stochastic flows on M and consistence sequences $\{P^{(n)} : n \geq 1\}$ of coalescing Feller transition probabilities. The sequence $\{P^{(n)} : n \geq 1\}$ defines distributions of finite-point motions of the flow.

Problem setting. We will be interested in the existence of a strong stochastic flow that corresponds to a consistent sequence $\{P^{(n)} : n \geq 1\}$ of coalescing Feller transition probabilities. By a strong flow we understand a stochastic flow ψ such that

for all $\omega \in \Omega, x \in M, r \leq s \leq t$,

$$\psi_{s,t}(\omega, \psi_{r,s}(\omega, x)) = \psi_{r,t}(\omega, x),\ \psi_{s,s}(\omega, x) = x.$$

Existence of a strong stochastic flow is well-known when its finite-point motions are families of solutions of an SDE with smooth enough coefficients (see [2]). In this case the flow is a flow of homeomorphisms of a corresponding manifold. On the contrary we will deal with flows in which coalescence occurs. In the case $M = \mathbb{R}$ existence of strong coalescing stochastic flows was proved in [3] for a large family of sequences $\{P^{(n)} : n \geq 1\}$ (see [4] and [5] for applications to the study of coalescing stochastic flows).

Main Result. Let M be a metric graph. By $\mathbb{P}^{(n)}_x$ we will denote the distribution of the Feller process $X^{(n)}$ in M^n that has transition probabilities $P^{(n)}$ and starts from $x \in M^n$.

Theorem. Assume that transition probabilities $P^{(n)}$ satisfy following properties:

- for any $x \in M$ and $\epsilon > 0$ $P^{(1)}_t(x, B(x, \epsilon)^c) = o(t),\ t \to 0+$;
- for any compact $K \subset M$ and any $t > 0$

$$\lim_{c\to\infty} \sup_{n\geq 1, x\in K^n} \mathbb{P}^{(n)}_x(\#X^{(n)}(t) \geq c) = 0.$$

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Then there exists a strong stochastic flow that corresponds to the sequence $\{P^{(n)} : n \geq 1\}$.

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