On topology of ambient manifolds admitting A-diffeomorphisms

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The report is devoted to exposition of a result obtained in collaboration with E.V. Zhuzhoma and V.S. Medvedev.

Let $M^n$, $n \geq 3$, be a closed orientable $n$-manifold and $\mathcal{G}(M^n)$ the set of A-diffeomorphisms $f : M^n \rightarrow M^n$ satisfying the following conditions:

- if a basic set belonging to nonwandering set $NW(f)$ of a diffeomorphism $f \in \mathcal{G}(M^n)$ is nontrivial (that is different from periodic orbit) then it is either an orientable codimension one expanding attractor or an orientable codimension one contracting repeller;
- invariant manifolds of isolated saddle periodic points intersects transversally;
- separatrices of isolated saddle periodic points with Morse index one can intersect only (n-1)-dimensional separatrices of other saddle isolated periodic orbits and separatrices of isolated saddle periodic points with Morse index (n-1) can intersect only one-dimensional separatrices of other saddle isolated periodic orbits.

Let us recall that Morse index of hyperbolic periodic point $p \in NW(f)$ of a diffeomorphism $f : M^n \rightarrow M^n$ is called the dimension of unstable manifold $W^u(p)$ of the point $p$.

For $f \in \mathcal{G}(M^n)$ denote $\mu_f \geq 0$ the number of all nodal periodic points (sinks and sources), $\nu_f \geq 0$ the number of isolated saddle periodic points with Morse index 1 or $n - 1$ and $\lambda_f \geq 0$ the number of all periodic points whose Morse index does not belong to the set $\{0, 1, n - 1, n\}$. Moreover denote by $k_f \geq 0$ the number of nontrivial basic sets and $\kappa_f$ the number all bunches belonging to union of all nontrivial basic sets of $f$.

Below, $S^n$ is an $m$-sphere, $T^n$ is an $n$-torus. Denote $T^n_m$ a manifold that is either empty set if $m = 0$ or is connected sum of $m \geq 1$ copies of $n$-torus $T^n$ if $m > 0$:

$$\underbrace{T^n \# \cdots \# T^n}_{m \geq 1}.$$  

Denote $S^n_m$ a manifold that is either the sphere $S^n$ if $m = 0$ or is connected sum of $m \geq 1$ copies of $S^{n-1} \times S^1$ if $m > 0$:

$$\underbrace{(S^{n-1} \times S^1) \# \cdots \# (S^{n-1} \times S^1)}_{m \geq 1}.$$  

Denote $N^n_m$ a manifold that is either empty set if $m = 0$ or is connected sum of simply-connected manifolds $N^n_i$ if $m > 0$:

$$N^n_0 \# \cdots \# N^n_m,$$

and each manifold $N_i$ admits polar Morse-Smale diffeomorphisms $f_i : N_i \rightarrow N_i$ such that $NW(f_i)$ does not contain saddle periodic points with Morse index 1 or $n - 1$.

**Theorem.** Let $M^n$ be a closed orientable $n$-manifold, $n \geq 3$, supporting a diffeomorphism $f \in \mathcal{G}(M^n)$. Then there are integers $k_f \geq 0$, $g_f \geq 0$, $l_f \geq 0$ such that $M^n$ is homeomorphic to connected sum:

$$T^{k_f \# S^{n}_f \# N_i}_f,$$

where $g_f \leq \kappa_f + \nu_f - 1$, $l_f \leq \lambda_f$.

**Remark** If $k_f = 0$ then $g_f = \frac{\nu_f - \mu_f + 2}{2}$.

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