Strong stochastic flows on metric graphs

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Introduction. Let M be a locally compact separable metric space. Assume that for each $n \geq 1$ a Feller transition function $(P_t^{(n)} : t \geq 0)$ on M^n is defined, and the sequence $(P^{(n)} : n \geq 1)$ is consistent, i.e. for all $n \geq 1$ and $\{i_1, \ldots, i_k\} \subset \{1, \ldots, n\}$ the following relation holds:

$$P_t^{(n)}(x, \pi_{i_1,\ldots,i_k}^{-1}(B)) = P_t^{(k)}(\pi_{i_1,\ldots,i_k}(x), B),$$

where $t \geq 0$, $x \in M^n$, $B \in \mathcal{B}(M^k)$ (a Borel subset of M^k), and $\pi_{i_1,\ldots,i_k} : M^n \to M^k$, $\pi_{i_1,\ldots,i_k}(x) = (x_{i_1}, \ldots, x_{i_k})$ is a projection. It is known (see [1]) that there exists a weak stochastic flow of kernels $(K_{s,t}(x, B) : s \leq t, x \in M, B \in \mathcal{B}(M))$, such that for all $n \geq 1$, $x \in M^n$ and $B_1, \ldots, B_n \in \mathcal{B}(M)$

$$P_t^{(n)}(x, B_1 \times B_n) = \mathbb{E} \prod_{i=1}^{n} K_{0,t}(x_i, B_i).$$

For $t_1 \leq \ldots \leq t_k$ stochastic kernels $K_{t_1,t_2}, \ldots, K_{t_{n-1},t_n}$ are independent, and the distribution of $K_{s,t}$ depends only on $t - s$. Stochastic flow $(K_{s,t} : s \leq t)$ is weak in the sense that for all $s \leq t \leq u$ and $x \in M$ the evolution property $K_{s,u}(x, \cdot) = \int_M K_{t,u}(y, \cdot) K_{s,t}(x, dy)$ holds almost surely. We address the question of existence of a measurable modification of the flow $(K_{s,t} : s \leq t)$ which is simultaneously a strong flow, i.e. the exceptional set where the evolution property fails does not depend on s, t, u, x. The positive answer to this question is known in the case when $(P_t^{(n)} : t \geq 0)$ is defined by solutions of an SDE with smooth enough coefficients [2], for a class of coalescing flows on the real line [3], for Howitt-Warren flows [4]. In this talk a new approach will be developed that allows to prove existence of strong stochastic flows for some consistent sequences of Feller transition functions on metric graphs. In particular, existence of a strong flow of coalescing independent Walsh Brownian motions on arbitrary metric graph will be proved.

References

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