A Katznelson-Tzafiri theorem for analytic Besov functions

Charles Batty [1]

The original Katznelson-Tzafriri theorem (1986), in its more general form, is as follows.

Let T be a power-bounded operator on a complex Banach space. Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ on the closed unit disc, where $\sum_{n=0}^{\infty} |a_n| < \infty$, and let $f(T) = \sum_{n=0}^{\infty} a_n T^n$. Assume that the function f is of spectral synthesis with respect to the set $\sigma_u(T) := \sigma(T) \cap \mathbb{T}$. Then $\lim_{n\to\infty} \|T^n f(T)\| = 0$.

The assumption of spectral synthesis is slightly stronger than assuming that f vanishes on $\sigma_u(T)$. If $\sigma_u(T)$ is countable, then it suffices to assume that f vanishes on $\sigma_u(T)$. Subsequently several variants of the theorem have been proved, where the assumption of spectral synthesis is replaced by the weaker assumption.

The functions f considered above form a Banach algebra W^+, and the map $f \mapsto f(T)$ is an algebra homomorphism, so it is a W^+-calculus for the operator T. Some power-bounded operators have a functional calculus with respect to a larger Baanch algebra $\mathcal{B}$ of "analytic Besov functions" on the unit disc—for example, power-bounded operators on Hilbert spaces and Ritt operators on arbitrary Banach spaces. I will describe a theorem of Katznelson-Tzafriri type, where T is assumed to have a $\mathcal{B}$-calculus, $f \in \mathcal{B}$, and f vanishes on $\sigma_u(T)$.

(Joint work with David Seifert)

[1]University of Oxford. Email: charles.batty@sjc.ox.ac.uk