On topology of ambient manifolds admitting A-diffeomorphisms

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The report is devoted to exposition of a result obtained in collaboration with E.V. Zhuzhoma and V.S. Medvedev.

Let M^n, $n \geq 3$, be a closed orientable n-manifold and $\mathbb{G}(M^n)$ the set of A-diffeomorphisms $f : M^n \to M^n$ satisfying the following conditions:

- if a basic set belonging to nonwandering set $NW(f)$ of a diffeomorphism $f \in \mathbb{G}(M^n)$ is nontrivial (that is different from periodic orbit) then it is either an orientable codimension one expanding attractor or an orientable codimension one contracting repeller;
- invariant manifolds of isolated saddle periodic points intersects transversally;
- separatrices of isolated saddle periodic points with Morse index one can intersect only (n-1)-dimensional separatrices of other saddle isolated periodic orbits and separatrices of isolated saddle periodic points with Morse index (n-1) can intersect only one-dimensional separatrices of other saddle isolated periodic orbits.

Let us recall that Morse index of hyperbolic periodic point $p \in NW(f)$ of a diffeomorphism $f : M^n \to M^n$ is called the dimension of unstable manifold $W^u(p)$ of the point p.

For $f \in \mathbb{G}(M^n)$ denote $\mu_f \geq 0$ the number of all nodal periodic points (sinks and sources), $\nu_f \geq 0$ the number of isolated saddle periodic points with Morse index 1 or $n-1$ and $\lambda_f \geq 0$ the number of all periodic points whose Morse index dose not belong to the set $\{0, 1, n-1, n\}$. Moreover denote by $k_f \geq 0$ the number of nontrivial basic sets and κ_f the number all bunches belonging to union of all nontrivial basic sets of f.

Below, $\mathbb{S}^m$ is an m-sphere, $\mathbb{T}^n$ is an n-torus. Denote $\mathcal{T}^n_m$ a manifold that is either empty set if $m = 0$ or is connected sum of $m \geq 1$ copies of n-torus $\mathbb{T}^n$ if $m > 0$

$$\underbrace{\mathbb{T}^n \sharp \cdots \sharp \mathbb{T}^n}_{m \geq 1}.$$

Denote $\mathcal{S}^n_m$ a manifold that is either the sphere $\mathbb{S}^n$ if $m = 0$ or is connected sum of $m \geq 1$ copies of $\mathbb{S}^{n-1} \times \mathbb{S}^1$ if $m > 0$:

$$\underbrace{\left(\mathbb{S}^{n-1} \times \mathbb{S}^1\right) \sharp \cdots \sharp \left(\mathbb{S}^{n-1} \times \mathbb{S}^1\right)}_{m \geq 1}.$$

Denote $\mathcal{N}_m$ a manifold that is either empty set if $m = 0$ or is connected sum of simply-connected manifolds N^n_i if $m > 0$:

$$N^n_1 \sharp \cdots \sharp N^n_m,$$

and each manifold N_i admits polar Morse-Smale diffeomorphisms $f_i : N_i \to N_i$ such that $NW(f_i)$ does not contain saddle periodic points with Morse index 1 or $n-1$.

Theorem. Let M^n be a closed orientable n-manifold, $n \geq 3$, supporting a diffeomorphism $f \in \mathbb{G}(M^n)$. Then there are interges $k_f \geq 0$, $g_f \geq 0$, $l_f \geq 0$ such that M^n is homeomorphic to connected sum:

$$\mathcal{T}^n_{k_f} \sharp\, \mathcal{S}^n_{g_f} \sharp\, \mathcal{N}_{l_f},$$

where $g_f \leq \kappa_f + \nu_f - 1$, $l_f \leq \lambda_f$.

Remark If $k_f = 0$ then $g_f = \frac{\nu_f - \mu_f + 2}{2}$.

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