Tensor invariants of dynamical systems with dissipation

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Introduction. It is known (see [1]) that...

We prove the integrability of certain classes of dynamical systems on the tangent bundles of finite-dimensional manifolds. The force field considered possessed so-called variable dissipation; they are generalizations of fields studied earlier. This paper continues earlier works of the author devoted to systems on the tangent bundles of two-and three-dimensional manifolds [1, 2].

Configuration spaces of of many dynamical systems are finite-dimensional smooth manifolds; naturally, their phase spaces are tangent bundles of these manifolds. For example, the motion of a five-dimensional generalized spherical pendulum in a nonconservative force field is described by a dynamical system on the tangent bundle of the four-dimensional sphere whose metric is induced by an additional symmetry group. In this case, dynamical systems that describe the motion of such a pendulum possess variable dissipation, and a complete list of first integrals consists of transcendental functions that can be expressed as finite combinations of elementary functions [3, 4].

In this activity, we prove the integrability (not only in first integrals, but in more general tensor invariants) of certain classes of dynamical systems on tangent bundles of smooth finite-dimensional manifolds in the case of systems with variable dissipation, which are generalizations of systems studied earlier. Similar results for manifolds of dimensions 2 and 3 were obtained by the author earlier [5].

References

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