The Decoherence-free Subalgebra of a Gaussian Quantum Markov Semigroup

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Introduction. Gaussian Quantum Markov semigroups (QMSs) have been used in the literature also under the name of quasi-free semigroups. They are semigroups on the set of bounded operators on the symmetric Fock space $\mathcal{H} = \Gamma_s(\mathbb{C}^d)$. Among these operators we count the Weyl operators $W(z)$, for $z \in \mathbb{C}^d$, which are unitary operators and whose linear combinations are dense in the whole $\mathcal{B}(\mathcal{H})$. Of notable interest are also annihilation and creation operators $a_j, a_j^\dagger$ for $j = 1, \dots, d$, which are not bounded but are used in many applications and in the very definition of gaussian QMSs.

Gaussian QMSs have been introduced either by their generator in the GKLS form

$$\mathcal{L}(x) = i[H, x] - \frac{1}{2} \sum_{\ell \geq 1} \left(L_\ell^* L_\ell x - 2 L_\ell^* x L_\ell + x L_\ell^* L_\ell \right),$$

with H a quadratic polynomial in $a_j, a_j^\dagger$ and L_ℓ a linear polynomial in $a_j, a_j^\dagger$, or via their explicit action on Weyl operators

$$\mathcal{T}_t(W(z)) = c_t(z) W(e^{tZ} z),$$

for some function $c_t(z)$ and real linear operator Z. In fact these definitions are equivalent and both correspond to the qualitative requirement for the QMS to preserve the set of so-called gaussian states. One of the main difficulties of achieving said result is to cope with the unbounded nature of annihilation and creation operators. The very same problem presents itself again when looking at the Decoherence-free subalgebra $\mathcal{N}(\mathcal{T})$.

By definition it is introduced as the set

$$\mathcal{N}(\mathcal{T}) = \{ x \in \mathcal{B}(\mathcal{H}) | \mathcal{T}_t(x^* x) = \mathcal{T}_t(x^*) \mathcal{T}_t(x), \mathcal{T}_t(x x^*) = \mathcal{T}_t(x) \mathcal{T}_t(x^*) \}.$$

A known result characterizes it as the commutator of the set

$$\{ \delta_H^n(L_\ell), \delta_H^n(L_\ell^*) : \ell \geq 1, n \geq 0 \}$$

where $\delta_H(X) = [H, X]$. Unfortunately this result holds for uniformly continuous QMSs where the operators H, L_ℓ are both bounded. In this talk we present a generalisation of this result and apply it to the case of gaussian QMSs where we have explicit expressions for H and L_ℓ. Up to unitary equivalence we will show that

$$\mathcal{N}(\mathcal{T}) = L^\infty(\mathbb{R}^{d_c}; \mathbb{C}) \overline{\otimes} \mathcal{B}(\mathbb{C}^{d_f}),$$

for some $d_c, d_f \geq 0$ with $d_c + d_f \geq d$.

References

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