Numerical research of the Barenblatt-Zheltov-Kochina model on the interval with wentzell boundary conditions

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Introduction. In terms of numerical investigation, we study Barenblatt – Zheltov – Kochina model, which describes dynamics of pressure of a filtered fluid in a fractured-porous medium with general Wentzell boundary conditions. Based on the theoretical results associated with Galerkin method, we developed an algorithm and implementation for the numerical solution of the Cauchy–Wentzell problem on the segment $[0,1]$. In particular, we examine the asymptotic approximation of the spectrum of the one-dimensional Laplace operator and present result of computational experiment. In the paper, these problems are solved under the assumption that the initial space is a contraction of the space $L^2(0,1)$.

Let us consider the Cauchy–Wentzell problem

$$\begin{aligned} &u(x,0) = u_0(x), x \in [0,1] \\ &u_{xx}(0,t) + \alpha_0 u_x(0,t) + \alpha_1 u(0,t) = 0, \\ &u_{xx}(1,t) + \beta_0 u_x(1,t) + \beta_1 u(1,t) = 0 \end{aligned} \tag{1}$$

for the BarenblattB–ZheltovB–Kochina equation on the interval $[0,1]$

$$\lambda u_t(x,t) - u_{txx}(x,t) = \alpha u_{xx}(x,t) + f(x,t), (x,t) \in [0,1] \times \mathbb{R}_+, \tag{2}$$

which describes dynamics of pressure of a filtered fluid in a fractured-porous medium. Here α and λ are the material parameters characterizing the environment; the parameter $\alpha \in \mathbb{R}_+$; the function $f = f(x,t)$ plays the role of external loading.

The purpose of this work is to show new approach for resolvability of problem (1)-(2) with Wentzell boundary conditions. Namely, according to the modified Galerkin method, describe the solution of the Cauchy–Wentzell problem.

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