Asymptotic behaviour of a class of random evolution problems with application to combinatorial and metric graphs

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We consider a family of graphs $\{G_k,\ k = 1, \dots, N\}$, each associated to the (discrete or continuous) *Laplacian* operator $\mathcal{L}_k$ acting on the function defined on the vertices (edges) of the graph.

Given a stochastic mechanism of switching the graphs during time, we get that the evolution is lead by an operator $\mathcal{L}_{X_k}$ (selected from the set $\{\mathcal{L}_1, \dots, \mathcal{L}_N\}$ according to some Markov chain X_k) during the (random) time interval $[T_k, T_{k+1})$

$$\begin{cases} \partial_t u(t,x) = \mathcal{L}_{X_k} u(t,x), & t \in [T_k, T_{k+1}), \\ u(0,x) = f(x). \end{cases} \tag{1}$$

We can associate to (1) the (random) evolution operator

$$S(t) = e^{(t-T_n)\mathcal{L}_{X_n}} \prod_{k=0}^{n-1} e^{(T_{k+1}-T_k)\mathcal{L}_{X_k}}, \qquad t \in [T_n, T_{n+1}).$$

Our main problem can be stated as follows:

(**P**) under which condition the random evolution operator $S(t)$ converges? towards which limit?

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References

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