Betti numbers and the spectrum of dynamics on metrizable compact spaces

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In topological dynamics, the study of properties of a dynamical system in terms of the underlying space is a central problem: In which way does the geometry of a space constrain the possible dynamics on it? And which information can one recover from a space by knowing that a dynamical system on it has certain properties? I will give an example of how these questions can be approached in terms of operator theory. This will be done by studying a certain quotient of a dynamical system, the *maximal equicontinuous factor* which is closely linked to a system's spectral properties. The starting point of the inquiry is the following result of Hauser and Jäger.

Theorem (Theorem 3.1, [1]). Suppose that ϕ is a homeomorphism of the two-torus $\mathbb{T}^2$. If the maximal equicontinuous factor of $(\mathbb{T}^2, \phi)$ is minimal[2], then it must be one of the following three:

(i) an irrational translation on the two-torus,

(ii) an irrational rotation on the circle,

(iii) the identity on a singleton.

Thus, the geometric properties of the two-torus imply that the maximal equicontinuous factor of a homeomorphism on it must have a relatively simple structure if it is minimal: It is a rotation on a compact abelian Lie group of dimension less than two. As it turns out, this is representative of the following general phenomenon.

Theorem. Let ϕ be a homeomorphism of a compact metric space K such that K is locally path-connected and the first Betti number $\mathrm{b}_1(K)$ is finite. If all ϕ-invariant functions $f \in \mathrm{C}(K)$ are constant, then every equicontinuous factor of (K, G) is isomorphic to a minimal flow on some compact abelian Lie group of dimension less than $\frac{\mathrm{b}_1(K)}{\mathrm{b}_0(K)}$.

We will see how operator theory can help prove this result and what it in turn means for dynamical systems in terms of their spectral theory.

References:

[1] T. Hauser, T. Jäger. Monotonicity of maximal equicontinuous factors and an application to toral flows. 2019 Proc. Amer. Math. Soc. V. 147 P. 4539-4554

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[2]i.e., there are no nontrivial, closed, invariant subsets