Asymptotic properties of dynamics of antibody levels

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Introduction.

The immune status is the concentration of specific antibodies, which appear after infection with a pathogen and remain in serum, providing protection against future attacks of that same pathogen. Over time the number of antibodies decreases until the next infection. During fighting the invader the immunity is boosted and then the immunity is gradually waning, etc. Our aim is to describe and study the evolution of probability density of the distribution of the process using the semigroup approach [4]. The model is based on three factors: occasional boosting and continuous waning of immunity and a general description of the period between subsequent boosting events [5]. The density of antibody concentration distribution satisfies some partial differential equation with an integral boundary condition. We check that this system generates a stochastic semigroup. We give a new theoretical result on asymptotic stability of stochastic semigroups. This result is based on a decomposition theorem of a stochastic semigroup into asymptotically stable and sweeping components [2], [3]. We apply the general result concerning asymptotic stability of stochastic semigroups [1], [2], [3] to the model. We analyze versions of the model with specially chosen functions describing the model, e.g. when immunity decreases exponentially; with constant increase of antibodies after infection; with a threshold concentration of antibodies at the re-infection; and with seasonal infections.

References

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