Markovian embedding semigroup for non-Markovian quantum collision models with correlated environment

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Problem setting. Quantum collision model describes an open quantum system interacting sequentially with individual subenvironments. The system dynamics is known to be described by a Markovian semigroup if all the subenvironments are initially identical and there are no correlations among them [1]. In the case of the correlated (or entangled) environment, the system dynamics may become strictly non-Markovian (positive indivisible), which was originally observed in [2] and later developed in [3], with some particular exactly solvable scenarios being considered. However, a general solution to the problem has remained unknown. Here we present a general solution to the problem, which is based on representing the environment in terms of the tensor network called matrix product state [4]. We find a natural Markovian embedding for the system and bond degrees of freedom within the tensor network.

Main result. Suppose the environment is in the matrix product pure state of n correlated d-dimensional particles given by formula

$$\begin{aligned} \psi_E &= \sum_{i_1,\dots,i_n=1}^{d} \sum_{a_1,\dots,a_{n-1}} B^{[1],i_1}_{a_1} B^{[2],i_2}_{a_1,a_2} B^{[3],i_3}_{a_2,a_3} \cdots B^{[n],i_n}_{a_{n-1}} i_1 i_2 i_3 \dots i_n \\ &= \sum_{i_1,\dots,i_n=1}^{d} B^{[1],i_1} B^{[2],i_2} B^{[3],i_3} \cdots B^{[n],i_n} i_1 i_2 i_3 \dots i_n, \end{aligned}$$

which has the right-cannonical form implying $\sum_{i_k=1}^{d} B^{[k],i_k}(B^{[k],i_k})^\dagger = I_{k-1}$, where I_{k-1} is the $|\{a_{k-1}\}| \times |\{a_{k-1}\}|$ identity matrix. Suppose the k-th system-subenvironment collision lasts time τ and is described by a unitary operator $U_{Sk} : \mathcal{H}_S \otimes \mathcal{H}_k \to \mathcal{H}_S \otimes \mathcal{H}_k$. We formally introduce an auxiliary Hilbert space $\mathcal{H}_{\mathrm{bond}\#k}$ spanning orthonormal vectors $\{a_k\}$.

Theorem. The system density operator dynamics $\varrho_S(k\tau) = \mathrm{tr}_{\mathrm{bond}\#k}[R(k\tau)]$ is governed by the Markovian embedding $R(k\tau) = \mathcal{E}^{[k]}\left[R\big((k-1)\tau\big)\right]$, $k = 1, 2, \dots$, $R(0) = \varrho_S(0)$, where each map $\mathcal{E}^{[k]}[\bullet] = \sum_{j_k} A_{j_k} \bullet (A_{j_k})^\dagger$ is a quantum channel (completely positive and trace decreasing map) with Kraus operators $A_{j_k} = \sum_{i_k} j_k U_{Sk} i_k \otimes (B^{[k],i_k})^\top$.

Discussion. The developed formalism is straightforward to generalize to the case of a mixed environment state. If matrices U_{Sk} and tensors $B^{[k]}$ do not depend on k, then the Markovian embedding is a discrete semigroup.

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