On admissible singular drifts of symmetric α-stable process

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We consider the problem of existence of a (unique) weak solution to the SDE describing symmetric α-stable process with a locally unbounded drift $b : \mathbb{R}^d \to \mathbb{R}^d$, $d \geq 3$, $1 < \alpha < 2$. In this talk, b belongs to the class of weakly form-bounded vector fields, the class providing the L^2 theory of the non-local operator behind the SDE, i.e. $(-\Delta)^{\frac{\alpha}{2}} + b \cdot \nabla$. It contains as proper sub-classes other classes of singular vector fields studied in the literature in connection with this operator, such as the Kato class, the weak $L^{d/(\alpha-1)}$ class and the Campanato-Morrey class (in general, such b makes invalid the standard heat kernel estimates in terms of the heat kernel of the fractional Laplacian). We show that the operator $(-\Delta)^{\frac{\alpha}{2}} + b \cdot \nabla$ with weakly form-bounded b admits a realization as (minus) Feller generator, and that the probability measures determined by the Feller semigroup (uniquely in appropriate sense) admit description as weak solutions to the corresponding SDE. The proof is based on detailed regularity theory of $(-\Delta)^{\frac{\alpha}{2}} + b \cdot \nabla$ in L^p, $p > d - \alpha + 1$.

The talk is based on joint work with Damir Kinzebulatov (Université Laval).

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