LARGE-TIME ASYMPTOTICS
OF FUNDAMENTAL SOLUTIONS FOR DIFFUSION EQUATIONS IN
PERIODIC MEDIA
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The diffusion equation is considered in an infinite 1-periodic medium of $\mathbb{R}^d$. We find large-time approximations for its fundamental solution. The approximation precision has pointwise and integral estimates of orders $O(t^{-(d+j+1)/2})$ and $O(t^{-(j+1)/2})$, $j = 0, 1, 2, \ldots$, respectively. The approximations are constructed on the base of the known fundamental solution of the homogenized equation with constant coefficients, its derivatives, and solutions of a family of auxiliary problems on the periodicity cell which is the unit cube in $\mathbb{R}^d$. The family of problems on the cell is generated recurrently. These results are used to construct approximations of the operator exponential of the diffusion equation with precision estimates in operator norms in $L^p$-spaces, $1 \leq p \leq \infty$. For the analogous equation in an $\varepsilon$-periodic medium, where $\varepsilon$ is a small parameter, we obtain approximations of the operator exponential in $L^p$-operator norms for a fixed time with precision of order $O(\varepsilon^n)$, $n = 1, 2, 3, \ldots$

To obtain these results, we use spectral approach based on the Bloch–Gelfand transformation.

References:

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