



Substochastic semigroups for initial-boundary value problems

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We study well-posedness of linear evolution equations on L^1 of the form

$$u'(t) = (A + B)u(t), \quad \Psi_0 u(t) = \Psi u(t), \quad t > 0, \quad u(0) = f,$$

where B, Ψ_0, Ψ are positive (unbounded) linear operators. Such initial-boundary value problems describe in particular the evolution of densities for piecewise deterministic Markov processes [1]. We assume that A_0 defined as the restriction of the operator A to $\ker(\Psi_0)$ generates a substochastic semigroup of operators on L^1 , i.e. a C_0 -semigroup of positive contractions on L^1 . We provide sufficient conditions for the operator A to be the generator of a positive semigroup [2] and for $A+B$ to be the generator of a substochastic semigroup [1,3]. This extends the approach of Greiner [4] by considering unbounded Ψ and positive semigroups on L^1 . We illustrate our results with examples of age and size structured equations arising from a gene expression model and a cell-cycle model.

References

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