

Trace decreasing semigroup for an open quantum system interacting with a repeatedly measured ancilla

Sergey Filippov

¹Steklov Mathematical Institute of Russian Academy of Sciences

²Moscow Institute of Physics and Technology (National Research University)

³Valiev Institute of Physics and Technology of Russian Academy of Sciences

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R. Grimaudo, A. Messina, A. Sergi, N.V. Vitanov, S.N. Filippov.
Two-qubit entanglement generation through non-Hermitian
Hamiltonians induced by repeated measurements on an ancilla.
Entropy 22, 1184 (2020)

E-print arXiv:2009.10004 [quant-ph]

and

I. A. Luchnikov, S. N. Filippov. Quantum evolution in the
stroboscopic limit of repeated measurements. Phys. Rev. A 95,
022113 (2017)

E-print arXiv:1609.05501 [quant-ph]

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Non-Hermitian Hamiltonian

$$\varrho(t) = \frac{\rho(t)}{\text{Tr}[\rho(t)]} = \frac{e^{-iH_{\text{eff}}t}\rho(0)e^{iH_{\text{eff}}^\dagger t}}{\text{Tr}\left[e^{-iH_{\text{eff}}t}\rho(0)e^{iH_{\text{eff}}^\dagger t}\right]}$$

Non-Hermitian Hamiltonian

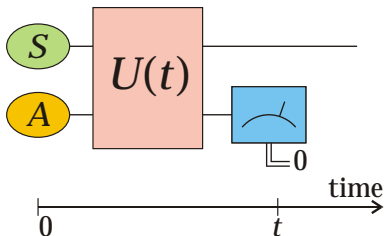
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$$\frac{d}{dt} \varrho(t) = -\frac{i}{\hbar} [H_0, \varrho(t)] - \frac{1}{\hbar} \{\Gamma, \varrho(t)\} + \frac{2}{\hbar} \text{Tr}[\Gamma \varrho(t)] \varrho(t)$$

$$\rho_S(0) \longrightarrow \rho_S^c(t) = \langle 0_A | U(t) [\rho_S(0) \otimes |0_A\rangle\langle 0_A|] U^\dagger(t) |0_A\rangle$$

$$= K(t)\rho_S(0)K^\dagger(t),$$

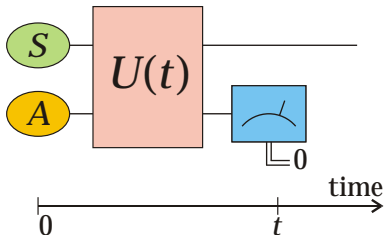
$$K(t) = \langle 0_A | U(t) |0_A\rangle$$



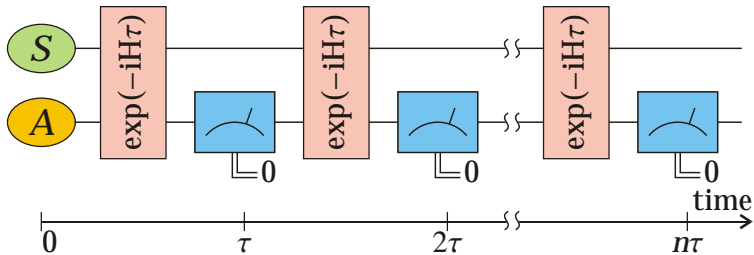
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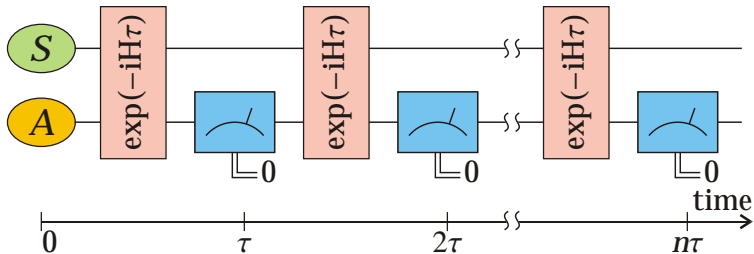
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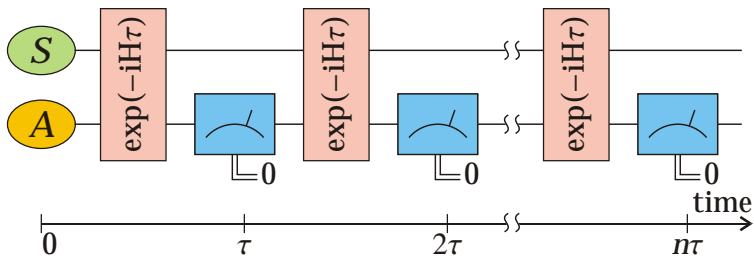


$$p_{\rho_S(0)}(t) = \text{tr}\rho_S^c(t), \quad \varrho_S^c(t) = \rho_S^c(t)/\text{tr}\rho_S^c(t) = \rho_S^c(t)/p_0(t)$$



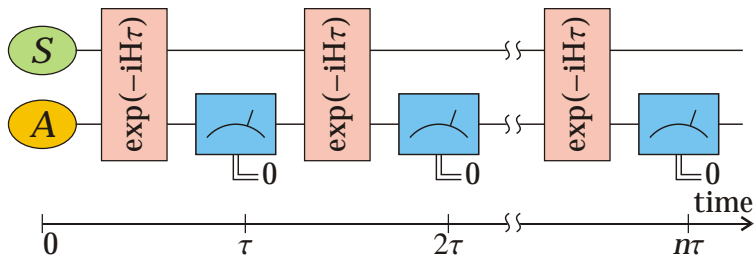


$$\rho_S(0) \longrightarrow \rho_S^c(n\tau) = K^n(\tau)\rho_S(0)(K^n(\tau))^\dagger$$



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$$K(\tau) = \langle 0_A | \exp(-iH\tau) | 0_A \rangle$$

$$\begin{aligned}
K(\tau) &= \langle 0_A | \left(I_{S+A} - i\tau H - \frac{\tau^2}{2} H^2 + o_{S+A}(\tau^2) \right) | 0_A \rangle \\
&= I_S - i\tau \langle 0_A | H | 0_A \rangle - \frac{\tau^2}{2} \langle 0_A | H^2 | 0_A \rangle + o_S(\tau^2) \\
&= \exp \left(-i\tau H_0^S - \frac{\tau^2}{2} \Gamma_S + o_S(\tau^2) \right),
\end{aligned}$$

$$\lim_{\tau \rightarrow 0} \|o(\tau^2)\|/\tau^2 = 0$$

$$H_0^S = (H_0^S)^\dagger = \langle 0_A | H | 0_A \rangle$$

$$\Gamma_S = \Gamma_S^\dagger = \langle 0_A | H^2 | 0_A \rangle - (H_0^S)^2 = \langle 0_A | H | 1_A \rangle \langle 1_A | H | 0_A \rangle \geq 0$$

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$$\frac{dp(t)}{dt} = \frac{d\text{tr}[\rho_S^c(t)]}{dt} = -\tau\text{tr}[\Gamma_S\rho_S^c(t)] \leq 0$$

$$\varrho_S^c(t) = \frac{\rho_S^c(t)}{\text{tr}[\rho_S^c(t)]} = \frac{K^n(t)\rho_S(0)(K^n)^\dagger(t)}{\text{tr}[K^n(t)\rho_S(0)(K^n)^\dagger(t)]} = \frac{e^{-iH_{\text{eff}}t}\rho_S(0)e^{+iH_{\text{eff}}^\dagger t}}{\text{tr}[e^{-iH_{\text{eff}}t}\rho_S(0)e^{+iH_{\text{eff}}^\dagger t}]}$$

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$$\frac{d\varrho_S^c(t)}{dt} = -i[H_0^S, \varrho_S^c(t)] - \frac{\tau}{2}\{\Gamma_S, \varrho_S^c(t)\} + \tau\text{tr}[\Gamma_S\varrho_S^c(t)]\varrho_S^c(t)$$

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$$i\frac{d|\psi_S(t)\rangle}{dt} = \left(H_0^S - \frac{i\tau}{2}\Gamma_S\right)|\psi_S(t)\rangle + \frac{i\tau}{2}\langle\psi_S(t)|\Gamma_S|\psi_S(t)\rangle|\psi_S(t)\rangle$$

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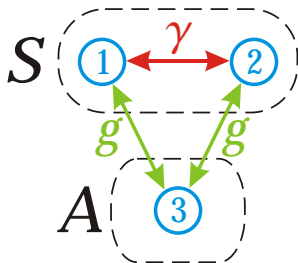
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$$H = \frac{1}{2}(H_{\text{eff}} + H_{\text{eff}}^\dagger) \otimes |0_A\rangle\langle 0_A| + \sqrt{cI + \frac{i}{\tau}(H_{\text{eff}} - H_{\text{eff}}^\dagger)} \otimes (|0_A\rangle\langle 1_A| + |1_A\rangle\langle 0_A|)$$

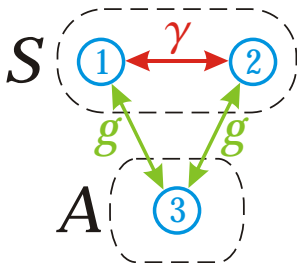
Example

$$H = \gamma_{xy}(\sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y) + \gamma_z \sigma_1^z \sigma_2^z + g_{xy}(\sigma_1^x \sigma_3^x + \sigma_1^y \sigma_3^y) + g_z \sigma_1^z \sigma_3^z + g_{xy}(\sigma_2^x \sigma_3^x + \sigma_2^y \sigma_3^y) + g_z \sigma_2^z \sigma_3^z$$



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$$H_{\text{eff}} = \gamma_{xy}(\sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y) + \gamma_z \sigma_1^z \sigma_2^z + g_z(\sigma_1^z + \sigma_2^z) - i\tau g_{xy}^2 (2I_{12} + \sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y - \sigma_1^z - \sigma_2^z)$$

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$$\mathcal{U}(t) = \begin{pmatrix} e^{-i(\gamma_z+2g_z)t/\hbar} e^{2\tau g_{xy}^2 t/\hbar} & 0 & 0 & 0 \\ 0 & \cos \alpha & -i \sin \alpha & 0 \\ 0 & -i \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & e^{-i(\gamma_z-2g_z)t/\hbar} e^{-2\tau g_{xy}^2 t/\hbar} \end{pmatrix}$$

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$$\rho_S^c(t) = \frac{1}{|\cos(\alpha)|^2 + |\sin(\alpha)|^2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |\cos \alpha|^2 & i \cos \alpha \sin \alpha^* & 0 \\ 0 & -i \cos \alpha^* \sin \alpha & |\sin \alpha|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\varrho_S^c(t \rightarrow \infty) = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = |\Psi^-\rangle\langle\Psi^-|, \quad |\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

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- ▶ If S is a composed system, then some interesting effects like entanglement generation (entanglement stabilization) are predicted
- ▶ Hermitian Hamiltonian dynamics for $S + A \implies$ map for S has Kraus rank = 1, so the conditional output states remain pure if they were pure in the beginning
- ▶ Open problem: What dynamics would be induced if the interaction between S and A is not unitary?