On equicontinuity and tightness of bi-continuous semigroups

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In many applications of semigroups of operators on Banach spaces the semigroups are not strongly continuous with respect to the norm of the Banach space but strongly continuous with respect to a weaker Hausdorff locally convex topology. Examples of such semigroups are adjoint semigroups of norm-strongly continuous semigroups, implemented semigroups, and transition and Koopman semigroups on the space of bounded continuous functions on a Polish space.

These examples belong to the general framework of bi-continuous semigroups [5,6]. In the context of perturbation theory of bi-continuous semigroups the notion of tightness emerged [1,2], which plays a similar role as equicontinuity in perturbation theory of strongly continuous semigroups on locally convex spaces [3]. We study the relation between tightness and equicontinuity with respect to the mixed topology and present sufficient conditions that guarantee their equivalence. This talk is based on [4].

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