Operator systems defined via one-parameter unitary groups in the quantum error correction theory

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Introduction. The operator system $\mathcal{V}$ is the linear subspace of some algebra with involution that is closed under conjugation $A \in \mathcal{V} \Rightarrow A^* \in \mathcal{V}$ and satisfying $I \in \mathcal{V}$. That type of spaces considered a quantum analog of the so-called confusability graphs of communication channels, so these spaces also called non-commutative operator graphs. The main condition defining operator systems interesting for a given application is really tiny, we are interested in operator systems that have the orthogonal projection P of rank$P \geq 2$ such that $\dim P\mathcal{V}P = 1$, given condition is called the Knill-Laflamme condition, projection P and the subspace ImP are called the quantum anticlique and the quantum error correcting code respectively. **Discussion.** We are interested in the question of how should be generated the graph $\mathcal{V}$ allow us to check the Knill-Laflamme condition in a more easy manner. Appears, that in the finite-dimensional case it is possible to give the sufficient condition of existing of quantum error-correcting code for the graphs generated by covariant positive operator-valued measure (POVM), these graphs have the following form

$$\mathcal{V} = \operatorname{span}\left\{U_g Q U_g^* \ g \in G\right\}, \tag{1}$$

where U_g is the projective unitary representation of the compact group G and Q is the positive operator.

This technique could be extended on the infinite-dimensional case, it is possible to introduce several examples of the non-commutative operator graphs that generated by unitary representations in the spirit of (1) possessing quantum anticliques, in most of that examples instead of the group $U_g,\ g \in G$ we take the group of operators $U_t = e^{-itH}, t \in \mathbb{R}$ defining the dynamics of some interesting quantum system with the Hamiltonian H. Such examples are given for the two-mode quantum oscillator and for the Jaynes-Cummings model of light-matter interaction, also introduced the example in the one-particle bosonic Fock space.

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