On the convergence of a novel time slicing approximation for Feynman path integrals

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We report on some recent results [4] concerning the properties of a sequence of approximate propagators for the Schrödinger equation, in the spirit of Feynman's path integrals.

Precisely, we consider Hamiltonian operators arising as the Weyl quantization of a quadratic form in phase space, plus a bounded potential perturbation in the form of a pseudodifferential operator with a rough symbol. It is known that the corresponding Schrödinger propagator is a generalized metaplectic operator [1,2]. Inspired by the standard perturbation method, this characterization naturally motivates the introduction of a manageable time slicing approximation consisting of operators of the same type.

By means of techniques and function spaces of time-frequency analysis it is possible to obtain several convergence results with precise rates in terms of the mesh size of the time slicing subdivision. In particular, we prove convergence in the norm operator topology in L^2, as well as pointwise convergence of the corresponding integral kernels for non-exceptional times. In this respect, it should be stressed that the standard Feynman-Trotter parametrices happen to be generalized metaplectic operators too, as shown in [3], but they are worse than the novel ones introduced here as far as the short-time approximation power is concerned - provided that the same assumptions on the Hamiltonian recalled above are considered.

We emphasize that the advantages of a time slicing approximation that is designed to best fit the features of the problem reflect into stronger convergence results. In particular, we are able to control the rate of pointwise convergence of integral kernels. This kind of result seems quite remarkable when compared to the analogous ones obtained for Feynman-Trotter. Indeed, the product formula just provides a qualitative strong convergence result that can be hardly improved or further detailed in the unitary setting.

References

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