Trace decreasing semigroup for an open quantum system interacting with a repeatedly measured ancilla

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Talk is based on


and

Non-Hermitian Hamiltonian

\[ \frac{d}{dt} |\Psi(t)\rangle = -iH_{\text{eff}} |\Psi(t)\rangle \]

\[ \frac{d}{dt} \rho(t) = -i\hbar \left( H_{\text{eff}} \rho(t) - \rho(t) H_{\text{eff}}^\dagger \right) \]

\[ H_0 = \frac{1}{2} \left( H_{\text{eff}} + H_{\text{eff}}^\dagger \right) \]

\[ i\Gamma = -\frac{1}{2} \left( H_{\text{eff}} - H_{\text{eff}}^\dagger \right) \]

\[ \frac{d}{dt} \text{Tr} [\rho(t)] = -2\hbar \text{Tr} [\Gamma \rho(t)] \]
Non-Hermitian Hamiltonian

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\[H_0 = \frac{(H_{\text{eff}} + H_{\text{eff}}^\dagger)}{2}\]

and \[i\Gamma = -\frac{(H_{\text{eff}} - H_{\text{eff}}^\dagger)}{2}\]
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\[ \frac{d}{dt} \rho(t) = -\frac{i}{\hbar} [H_0, \rho(t)] - \frac{1}{\hbar} \{\Gamma, \rho(t)\} \]

\[ \frac{d}{dt} \text{Tr} [\rho(t)] = -\frac{2}{\hbar} \text{Tr} [\Gamma \rho(t)] \]
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\[ \frac{d}{dt} \text{Tr} \left[ \rho(t) \right] = -\frac{2}{\hbar} \text{Tr} \left[ \Gamma \rho(t) \right] \]
Non-Hermitian Hamiltonian

\[ q(t) = \frac{\rho(t)}{\text{Tr} [\rho(t)]} = \frac{e^{-iH_{\text{eff}}t} \rho(0) e^{iH_{\text{eff}}^\dagger t}}{\text{Tr} \left[ e^{-iH_{\text{eff}}t} \rho(0) e^{iH_{\text{eff}}^\dagger t} \right]} \]
\[ \rho(t) = \frac{\rho(t)}{\text{Tr}[\rho(t)]} = \frac{e^{-iH_{\text{eff}} t} \rho(0) e^{iH_{\text{eff}}^\dagger t}}{\text{Tr}\left[ e^{-iH_{\text{eff}} t} \rho(0) e^{iH_{\text{eff}}^\dagger t} \right]} \]

\[ \frac{d}{dt} \rho(t) = -i \frac{\hbar}{\hbar} [H_0, \rho(t)] - \frac{1}{\hbar} \{ \Gamma, \rho(t) \} + \frac{2}{\hbar} \text{Tr} [\Gamma \rho(t)] \rho(t) \]
\[ \rho_S(0) \longrightarrow \rho_S^c(t) = \langle 0_A | U(t) \left[ \rho_S(0) \otimes |0_A\rangle\langle 0_A| \right] U^\dagger(t) |0_A\rangle = K(t)\rho_S(0)K^\dagger(t), \]

\[ K(t) = \langle 0_A | U(t) |0_A\rangle \]
\[ \rho_S(0) \rightarrow \rho^c_S(t) = \langle 0_A | U(t) [\rho_S(0) \otimes |0_A\rangle\langle 0_A|] U^\dagger(t) |0_A\rangle \]

\[ = K(t)\rho_S(0)K^\dagger(t), \]

\[ K(t) = \langle 0_A | U(t) |0_A\rangle \]

\[ p_{\rho_S(0)}(t) = \text{tr}\rho^c_S(t), \quad \rho^c_S(t) = \rho^c_S(t)/\text{tr}\rho^c_S(t) = \rho^c_S(t)/p_0(t) \]
\[ \rho_S(0) \rightarrow \rho_c = K_n(\tau) \rho_S(0) K_n^\dagger(\tau) \]

\[ \rho_c(\tau) = \langle 0_A | \exp(-iH\tau) | 0_A \rangle \]
\[ \rho_S(0) \rightarrow \rho_S^c(n\tau) = K^n(\tau)\rho_S(0)(K^n(\tau))^\dagger \]
\[ \rho_S(0) \rightarrow \rho_S^c(n\tau) = K^n(\tau)\rho_S(0)(K^n(\tau))^\dagger \]

\[ p(n\tau) = \text{tr}[K(\tau)\rho_S^c((n-1)\tau)K^\dagger(\tau)] = \text{tr}[K^n(\tau)\rho_S(0)(K^n(\tau))^\dagger] = \text{tr}\rho_S^c(n\tau) \]
\[ \rho_S(0) \longrightarrow \rho_S^c(n\tau) = K^n(\tau)\rho_S(0)(K^n(\tau))^\dagger \]

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\[ K(\tau) = \langle 0_A | \exp(-iH\tau) | 0_A \rangle \]
\[ K(\tau) = \langle 0_A | \left( I_{S+A} - i\tau H - \frac{\tau^2}{2} H^2 + o_{S+A}(\tau^2) \right) | 0_A \rangle \]

\[ = I_S - i\tau \langle 0_A | H | 0_A \rangle - \frac{\tau^2}{2} \langle 0_A | H^2 | 0_A \rangle + o_S(\tau^2) \]

\[ = \exp \left( -i\tau H_0^S - \frac{\tau^2}{2} \Gamma_S + o_S(\tau^2) \right), \]

\[ \lim_{\tau \to 0} \frac{\| o(\tau^2) \|}{\tau^2} = 0 \]

\[ H_0^S = (H_0^S)\dagger = \langle 0_A | H | 0_A \rangle \]

\[ \Gamma_S = \Gamma_S\dagger = \langle 0_A | H^2 | 0_A \rangle - (H_0^S)^2 = \langle 0_A | H | 1_A \rangle \langle 1_A | H | 0_A \rangle \geq 0 \]
\[ t = n \tau \]
\[ t = n\tau \]

\[
K^n(\tau) = \exp \left( -in\tau H^S_0 - \frac{n\tau^2}{2} \Gamma_S + o(\tau^2) \right) \\
= \exp \left[ -it \left( H^S_0 - \frac{i\tau}{2} \Gamma_S \right) + o(\tau^2) \right]
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\[
H_{\text{eff}} = H_0^S - \frac{i\tau}{2} \Gamma_S
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\[
H_{\text{eff}} = H_0^S - \frac{i\tau}{2} \Gamma_S
\]

\[
\frac{d\rho_c^S(t)}{dt} = -i \left( H_{\text{eff}} \rho_c^S(t) - \rho_c^S(t) H_{\text{eff}}^\dagger \right)
\]
\[ t = n\tau \]

\[
K^n(\tau) = \exp \left( -i n\tau H^S_0 - \frac{n\tau^2}{2} \Gamma_S + o(\tau^2) \right)
\]
\[
= \exp \left[ -i t \left( H^S_0 - \frac{i\tau}{2} \Gamma_S \right) + o(\tau^2) \right]
\]

\[ H_{\text{eff}} = H^S_0 - \frac{i\tau}{2} \Gamma_S \]

\[
\frac{d\rho^c_S(t)}{dt} = -i \left( H_{\text{eff}} \rho^c_S(t) - \rho^c_S(t) H_{\text{eff}}^\dagger \right)
\]
\[
\frac{dp(t)}{dt} = \frac{d \text{tr} [\rho^c_S(t)]}{dt} = -\tau \text{tr} [\Gamma_S \rho^c_S(t)] \leq 0
\]
$$\varrho^c_S(t) = \frac{\rho^c_S(t)}{\text{tr}[\rho^c_S(t)]} = \frac{K^n(t)\rho_S(0)(K^n)\dagger(t)}{\text{tr}[K^n(t)\rho_S(0)(K^n)\dagger(t)]} = \frac{e^{-iH_{\text{eff}}t}\rho_S(0)e^{+iH_{\text{eff}}\dagger t}}{\text{tr}[e^{-iH_{\text{eff}}t}\rho_S(0)e^{+iH_{\text{eff}}\dagger t}]}$$
\[
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\]

\[
\frac{d\rho^c_S(t)}{dt} = -i[H_0^S, \rho^c_S(t)] - \frac{\tau}{2} \{\Gamma_S, \rho^c_S(t)\} + \tau \text{tr}[\Gamma_S \rho^c_S(t)] \rho^c_S(t)
\]
\[ D^{c}_S(t) = \frac{\rho_{S}^{c}(t)}{\text{tr}[\rho_{S}^{c}(t)]} = \frac{K^{n}(t)\rho_{S}(0)(K^{n})^\dagger(t)}{\text{tr}[K^{n}(t)\rho_{S}(0)(K^{n})^\dagger(t)]} = \frac{e^{-iH_{\text{eff}}t}\rho_{S}(0)e^{+iH_{\text{eff}}^\dagger t}}{\text{tr}[e^{-iH_{\text{eff}}t}\rho_{S}(0)e^{+iH_{\text{eff}}^\dagger t}]} \]

\[ \frac{d\rho_{S}^{c}(t)}{dt} = -i[H_{S}^{0}, \rho_{S}^{c}(t)] - \frac{\tau}{2} \{ \Gamma_{S}, \rho_{S}^{c}(t) \} + \tau \text{tr}[\Gamma_{S} \rho_{S}^{c}(t)] \rho_{S}^{c}(t) \]

\[ i \frac{d|\psi_{S}(t)\rangle}{dt} = \left( H_{S}^{0} - \frac{i\tau}{2} \Gamma_{S} \right) |\psi_{S}(t)\rangle + \frac{i\tau}{2} \langle \psi_{S}(t)|\Gamma_{S}|\psi_{S}(t)\rangle |\psi_{S}(t)\rangle \]
Non-Hermitian Hamiltonian Engineering

Arbitrary non-Hermitian Hamiltonian $H_{\text{eff}}$

Denote by $f$ the maximum Bohr frequency of $H_2$. Fix $\tau$ in such a way that $f\tau \ll 1$, e.g., $\tau = 10^{-2} f^{-1}$.

$c = \max(0, -M)$, where $M$ is the minimum eigenvalue of the operator $i \tau (H_{\text{eff}} - H_{\text{eff}}^\dagger)$.

$cI + i \tau (H_{\text{eff}} - H_{\text{eff}}^\dagger) \geq 0$ and corresponds to $\Gamma_S H_{\text{eff}} = \frac{1}{2} (H_{\text{eff}} + H_{\text{eff}}^\dagger) \otimes |0_A \rangle \langle 0_A| + \sqrt{cI + i \tau (H_{\text{eff}} - H_{\text{eff}}^\dagger)} \otimes (|0_A \rangle \langle 1_A| + |1_A \rangle \langle 0_A|)$.
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\[ cI + \frac{i}{\tau}(H_{\text{eff}} - H_{\text{eff}}^\dagger) \geq 0 \] and corresponds to $\Gamma_S$

\[ H = \frac{1}{2}(H_{\text{eff}} + H_{\text{eff}}^\dagger) \otimes |0_A\rangle\langle 0_A| + \sqrt{cI + \frac{i}{\tau}(H_{\text{eff}} - H_{\text{eff}}^\dagger)} \otimes (|0_A\rangle\langle 1_A| + |1_A\rangle\langle 0_A|) \]
\[ H = \gamma_{xy} (\sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y) + \gamma_z \sigma_1^z \sigma_2^z + g_{xy} (\sigma_1^x \sigma_3^x + \sigma_1^y \sigma_3^y) \\
+ g_z \sigma_1^z \sigma_3^z + g_{xy} (\sigma_2^x \sigma_3^x + \sigma_2^y \sigma_3^y) + g_z \sigma_2^z \sigma_3^z \]
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\]

\[
H_{\text{eff}} = \gamma_{xy} (\sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y) + \gamma_z \sigma_1^z \sigma_2^z + g_z (\sigma_1^z + \sigma_2^z) \\
- i\tau g_{xy}^2 (2I_{12} + \sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y - \sigma_1^z - \sigma_2^z)
\]
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\[ \mathcal{U}(t) = \begin{pmatrix}
    e^{-i(\gamma_z + 2g_z)t/\hbar} & e^{2\tau g_{xy}^2 t/\hbar} & 0 & 0 \\
    0 & \cos \alpha & -i \sin \alpha & 0 \\
    0 & -i \sin \alpha & \cos \alpha & 0 \\
    0 & 0 & 0 & e^{-i(\gamma_z - 2g_z)t/\hbar} e^{-2\tau g_{xy}^2 t/\hbar}
\end{pmatrix} \]

with \( \alpha = 2(\gamma_{xy} - i\tau g_{xy}^2)t \equiv (\gamma - ig)t \)
\[ H'_{\text{eff}} = H_{\text{eff}} + 2i\tau g_{xy}^2 I_{12} \]

\[
U(t) = \begin{pmatrix}
  e^{-i(\gamma z + 2g_z)t/\hbar} & e^{2\tau g_{xy}^2 t/\hbar} & 0 & 0 \\
  0 & \cos \alpha & -i \sin \alpha & 0 \\
  0 & -i \sin \alpha & \cos \alpha & 0 \\
  0 & 0 & 0 & e^{-i(\gamma z - 2g_z)t/\hbar} e^{-2\tau g_{xy}^2 t/\hbar}
\end{pmatrix}
\]

with \( \alpha = 2(\gamma_{xy} - i\tau g_{xy}^2) t \equiv (\gamma - ig) t \)

\[ \rho^c_S(t) = \frac{\rho^c_S(t)}{\text{Tr}\{\rho^c_S(t)\}} = \frac{U(t)\rho_S(0)U^\dagger(t)}{\text{Tr}\{U(t)\rho_S(0)U^\dagger(t)\}} \]
\[ \rho_S(0) = |01\rangle\langle 01| \]
\[
\rho_S(0) = |01\rangle\langle 01|
\]

\[
\rho^c_S(t) = \frac{1}{|\cos(\alpha)|^2 + |\sin(\alpha)|^2} \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & |\cos(\alpha)|^2 & i \cos(\alpha) \sin(\alpha) & 0 \\
0 & -i \cos(\alpha)^* \sin(\alpha) & |\sin(\alpha)|^2 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

with \( \alpha = 2(\gamma_{xy} - i\tau g_{xy}^2)t \equiv (\gamma - ig)t \)
\[
\rho_{S}(0) = \ket{01}\bra{01}
\]

\[
\rho_{S}^{c}(t) = \frac{1}{|\cos(\alpha)|^2 + |\sin(\alpha)|^2} \begin{pmatrix}
0 & 0 & i \cos \alpha \sin \alpha^* & 0 \\
0 & -i \cos \alpha^* \sin \alpha & |\sin \alpha|^2 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

with \( \alpha = 2(\gamma_{xy} - i \tau g_{xy})t \equiv \gamma - ig \)

\[
\rho_{S}^{c}(t \rightarrow \infty) = \frac{1}{2} \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 \\
0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} = \ket{\Psi^-}\bra{\Psi^-}, \quad \ket{\Psi^-} = \frac{\ket{01} - \ket{10}}{\sqrt{2}}
\]
Summary

- Derivation of a quantum non-Hermitian Hamiltonian in a physically motivated scenario

- Trace decreasing semigroup for subnormalized density operators

- Inverse problem (non-Hermitian Hamiltonian engineering) is resolved

- If $S$ is a composed system, then some interesting effects like entanglement generation (entanglement stabilization) are predicted

- Hermitian Hamiltonian dynamics for $S + A = \Rightarrow$ map for $S$ has Kraus rank = 1, so the conditional output states remain pure if they were pure in the beginning

- Open problem: What dynamics would be induced if the interaction between $S$ and $A$ is not unitary?
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