Disordering in a discrete-time stochastic flows

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Introduction.

The discrete-time approximation of the Arratia flow [1] are considered. This approximations $\{x_k^n(u), k = 0, \ldots, n\}$ are given by a difference equation with random perturbation generated by a sequence of independent stationary Gaussian processes $\{\xi_k^n(u),\, u \in \mathbb{R},\, k = 0, \ldots, n\}$ with covariance function Γ_n:

$$x_{k+1}^n(u) = x_k^n(u) + \frac{1}{\sqrt{n}}\xi_{k+1}^n(x_k^n(u)),\; x_0^n(u) = u,\; u \in \mathbb{R}.$$

Define the random process $\tilde{x}_n(u, \cdot)$ on $[0, 1]$ as the polygonal line with edges $\left(\frac{k}{n}, x_k^n(u)\right)$, $k = 0, \ldots, n$. It was proved in [3] that if the covariance Γ_n approximates in some sense the function $\mathbb{I}_{\{0\}}$ then $m-$point motion of $\tilde{x}_n$ weakly converges to the $m-$point motion of the Arratia flow.

Results.

We obtain an explicit form of the Ito-Wiener expansion for $f(x_n(u_1), \ldots, x_n(u_m))$ with respect to noise that produced by the processes $\{\xi_k^n(u),\, u \in \mathbb{R},\, k = 0, \ldots, n\}_{n \geq 1}$. This expansion can be regarded as a discrete-time analogue of the Krylov-Veretennikov representation formula [4].

In contrasts to the flow of Brownian particles on the line, in the discrete-time approximations the order between particles can change in time. We define a measure of disordering for 2-point motion as follows

$$\Phi_n = \int_0^1 \mathbb{I}_{\{\tilde{x}_n(u_2,s) - \tilde{x}_n(u_1,s) < 0\}} ds,$$

where $u_1 < u_2$. If the discrete-time flow approximates the Arratia flow then the following asymptotics holds [5]:

$$\overline{\lim_{n\to\infty}} \frac{2C_n}{n} \ln \mathbb{P}\{\Phi_n > 0\} \leq -1$$

$$\underline{\lim_{n\to\infty}} \frac{2C_n}{n} \ln \mathbb{P}\{\Phi_n > \varepsilon\} \geq -K^2,$$

where $C_n = \sup_{\mathbb{R}} \frac{2-2\Gamma_n(x)}{x^2}$ and $K > 0$.

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