Operator splitting for abstract Cauchy problems with dynamical boundary conditions

P. Csomós[1]

Keywords: operator splitting; Lie and Strang splitting; Trotter product; abstract dynamical boundary problems; error bound.

MSC2010 codes: 47D06, 47N40, 34G10, 65J08, 65M12, 65M15

In the present work we focus on the abstract setting of coupled Cauchy problems, where one of the sub-problems provides a extra condition, of boundary type, to the other. We consider equations of the form:

$$\begin{cases} \dot{u}(t) = Au(t) & \text{for } t \geq 0, \quad u(0) = u_0 \in E, \\ \dot{v}(t) = Bv(t) & \text{for } t \geq 0, \quad v(0) = v_0 \in F, \\ Lu(t) = v(t) & \text{for } t \geq 0, \end{cases}$$

where E and F are Banach spaces over the complex field $\mathbb{C}$, A and B are (unbounded) linear operators on E and F, respectively. The coupling of the two problems involves the unbounded linear operator L acting between E and F. Moreover, the coupling is such that one of the problems prescribes a "boundary type" extra condition for the other one.

In order to give an approximate solution to this problem, we study operator splitting methods. The theory of one-sided coupled operator matrices (see [1], [3], [4]) provides an excellent framework to study the well-posedness of such problems. We show that with this machinery even operator splitting methods can be treated conveniently and rather efficiently. We consider three specific examples: the Lie (sequential), the Strang, and the weighted splitting, and prove the convergence of these methods along with error bounds under fairly general assumptions. Simple numerical examples show that the obtained theoretical bounds can be computationally realised.

The talk is based on the joint work [2] with B. Farkas and M. Ehrhardt (Wuppertal).

Acknowledgment. Supported by the ÚNKP-20-5 New National Excellence Program of the Ministry for Innovation and Technology from the source of the National Research, Development and Innovation Fund.

References

[1] V. Casarino, K.-J. Engel, R. Nagel, G. Nickel. A semigroup approach to boundary feedback systems. // Integral Equations and Operator Theory. 2003. V. 47. No. 3. P. 289–306

[2] P. Csomós, M. Ehrhardt, B. Farkas. Operator splitting for abstract Cauchy problems with dynamical boundary conditions. // To appear in Operators and Matrices (2021).

[3] K.-J. Engel. Spectral theory and generator property for one-sided coupled operator matrices. // Semigroup Forum. 1999. V. 58. No. 2. P. 267–295

[4] G. Greiner. Perturbing the boundary conditions of a generator. // Houston J. Math. 1987. V. 13. No. 2. P. 213–229.

[1]ELTE Eötvös Loránd University, Department of Applied Analysis and Computational Mathematics, Budapest, Hungary. Email: petra.csomos@ttk.elte.hu