Soliton and breather dynamics in the modular KdV equation

E. N. Pelinovsky[1], A. V. Kokorina[2], A. V. Slunyaev[3], T. V. Tarasova[4]

Keywords: modular KdV equation; solitons; breathers; cnoidal waves; extreme events; rogue waves.

MSC2010 codes: 00A79, 35C08, 35Q53, 35Q55

In this work we study the dynamics of nonlinear waves governed by the so-called modular Korteweg–de Vries (KdV) equation in the form

$$\frac{\partial u}{\partial t} + 6|u|\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0, \tag{1}$$

where the real-valued function $u(x,t) \in \Re$ is defined in infinite line. Equation (1) was discussed in [1]; it differs from the classical KdV equation only in the absolute value operation.

The equation has an interesting feature that its solutions of strictly one sign (i.e., $u \geq 0$, $\forall(x,t)$) coincide with the ones of the classical KdV equation. In particular, this is true for single- and multi-soliton solutions, which may be either positive or negative. Hence, a solitonic wave regime may be called integrable by the Inverse Scattering Technique (IST). However, if for any $u(x, t = t_0)$ solution of the associated scattering problem for the KdV equation contains non-empty continuous spectrum, then the integrability property breaks. Moreover, it is obvious that due to the modulus in (1), sign-changing solutions may contain singular derivatives of high orders.

The described difficulties motivate the study using numerical approaches, on the one hand. On the other hand, the code for numerical simulations should be characterized by high stability. In this work for the direct simulation of the wave evolution within the equation (1) we use the pseudo-spectral method with implicit integration over time and the Crank-Nicholson scheme, see the description and examples in [2]. A “standard set” of model problems for pulse-like perturbations has been considered: interactions between solitons and the Cauchy problem.

As discussed above, collisions between solitons of the same sign are described by the KdV analytic solutions, hence they are fully elastic. Collisions between solitons of different polarities result in radiation (see Fig. 1), which becomes more intense when the solitons’ amplitudes are close, however remains small even after a great number of consecutive collisions due to the periodic boundary conditions in the spatial (x) domain. Interactions of solitons of different polarities are characterized by non-classical phase shifts; similar situations were found in our previous work on compacton dynamics within sub-linear KdV equation [3]. Hence, despite the uneven modular term in (1), soliton interactions are generally weakly inelastic. At the same time, the regimes when solitons of opposite signs get bounded and form breathers were found. This situation is typical for weakly non-integrable equations which possess solitons of two signs (such as perturbed modified KdV equation); the latter tend to couple in such systems. However, non-classical effects of this dynamics are found within the modular KdV framework: modular KdV breathers can de-couple in some cases.

[1]HSE University, Laboratory of Dynamical Systems and Applications, Russia, N. Novgorod. Institute of Applied Physics RAS, Group for modeling of extreme wave phenomena in the ocean, Russia, N. Novgorod

[2]Institute of Applied Physics RAS, Group for modeling of extreme wave phenomena in the ocean, Russia, N. Novgorod

[3]HSE University, Laboratory of Dynamical Systems and Applications, Russia, N. Novgorod. Institute of Applied Physics RAS, Group for modeling of extreme wave phenomena in the ocean, Russia, N. Novgorod. Email: slunyaev@appl.sci-nnov.ru

[4]HSE University, Faculty of Informatics, Mathematics, and Computer Science, Russia, N. Novgorod. Institute of Applied Physics RAS, Group for modeling of extreme wave phenomena in the ocean, Russia, N. Novgorod.